Online Supplements to "Value and Design of Traceability-Driven Blockchains"

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SA. Value of Blockchain in a Serial Supply Chain: Proofs

Proof of Proposition 1. We use backward induction to solve the game. Recall that the game consists of three stages. First, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in previous stages. Specifically, for supplier 1, the first-order condition of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ is $\frac{d\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1}\Big|_{q_1=\tilde{q}_1(w_1,w_2,q_2)} = (w_1 - w_2)q_2 - C'(\tilde{q}_1(w_1,w_2,q_2)) = (w_1-w_2)q_2 - \theta\gamma(\tilde{q}_1(w_1,w_2,q_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_1}(w_2,q_1|w_1,q_2)$ w.r.t. q_1 yields $\frac{d^2\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1^2} = -C''(q_1) = -\theta\gamma(\gamma-1)q_1^{\gamma-2} < 0$. Thereby, supplier 1's optimal quality in response to w_1 and w_2 is $\tilde{q}_1(w_1,w_2,q_2) = \left[\frac{(w_1-w_2)q_2}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. On the other hand, for supplier 2, the first-order condition of $\pi_{S_2}(q_2|w_2,q_1)$ is $\frac{d\pi_{S_2}(q_2|w_2,q_1)}{dq_2}\Big|_{q_2=\tilde{q}_2(w_2,q_1)} = w_2q_1 - C'(\tilde{q}_2(w_2,q_1)) = w_2q_1 - \theta\gamma(\tilde{q}_2(w_2,q_1))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_2}(q_2|w_2,q_1)$ w.r.t. q_2 yields $\frac{d^2\pi_{S_2}(q_2|w_2,q_1)}{dq_2^2} = -C''(q_2) = -\theta\gamma(\gamma-1)q_2^{\gamma-2} < 0$. Thereby, supplier 2's optimal quality in response to w_2 is $\tilde{q}_2(w_2,q_1) = \left(\frac{w_2q_1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1,w_2) = \left[\frac{(w_1-w_2)^{\gamma-1}w_2}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SA.1}$$

$$\tilde{q}_2(w_1,w_2) = \left[\frac{(w_1-w_2)w_2^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SA.2}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2,\tilde{q}_1(w_1,w_2)|w_1,\tilde{q}_2(w_1,w_2))$, given w_1 decided in stage 1. Plugging (SA.1) and (SA.2) into $\pi_{S_2}(q_2|w_2,q_1)$, we have $\pi_{S_2}(\tilde{q}_2(w_1,w_2)|w_2,\tilde{q}_1(w_1,w_2)) = w_2\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - \theta(\tilde{q}_2(w_1,w_2))^\gamma = (\gamma-1)(w_1-w_2)^{\frac{1}{\gamma-2}}w_2^{\frac{\gamma-1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} \geqslant 0$ for any $w_1 \geqslant w_2 \geqslant 0$. Thus, IR_2 is always satisfied. Then, plugging (SA.1) and (SA.2) into (1), we have supplier 1's problem as follows:

$$\max_{w_2}\ \pi_{S_1}(w_2|w_1) = w_1\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - \theta(\tilde{q}_1(w_1,w_2))^\gamma - w_2\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2)$$
$$= (\gamma-1)(w_1-w_2)^{\frac{\gamma-1}{\gamma-2}}w_2^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}}.$$

We now analyze supplier 1's optimal contracting decision. Taking the first-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\frac{d\pi_{S_1}(w_2|w_1)}{dw_2} = \frac{\gamma-1}{\gamma-2}(w_1-w_2)^{\frac{1}{\gamma-2}} w_2^{\frac{3-\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} (w_1-\gamma w_2). \tag{SA.3}$$

Solving (SA.3) yields the solution of supplier 1's first-order condition as follows:

$$\tilde{w}_2(w_1) = \frac{w_1}{\gamma}. \tag{SA.4}$$

Then, we need to show that $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. In particular, we will prove that the sufficient condition of the local maximum is able to guarantee the unique global maximum, the underlying idea of which was used by Petruzzi and Dada (1999) and Aydin and Porteus (2008). Taking the second-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\begin{aligned}\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2} = {} & \frac{\gamma-1}{\gamma-2}(w_1-w_2)^{\frac{3-\gamma}{\gamma-2}} w_2^{\frac{5-2\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} \\ & \cdot \left[-\gamma w_2(w_1-w_2) + \frac{3-\gamma}{\gamma-2}(w_1-\gamma w_2)(w_1-w_2) - \frac{1}{\gamma-2}(w_1-\gamma w_2)w_2 \right].\end{aligned}$$

By Assumption 2 and $\tilde{w}_2(w_1) < w_1$, we can show that $\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2}\Big|_{\tilde{w}_2(w_1)} < 0$. Hence, $\tilde{w}_2(w_1)$ is a strict local maximum. Suppose now that there exist more than one, say two, interior stationary points for the function $\pi_{S_1}(w_2|w_1)$. Because both points need to be local maxima, the function should also have an interior local minimum somewhere in between, which is a contradiction to the result that all interior stationary points are local maxima. Consequently, we can conclude that there exists only one stationary point $\tilde{w}_2(w_1)$ that satisfies (SA.4), which is the unique local maximum, and also the unique global maximum. Thus, $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. Then, plugging (SA.4) into (SA.1) and (SA.2), we have

$$\tilde{q}_1(w_1) = \left[\frac{(\gamma-1)^{\gamma-1} w_1^{\gamma}}{\theta^{\gamma}\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SA.5}$$

$$\tilde{q}_2(w_1) = \left[\frac{(\gamma-1) w_1^{\gamma}}{\theta^{\gamma}\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SA.6}$$

Comparing (SA.5) and (SA.6) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, to maximize $\pi_B(w_1|\tilde{q}_1(w_1,\tilde{w}_2(w_1)),\tilde{q}_2(w_1,\tilde{w}_2(w_1)))$. Plugging (SA.4) into $\pi_{S_1}(w_2|w_1)$, we have $\pi_{S_1}(\tilde{w}_2(w_1)|w_1) = (\gamma-1)\big[w_1-\tilde{w}_2(w_1)\big]^{\frac{\gamma-1}{\gamma-2}}\big[\tilde{w}_2(w_1)\big]^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} = (\gamma-1)^{\frac{2\gamma-3}{\gamma-2}} w_1^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} \geqslant 0$ for any $w_1 \geqslant 0$. Thus, IR_1 is always satisfied. Then, plugging (SA.4), (SA.5) and (SA.6) into (1), we have the buyer's problem as follows:

$$\max_{w_1} \ \pi_B(w_1) = (p+l)\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1)$$
$$= (p+l-w_1)(\gamma-1)^{\frac{1}{\gamma-2}} w_1^{\frac{2}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} - l.$$

We now analyze the buyer's optimal contracting decision. Taking the first-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d\pi_B(w_1)}{dw_1} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}} w_1^{\frac{4-\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} \Big[2(p+l) - \gamma w_1\Big]. \tag{SA.7}$$

Solving (SA.7) yields the solution of the buyer's first-order condition as follows:

$$w_1^{N\ddagger} = \frac{2(p+l)}{\gamma}. \tag{SA.8}$$

Then, we need to show that $w_1^{N\ddagger}$ is the buyer's optimal contracting decision. Similar to the previous proof, if the stationary point characterized in (SA.8) is a strict local maximum, then $w_1^{N\ddagger}$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d^2\pi_B(w_1)}{dw_1^2} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}} w_1^{\frac{6-2\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} \left[\left(\frac{4-\gamma}{\gamma-2}\right)\Big[2(p+l) - \gamma w_1\Big] - \gamma w_1\right].$$

By Assumption 2, we can show that $\frac{d^2\pi_B(w_1)}{dw_1^2}\Big|_{w_1^{N\ddagger}} < 0$. Thus, $w_1^{N\ddagger}$ is the buyer's optimal contracting decision.

Finally, plugging $w_1^{N\ddagger}$ into (SA.4), (SA.5) and (SA.6), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{N\ddagger} = \frac{2(p+l)}{\gamma^2}$, $q_1^{N\ddagger} = \left[\frac{2(p+l)(\gamma-1)^{\frac{\gamma-1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$, $q_2^{N\ddagger} = \left[\frac{2(p+l)(\gamma-1)^{\frac{1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{N\ddagger} \in (0,1)$ for $i \in \{1,2\}$. Moreover, we can show that $w_1^{N\ddagger}/w_2^{N\ddagger} = \gamma \geqslant 2$ and $q_1^{N\ddagger}/q_2^{N\ddagger} = (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$. □

Proof of Proposition 2. We use backward induction to solve the game. Recall that the game consists of three stages. First, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in previous stages. Specifically, for supplier 1, the first-order condition of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ is $\frac{d\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1}\Big|_{q_1=\tilde{q}_1(w_1,q_2)} = w_1q_2 - C'(\tilde{q}_1(w_1,q_2)) = w_1q_2 - \theta\gamma(\tilde{q}_1(w_1,q_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_1}(w_2,q_1|w_1,q_2)$ w.r.t. q_1 yields $\frac{d^2\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1^2} = -C''(q_1) = -\theta\gamma(\gamma-1)q_1^{\gamma-2} < 0$. Thereby, supplier 1's optimal quality in response to w_1 is $\tilde{q}_1(w_1,q_2) = \left(\frac{w_1q_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. On the other hand, for supplier 2, the first-order condition of $\pi_{S_2}(q_2|w_2)$ is $\frac{d\pi_{S_2}(q_2|w_2)}{dq_2}\Big|_{q_2=\tilde{q}_2(w_2)} = w_2 - C'(\tilde{q}_2(w_2)) = w_2 - \theta\gamma(\tilde{q}_2(w_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_2}(q_2|w_2)$ w.r.t. q_2 yields $\frac{d^2\pi_{S_2}(q_2|w_2)}{dq_2^2} = -C''(q_2) = -\theta\gamma(\gamma-1)q_2^{\gamma-2} < 0$. Thereby, supplier 2's optimal quality in response to w_2 is $\tilde{q}_2(w_2) = \left(\frac{w_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1, w_2) = \left(\frac{w_1^{\gamma-1} w_2}{\theta^\gamma \gamma^\gamma}\right)^{\frac{1}{(\gamma-1)^2}}, \tag{SA.9}$$

$$\tilde{q}_2(w_2) = \left(\frac{w_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SA.10}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1, w_2)|w_1, \tilde{q}_2(w_2))$, given w_1 decided in stage 1. Plugging (SA.10) into $\pi_{S_2}(q_2|w_2)$, we have $\pi_{S_2}(\tilde{q}_2(w_2)|w_2) = w_2\tilde{q}_2(w_2) - \theta(\tilde{q}_2(w_2))^\gamma = (\gamma-1)w_2^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \geqslant 0$ for any $w_2 \geqslant 0$. Thus, IR_2 is always satisfied. Then, plugging (SA.9) and (SA.10) into (2), we have supplier 1's problem as follows:

$$\begin{aligned}\max_{w_2}\ \pi_{S_1}(w_2|w_1) &= w_1\tilde{q}_1(w_1, w_2)\tilde{q}_2(w_2) - \theta(\tilde{q}_1(w_1, w_2))^\gamma - w_2\tilde{q}_2(w_2)\\ &= (\gamma-1)w_1^{\frac{\gamma}{\gamma-1}} w_2^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{2\gamma-1}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2}{(\gamma-1)^2}} - w_2^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}.\end{aligned}$$

We now analyze supplier 1's optimal contracting decision. Taking the first-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\frac{d\pi_{S_1}(w_2|w_1)}{dw_2} = \frac{\gamma}{\gamma-1} w_2^{\frac{3\gamma-\gamma^2-1}{(\gamma-1)^2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[w_1^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2-\gamma+1}{(\gamma-1)^2}} - w_2^{\frac{\gamma(\gamma-2)}{(\gamma-1)^2}}\right]. \tag{SA.11}$$

Solving (SA.11) yields the solution of supplier 1's first-order condition as follows:

$$\tilde{w}_2(w_1) = \left[\frac{w_1^{\gamma(\gamma-1)}}{\theta^\gamma \gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SA.12}$$

Then, we need to show that $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. In similar fashion to the proof of Proposition 1, if the stationary point characterized in (SA.12) is a strict local maximum, then $\tilde{w}_2(w_1)$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\begin{aligned}\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2} &= \frac{-1}{(\gamma-1)^3} w_2^{\frac{-2\gamma^2+5\gamma-2}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\\ &\quad\cdot\left[\gamma(\gamma-2)w_2^{\frac{\gamma(\gamma-2)}{(\gamma-1)^2}} + (\gamma^2-3\gamma+1)\left[w_1^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2-\gamma+1}{(\gamma-1)^2}} - w_2^{\frac{\gamma(\gamma-2)}{(\gamma-1)^2}}\right]\right].\end{aligned}$$

By Assumption 2, we can show that $\left.\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2}\right|_{\tilde{w}_2(w_1)} < 0$. Thus, $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. Then, plugging (SA.12) into (SA.9) and (SA.10), we have

$$\tilde{q}_1(w_1) = \left[\frac{w_1^\gamma}{\theta^\gamma \gamma^{\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SA.13}$$

$$\tilde{q}_2(w_1) = \left[\frac{w_1^{\gamma}}{\theta^{\gamma}\gamma^{2\gamma-1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SA.14}$$

Comparing (SA.13) and (SA.14) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = \gamma^{\frac{1}{\gamma}} > 1$.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, to maximize $\pi_B(w_1|\tilde{q}_1(w_1, \tilde{w}_2(w_1)), \tilde{q}_2(\tilde{w}_2(w_1)))$. Plugging (SA.12) into $\pi_{S_1}(w_2|w_1)$, we have $\pi_{S_1}(\tilde{w}_2(w_1)|w_1) = (\gamma-1)w_1^{\frac{\gamma}{\gamma-1}}\left[\tilde{w}_2(w_1)\right]^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{2\gamma-1}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2}{(\gamma-1)^2}} - \left[\tilde{w}_2(w_1)\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} = (\gamma-2)w_1^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma+1}{\gamma-2}} \geqslant 0$ for any $w_1 \geqslant 0$. Thus, IR_1 is always satisfied. Then, plugging (SA.12), (SA.13) and (SA.14) into (2), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1}\ \pi_B(w_1) &= (p+l)\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1)\\ &= (p+l-w_1)w_1^{\frac{2}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} - l.\end{aligned}$$

We now analyze the buyer's optimal contracting decision. Taking the first-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d\pi_B(w_1)}{dw_1} = \frac{1}{\gamma-2}w_1^{\frac{4-\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}}\Big[2(p+l) - \gamma w_1\Big]. \tag{SA.15}$$

Solving (SA.15) yields the solution of the buyer's first-order condition as follows:

$$w_1^{T\ddagger} = \frac{2(p+l)}{\gamma}. \tag{SA.16}$$

Then, we need to show that $w_1^{T\ddagger}$ is the buyer's optimal contracting decision. Similar to the previous proof, if the stationary point characterized in (SA.16) is a strict local maximum, then $w_1^{T\ddagger}$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d^2\pi_B(w_1)}{dw_1^2} = \frac{1}{\gamma-2}w_1^{\frac{6-2\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}}\left[\left(\frac{4-\gamma}{\gamma-2}\right)\Big[2(p+l) - \gamma w_1\Big] - \gamma w_1\right].$$

By Assumption 2, we can show that $\frac{d^2\pi_B(w_1)}{dw_1^2}\Big|_{w_1^{T\ddagger}} < 0$. Thus, $w_1^{T\ddagger}$ is the buyer's optimal contracting decision.

Finally, plugging $w_1^{T\ddagger}$ into (SA.12), (SA.13) and (SA.14), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{T\ddagger} = \left[2(p+l)\right]^{\frac{\gamma-1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma^2-2\gamma+1}{\gamma(\gamma-2)}}$, $q_1^{T\ddagger} = \left[\frac{2(p+l)}{\theta\gamma^{2+\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}}$, $q_2^{T\ddagger} = \left[\frac{2(p+l)}{\theta\gamma^{3-\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{T\ddagger} \in (0,1)$ for $i \in \{1,2\}$. Moreover, we can show that $w_1^{T\ddagger}/w_2^{T\ddagger} = \gamma/q_1^{T\ddagger} > \gamma$ and $q_1^{T\ddagger}/q_2^{T\ddagger} = \gamma^{\frac{1}{\gamma}} > 1$. $\square$

Proof of Theorem 1. The theorem follows from comparing the equilibrium contracts and suppliers' quality decisions characterized in Propositions 1 and 2. By Assumptions 1 and 2, we have

$$w_2^{T\ddagger} < w_2^{N\ddagger} \Leftrightarrow \left[2(p+l)\right]^{\frac{\gamma-1}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma^2-2\gamma+1}{\gamma(\gamma-2)}} < \frac{2(p+l)}{\gamma^2} \Leftrightarrow \left[\frac{2(p+l)}{\theta\gamma^{2+\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}} < 1,$$
$$q_1^{T\ddagger} > q_1^{N\ddagger} \Leftrightarrow \left[\frac{2(p+l)}{\theta\gamma^{2+\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}} > \left[\frac{2(p+l)(\gamma-1)^{\frac{\gamma-1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}} \Leftrightarrow \gamma^{\frac{\gamma-1}{\gamma}} > (\gamma-1)^{\frac{\gamma-1}{\gamma}},$$
$$q_2^{T\ddagger} > q_2^{N\ddagger} \Leftrightarrow \left[\frac{2(p+l)}{\theta\gamma^{3-\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}} > \left[\frac{2(p+l)(\gamma-1)^{\frac{1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}} \Leftrightarrow \gamma^{\frac{1}{\gamma}} > (\gamma-1)^{\frac{1}{\gamma}},$$
$$\frac{w_1^{T\ddagger}}{w_2^{T\ddagger}} > \frac{w_1^{N\ddagger}}{w_2^{N\ddagger}} \Leftrightarrow \frac{\gamma}{q_1^{T\ddagger}} > \gamma, \qquad \frac{q_1^{T\ddagger}}{q_2^{T\ddagger}} > \frac{q_1^{N\ddagger}}{q_2^{N\ddagger}} \Leftrightarrow \gamma^{\frac{1}{\gamma}} > (\gamma-1)^{\frac{1}{\gamma}}.$$

By Assumptions 1 and 2, the above inequalities always hold. Hence, the theorem is proved. □

Proof of Theorem 2. Consider the case without traceability. Based on the equilibrium characterized in Proposition 1, we obtain the equilibrium expected profits for the buyer, the downstream and the upstream suppliers, and the entire supply chain as follows:

$$\pi_B^{N\ddagger} = (\gamma-2)(\gamma-1)^{\frac{1}{\gamma-2}} 2^{\frac{2}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+4}{\gamma-2}} - l,$$
$$\pi_{S_1}^{N\ddagger} = (\gamma-1)^{\frac{2\gamma-3}{\gamma-2}} 2^{\frac{\gamma}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3\gamma}{\gamma-2}},$$
$$\pi_{S_2}^{N\ddagger} = (\gamma-1)^{\frac{\gamma-1}{\gamma-2}} 2^{\frac{\gamma}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3\gamma}{\gamma-2}},$$
$$\pi_{SC}^{N\ddagger} = \pi_B^{N\ddagger} + \pi_{S_1}^{N\ddagger} + \pi_{S_2}^{N\ddagger}.$$

Consider the case with traceability. Based on the equilibrium characterized in Proposition 2, we obtain the equilibrium expected profits for the buyer, the downstream and the upstream suppliers, and the entire supply chain as follows:

$$\pi_B^{T\ddagger} = (\gamma-2) 2^{\frac{2}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+3}{\gamma-2}} - l,$$
$$\pi_{S_1}^{T\ddagger} = (\gamma-2) 2^{\frac{\gamma}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma+1}{\gamma-2}},$$
$$\pi_{S_2}^{T\ddagger} = (\gamma-1) 2^{\frac{\gamma}{\gamma-2}} (p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3\gamma-1}{\gamma-2}},$$
$$\pi_{SC}^{T\ddagger} = \pi_B^{T\ddagger} + \pi_{S_1}^{T\ddagger} + \pi_{S_2}^{T\ddagger}.$$

We next compare the buyer's equilibrium expected profits with and without traceability. By Assumptions 1 and 2, it is easy to see that $\pi_B^{T\ddagger} > \pi_B^{N\ddagger}$ always hold.

We then compare the downstream supplier's equilibrium expected profits with and without traceability. By Assumptions 1 and 2, we have

$$\pi_{S_1}^{T\ddagger} > \pi_{S_1}^{N\ddagger} \Leftrightarrow (\gamma-2)2^{\frac{\gamma}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma+1}{\gamma-2}} > (\gamma-1)^{\frac{2\gamma-3}{\gamma-2}}2^{\frac{\gamma}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3\gamma}{\gamma-2}}$$
$$\Leftrightarrow (\gamma-2)^{\gamma-2}\gamma^{\gamma-1} > (\gamma-1)^{2\gamma-3}$$
$$\Leftrightarrow (\gamma-2)\log(\gamma-2)+(\gamma-1)\log\gamma-(2\gamma-3)\log(\gamma-1) > 0.$$

Denote $D_1(\gamma) \equiv (\gamma-2)\log(\gamma-2)+(\gamma-1)\log\gamma-(2\gamma-3)\log(\gamma-1)$. Taking the first- and second-order derivatives of $D_1(\gamma)$ w.r.t. γ, we have $D_1'(\gamma) = \log\left[\frac{\gamma(\gamma-2)}{(\gamma-1)^2}\right]+\frac{1}{\gamma(\gamma-1)}$ and $D_1''(\gamma) = \frac{3\gamma-2}{\gamma^2(\gamma-1)^2(\gamma-2)}$. Since $D_1''(\gamma) > 0$ by Assumption 2, we know that $D_1'(\gamma)$ increases in γ, and thus, $D_1'(\gamma) < \lim_{\gamma\to+\infty} D_1'(\gamma) = \log 1 = 0$, for any $\gamma \geqslant 2$. Hence, $D_1(\gamma)$ decreases in γ, and thus, $D_1(\gamma) > \lim_{\gamma\to+\infty} D_1(\gamma) = \lim_{\gamma\to+\infty}\left[\gamma\log\left(\frac{\gamma-2}{\gamma-1}\right)+\gamma\log\left(\frac{\gamma}{\gamma-1}\right)+2\log\left(\frac{\gamma-1}{\gamma-2}\right)+\log\left(\frac{\gamma-1}{\gamma}\right)\right] = \lim_{\gamma\to+\infty}\left[-\frac{\gamma^2}{(\gamma-2)(\gamma-1)}+\frac{\gamma}{\gamma-1}\right] = 0$. Therefore, we can see that $\pi_{S_1}^{T\ddagger} > \pi_{S_1}^{N\ddagger}$ always holds.

We then compare the upstream supplier's equilibrium expected profits with and without traceability. By Assumptions 1 and 2, it is easy to show that $\pi_{S_2}^{T\ddagger} > \pi_{S_2}^{N\ddagger}$ always holds. Therefore, we can also see that $\pi_{SC}^{T\ddagger} > \pi_{SC}^{N\ddagger}$ always holds.

Finally, we compare the changes of the two suppliers' equilibrium expected profits due to traceability. By Assumptions 1 and 2, we have

$$\pi_{S_2}^{T\ddagger}-\pi_{S_2}^{N\ddagger} > \pi_{S_1}^{T\ddagger}-\pi_{S_1}^{N\ddagger} \Leftrightarrow (\gamma-1)\left(\frac{1}{\gamma}\right)^{\frac{3\gamma-1}{\gamma-2}}-(\gamma-1)^{\frac{\gamma-1}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3\gamma}{\gamma-2}} > (\gamma-2)\left(\frac{1}{\gamma}\right)^{\frac{2\gamma+1}{\gamma-2}}-(\gamma-1)^{\frac{2\gamma-3}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3\gamma}{\gamma-2}}$$
$$\Leftrightarrow \left(\frac{\gamma^2-3\gamma+2}{\gamma^2-3\gamma+1}\right)^{\gamma-2} > \frac{\gamma}{\gamma-1}$$
$$\Leftrightarrow (\gamma-2)\log\left(\frac{\gamma^2-3\gamma+2}{\gamma^2-3\gamma+1}\right)-\log\left(\frac{\gamma}{\gamma-1}\right) > 0.$$

Denote $D_2(\gamma) \equiv (\gamma-2)\log\left(\frac{\gamma^2-3\gamma+2}{\gamma^2-3\gamma+1}\right)-\log\left(\frac{\gamma}{\gamma-1}\right)$. Taking the first- and second-order derivatives of $D_2(\gamma)$ w.r.t. γ, we have $D_2'(\gamma) = \log\left(\frac{\gamma^2-3\gamma+2}{\gamma^2-3\gamma+1}\right)-\frac{\gamma+1}{\gamma(\gamma^2-3\gamma+1)}$ and $D_2''(\gamma) = \frac{2\gamma-3}{\gamma^2-3\gamma+1}\left[\frac{1}{\gamma^2-3\gamma+1}-\frac{1}{\gamma^2-3\gamma+2}\right]+\frac{3\gamma(\gamma-2)+1}{\gamma^2(\gamma^2-3\gamma+1)^2}$. Since $D_2''(\gamma) > 0$ by Assumption 2, we know that $D_2'(\gamma)$ increases in γ, and thus, $D_2'(\gamma) < \lim_{\gamma\to+\infty} D_2'(\gamma) = \log 1 = 0$, for any $\gamma \geqslant 2$. Hence, $D_2(\gamma)$ decreases in γ, and thus, $D_2(\gamma) > \lim_{\gamma\to+\infty} D_2(\gamma) = \lim_{\gamma\to+\infty}\frac{\log\left(\frac{\gamma^2-3\gamma+2}{\gamma^2-3\gamma+1}\right)}{\frac{1}{\gamma-2}}-\log 1 = \lim_{\gamma\to+\infty}\frac{(2\gamma-3)(\gamma-2)}{(\gamma^2-3\gamma+1)(\gamma-1)} = 0$. Therefore, we can see that $\pi_{S_2}^{T\ddagger}-\pi_{S_2}^{N\ddagger} > \pi_{S_1}^{T\ddagger}-\pi_{S_1}^{N\ddagger}$ always holds. $\square$

SB. Value of Blockchain in a Parallel Supply Chain: Proofs

Proof of Proposition 3. We first derive the suppliers' optimal quality decisions. Given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i,q_{-i})$. For supplier i, the first-order condition of $\pi_{S_i}(q_i|w_i,q_{-i})$ is $\frac{d\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i}\Big|_{q_i=\tilde{q}_i(w_i,q_{-i})} = w_iq_{-i}-C'(\tilde{q}_i(w_i,q_{-i})) = w_iq_{-i}-\theta\gamma(\tilde{q}_i(w_i,q_{-i}))^{\gamma-1} = 0$.

Taking the second-order derivative of $\pi_{S_i}(q_i|w_i,q_{-i})$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i^2} = -C''(q_i) = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, supplier i's optimal quality in response to w_i is $\tilde{q}_i(w_i,q_{-i}) = \left(\frac{w_i q_{-i}}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i,w_{-i}) = \left(\frac{w_i^{\gamma-1}w_{-i}}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}. \tag{SB.1}$$

Next, consider the buyer's problem. Plugging (SB.1) into $\pi_{S_i}(q_i|w_i,q_{-i})$, we have $\pi_{S_i}(\tilde{q}_i(w_i,w_{-i})|w_i,\tilde{q}_{-i}(w_i,w_{-i})) = w_i\tilde{q}_i(w_i,w_{-i})\tilde{q}_{-i}(w_i,w_{-i}) - \theta(\tilde{q}_i(w_i,w_{-i}))^\gamma = (\gamma-1)w_i^{\frac{\gamma-1}{\gamma-2}}w_{-i}^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} \geqslant 0$ for any $w_i \geqslant 0$ and $w_{-i} \geqslant 0$. Thus, IR_i is always satisfied. Then, plugging (SB.1) into (3), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1,w_2}\ \pi_B(w_1,w_2) &= (p+l)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l - (w_1+w_2)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2)\\ &= (p+l-w_1-w_2)w_1^{\frac{1}{\gamma-2}}w_2^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} - l.\end{aligned}$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_1} = w_1^{\frac{3-\gamma}{\gamma-2}}w_2^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2} - w_1\right], \tag{SB.2}$$

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_2} = w_1^{\frac{1}{\gamma-2}}w_2^{\frac{3-\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2} - w_2\right]. \tag{SB.3}$$

Solving (SB.2) and (SB.3) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{N\dagger} = w_2^{N\dagger} = \frac{p+l}{\gamma}. \tag{SB.4}$$

Then, we need to show that $(w_1^{N\dagger}, w_2^{N\dagger})$ are the buyer's optimal contracting decisions. In particular, we will prove that the sufficient conditions of the local maximum are able to guarantee the unique global maximum, the underlying idea of which was used by Petruzzi and Dada (1999) and Aydin and Porteus (2008). Taking the second-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\begin{aligned}\frac{\partial^2\pi_B(w_1,w_2)}{\partial w_1^2} &= \frac{1}{\gamma-2}w_1^{\frac{5-2\gamma}{\gamma-2}}w_2^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2} - 2w_1\right],\\ \frac{\partial^2\pi_B(w_1,w_2)}{\partial w_2^2} &= \frac{1}{\gamma-2}w_1^{\frac{1}{\gamma-2}}w_2^{\frac{5-2\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2} - 2w_2\right],\\ \frac{\partial^2\pi_B(w_1,w_2)}{\partial w_1\partial w_2} &= \frac{1}{\gamma-2}w_1^{\frac{3-\gamma}{\gamma-2}}w_2^{\frac{3-\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2} - w_1 - w_2\right].\end{aligned}$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1,w_2)$ is negative definite in the neighborhood of $(w_1^{N\dagger}, w_2^{N\dagger})$. Hence, $(w_1^{N\dagger}, w_2^{N\dagger})$ is a strict local maximum. Suppose now that there exist more than one, say two, interior stationary points for the function $\pi_B(w_1, w_2)$. Because both points need to be local maxima, the function should also have an interior local minimum somewhere in between, which is a contradiction to the result that all interior stationary points are local maxima. Consequently, we can conclude that there exists only one stationary point $(w_1^{N\dagger}, w_2^{N\dagger})$ that satisfies (SB.4), which is the unique local maximum, and also the unique global maximum. Thus, $(w_1^{N\dagger}, w_2^{N\dagger})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{N\dagger}, w_2^{N\dagger})$ into (SB.1), we obtain the suppliers' optimal quality decisions: $q_1^{N\dagger} = q_2^{N\dagger} = \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{N\dagger} \in (0,1)$ for $i \in \{1,2\}$. $\square$

Proof of Proposition 4. We first derive the suppliers' optimal quality decisions. Given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i)$. For supplier i, the first-order condition of $\pi_{S_i}(q_i|w_i)$ is $\frac{d\pi_{S_i}(q_i|w_i)}{dq_i}\Big|_{q_i=\tilde{q}_i(w_i)} = w_i - C'(\tilde{q}_i(w_i)) = w_i - \theta\gamma(\tilde{q}_i(w_i))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_i}(q_i|w_i)$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i)}{dq_i^2} = -C''(q_i) = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, the solution of the first-order condition is supplier i's optimal quality in response to w_i. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i) = \left(\frac{w_i}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SB.5}$$

Next, consider the buyer's problem. Plugging (SB.5) into $\pi_{S_i}(q_i|w_i)$, we have $\pi_{S_i}(\tilde{q}_i(w_i)|w_i) = w_i\tilde{q}_i(w_i) - \theta(\tilde{q}_i(w_i))^\gamma = (\gamma-1)w_i^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \geqslant 0$ for any $w_i \geqslant 0$ and $w_{-i} \geqslant 0$. Thus, IR_i is always satisfied. Then, plugging (SB.5) into (4), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1,w_2}\ \pi_B(w_1,w_2) &= p\tilde{q}_1(w_1)\tilde{q}_2(w_2) + \frac{1}{2}(p-l)\tilde{q}_1(w_1)\Big[1-\tilde{q}_2(w_2)\Big] + \frac{1}{2}(p-l)\tilde{q}_2(w_2)\Big[1-\tilde{q}_1(w_1)\Big] \\ &\quad - l\Big[1-\tilde{q}_1(w_1)\Big]\Big[1-\tilde{q}_2(w_2)\Big] - w_1\tilde{q}_1(w_1) - w_2\tilde{q}_2(w_2) \\ &= \frac{1}{2}(p+l)\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[w_1^{\frac{1}{\gamma-1}} + w_2^{\frac{1}{\gamma-1}}\right] - l - w_1^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - w_2^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}.\end{aligned}$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1, w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_1} = \frac{1}{\gamma-1}w_1^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{1}{2}(p+l) - \gamma w_1\right], \tag{SB.6}$$

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_2} = \frac{1}{\gamma-1}w_2^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{1}{2}(p+l) - \gamma w_2\right]. \tag{SB.7}$$

Solving (SB.6) and (SB.7) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{T\dagger} = w_2^{T\dagger} = \frac{p+l}{2\gamma}. \tag{SB.8}$$

Then, we need to show that $(w_1^{T\dagger}, w_2^{T\dagger})$ are the buyer's optimal contracting decisions. In similar fashion to the proof of Proposition 3, if the stationary point characterized in (SB.8) is a strict local maximum, then $(w_1^{T\dagger}, w_2^{T\dagger})$ must be the unique global maximum, proved by contradiction. Taking the second-order derivatives of $\pi_B(w_1, w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1^2} = \frac{1}{2(\gamma-1)^2} w_1^{\frac{3-2\gamma}{\gamma-1}} \left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} \Big[(2-\gamma)(p+l) - 2\gamma w_1\Big],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_2^2} = \frac{1}{2(\gamma-1)^2} w_2^{\frac{3-2\gamma}{\gamma-1}} \left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} \Big[(2-\gamma)(p+l) - 2\gamma w_2\Big],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1 \partial w_2} = 0.$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1, w_2)$ is negative definite in the neighborhood of $(w_1^{T\dagger}, w_2^{T\dagger})$. Thus, $(w_1^{T\dagger}, w_2^{T\dagger})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{T\dagger}, w_2^{T\dagger})$ into (SB.5), we obtain the suppliers' optimal quality decisions: $q_1^{T\dagger} = q_2^{T\dagger} = \left(\frac{p+l}{2\theta\gamma^2}\right)^{\frac{1}{\gamma-1}}$. By Assumptions 1 and 2, we have $q_i^{T\dagger} \in (0,1)$ for $i \in \{1,2\}$. □

Proof of Theorem 3. The theorem follows from comparing the equilibrium contracts and suppliers' quality decisions characterized in Propositions 3 and 4. First, it is easy to see that $w_i^{T\dagger} < w_i^{N\dagger}$ always holds. Second, by Assumptions 1 and 2, we have

$$q_i^{T\dagger} > q_i^{N\dagger} \Leftrightarrow \left(\frac{p+l}{2\theta\gamma^2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}} \Leftrightarrow \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{1}{(\gamma-2)(\gamma-1)}} \Leftrightarrow \frac{\gamma^2}{2^{\gamma-2}} > \frac{p+l}{\theta}.$$

Thus, the comparison between $q_i^{T\dagger}$ and $q_i^{N\dagger}$ can be characterized by thresholds $\bar{l}$, or $\bar{p}$, or $\bar{\theta}$, or $\bar{\gamma}$ such that $q_i^{T\dagger} > q_i^{N\dagger}$ if $l < \bar{l}$, or $p < \bar{p}$, or $\theta > \bar{\theta}$, or $\gamma < \bar{\gamma}$; whereas $q_i^{T\dagger} < q_i^{N\dagger}$ if $l > \bar{l}$, or $p > \bar{p}$, or $\theta < \bar{\theta}$, or $\gamma > \bar{\gamma}$, where

$$\bar{l} \equiv \frac{\theta\gamma^2}{2^{\gamma-2}} - p,\ \bar{p} \equiv \frac{\theta\gamma^2}{2^{\gamma-2}} - l,\ \bar{\theta} \equiv \frac{2^{\gamma-2}(p+l)}{\gamma^2},\ \bar{\gamma} \equiv \begin{cases} \bar{\gamma}_0 & \text{if } \frac{p+l}{\theta} \leqslant 4, \\ 2 & \text{if } \frac{p+l}{\theta} > 4, \end{cases}$$

and $\bar{\gamma}_0$ is the unique solution to $\frac{\gamma^2}{2^{\gamma-2}} = \frac{p+l}{\theta}$ in the range of $\gamma > 2$. Besides, $\bar{\gamma}_0 > 4$, and it is decreasing in p and l, while increasing in θ. Hence, the theorem is proved. □

Proof of Theorem 4. Consider the case without traceability. Based on the equilibrium characterized in Proposition 3, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{N\dagger} = (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l,$$
$$\pi_{S_i}^{N\dagger} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}},$$
$$\pi_{SC}^{N\dagger} = (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l.$$

Consider the case with traceability. Based on the equilibrium characterized in Proposition 4, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{T\dagger} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+1}{\gamma-1}} - l,$$

$$\pi_{S_i}^{T\dagger} = (\gamma-1)\left(\frac{p+l}{2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}},$$

$$\pi_{SC}^{T\dagger} = (\gamma^2-1)(p+l)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l.$$

We first compare the buyer's equilibrium expected profits with and without traceability. By Assumptions 1 and 2, we have

$$\pi_B^{T\dagger} > \pi_B^{N\dagger} \Leftrightarrow (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+1}{\gamma-1}} - l > (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l$$
$$\Leftrightarrow \left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} \gamma^{\frac{2}{(\gamma-2)(\gamma-1)}} > \left(\frac{\gamma-2}{\gamma-1}\right)\left(\frac{p+l}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}.$$

The last inequality always holds since $\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} \geqslant 1$, $\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}} > 1$, $\frac{\gamma-2}{\gamma-1} < 1$, and $\left(\frac{p+l}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}} < 1$. Hence, we can see that $\pi_B^{T\dagger} > \pi_B^{N\dagger}$ always holds.

We then compare the suppliers' equilibrium expected profits with and without traceability. By Assumptions 1 and 2, we have

$$\pi_{S_i}^{T\dagger} > \pi_{S_i}^{N\dagger} \Leftrightarrow (\gamma-1)\left(\frac{p+l}{2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} > (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}}$$
$$\Leftrightarrow \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} > \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}} \Leftrightarrow \frac{\gamma^2}{2^{\gamma-2}} > \frac{p+l}{\theta}.$$

Thus, the comparison between $\pi_{S_i}^{T\dagger}$ and $\pi_{S_i}^{N\dagger}$ can be characterized by thresholds $\bar{l}$, or $\bar{p}$, or $\bar{\theta}$, or $\bar{\gamma}$ such that $\pi_{S_i}^{T\dagger} > \pi_{S_i}^{N\dagger}$ if $l < \bar{l}$, or $p < \bar{p}$, or $\theta > \bar{\theta}$, or $\gamma < \bar{\gamma}$; whereas $\pi_{S_i}^{T\dagger} < \pi_{S_i}^{N\dagger}$ if $l > \bar{l}$, or $p > \bar{p}$, or $\theta < \bar{\theta}$, or $\gamma > \bar{\gamma}$, where the thresholds $\bar{l}$, $\bar{p}$, $\bar{\theta}$, and $\bar{\gamma}$ are characterized in the proof of Theorem 3.

Finally, we compare the equilibrium total supply chain profits with and without traceability. By Assumptions 1 and 2, we have

$$\pi_{SC}^{T\dagger} > \pi_{SC}^{N\dagger} \Leftrightarrow (\gamma^2-1)(p+l)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l > (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l$$
$$\Leftrightarrow \left(\frac{\gamma^2-1}{\gamma^2-2}\right)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}$$
$$\Leftrightarrow \left(\frac{\gamma^2-1}{\gamma^2-2}\right)\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}}\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}.$$

The last inequality always holds since $\frac{\gamma^2-1}{\gamma^2-2}>1$, $\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}}>1$, $\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}}\geqslant 1$, and $\left(\frac{p+l}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}<1$. Hence, we can see that $\pi_{SC}^{T\dagger}>\pi_{SC}^{N\dagger}$ always holds. □

SC. Data Permission and Consensus Mechanism: Proofs

Proof of Proposition 5. The game consists of two stages. First, in stage 2, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in stage 1. Specifically, for supplier $i\in\{1,2\}$, the first-order condition of $\pi_{S_i}(q_i|w_i)$ is $\frac{d\pi_{S_i}(q_i|w_i)}{dq_i}\Big|_{q_i=\tilde{q}_i(w_i)}=w_i-\theta\gamma(\tilde{q}_i(w_i))^{\gamma-1}=0$. Taking the second-order derivative of $\pi_{S_i}(q_i|w_i)$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i)}{dq_i^2}=-\theta\gamma(\gamma-1)q_i^{\gamma-2}<0$. Thereby, the solution of the first-order condition is supplier i's optimal quality in response to w_i and t_i. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i)=\left(\frac{w_i}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SC.1}$$

Next, consider stage 1, where the buyer chooses the contract to offer to suppliers 1 and 2, w_1 and w_2, to maximize $\pi_B(w_1,w_2|\tilde{q}_1(w_1),\tilde{q}_2(w_2))$. Plugging (SC.1) into $\pi_{S_i}(q_i|w_i)$, we have $\pi_{S_i}(\tilde{q}_i(w_i)|w_i)=w_i\tilde{q}_i(w_i)-\theta(\tilde{q}_i(w_i))^\gamma=(\gamma-1)w_i^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\geqslant 0$ for any $w_i\geqslant 0$. Thus, IR_i is always satisfied. Then, plugging (SC.1) into (C.1), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1,w_2}\ \pi_B(w_1,w_2)&=(p+l)\tilde{q}_1(w_1)\tilde{q}_2(w_1)-l-w_1\tilde{q}_1(w_1)-w_2\tilde{q}_2(w_1)\\&=(p+l)w_1^{\frac{1}{\gamma-1}}w_2^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-1}}-w_1^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}-w_2^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}-l.\end{aligned}$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_1}=\frac{1}{\gamma-1}w_1^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[(p+l)w_2^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}-\gamma w_1\right], \tag{SC.2}$$

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_2}=\frac{1}{\gamma-1}w_2^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[(p+l)w_1^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}-\gamma w_2\right]. \tag{SC.3}$$

Solving (SC.2) and (SC.3) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{R\ddagger}=w_2^{R\ddagger}=(p+l)^{\frac{\gamma-1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}}. \tag{SC.4}$$

Then, we need to show that $(w_1^{R\ddagger},w_2^{R\ddagger})$ are the buyer's optimal contracting decisions. In similar fashion to the proof of Proposition 3, by Assumption 2, we can show that the Hessian of $\pi_B(w_1,w_2)$ is negative definite in the neighborhood of $(w_1^{R\ddagger},w_2^{R\ddagger})$. Thus, $(w_1^{R\ddagger},w_2^{R\ddagger})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{R\ddagger}, w_2^{R\ddagger})$ into (SC.1), we obtain the suppliers' optimal quality decisions: $q_1^{R\ddagger} = q_2^{R\ddagger} = \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{R\ddagger} \in (0,1)$ for $i \in \{1,2\}$. $\square$

Proof of Theorem 5 and Proposition 6. Under restricted data permission, based on the equilibrium characterized in Proposition 5, we obtain the equilibrium expected profits for the buyer, the downstream and the upstream suppliers, and the entire supply chain as follows:

$$\pi_B^{R\ddagger} = (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l,$$
$$\pi_{S_1}^{R\ddagger} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}},$$
$$\pi_{S_2}^{R\ddagger} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}},$$
$$\pi_{SC}^{R\ddagger} = \pi_B^{R\ddagger} + \pi_{S_1}^{R\ddagger} + \pi_{S_2}^{R\ddagger} = (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l.$$

The equilibrium profits under unrestricted data permission are characterized in the proof of Theorem 2.

We first compare the equilibrium total supply chain profits under restricted and unrestricted data permission. By Assumptions 1 and 2, we have

$$\pi_{SC}^{R\ddagger} > \pi_{SC}^{T\ddagger} \Leftrightarrow (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l > (\gamma^3-2\gamma-2)2^{\frac{2}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3\gamma-1}{\gamma-2}} - l$$
$$\Leftrightarrow (\gamma^2-2)\gamma^{\frac{\gamma-1}{\gamma-2}} > (\gamma^3-2\gamma-2)2^{\frac{2}{\gamma-2}}.$$

Thus, the comparison between $\pi_{SC}^{R\ddagger}$ and $\pi_{SC}^{T\ddagger}$ can be characterized by threshold $\bar{\gamma}_1$, which is the unique solution to $(\gamma^2-2)\gamma^{\frac{\gamma-1}{\gamma-2}} = (\gamma^3-2\gamma-2)2^{\frac{2}{\gamma-2}}$ in the range of $\gamma > 2$, such that $\pi_{SC}^{R\ddagger} > \pi_{SC}^{T\ddagger}$ if $\gamma > \bar{\gamma}_1$; whereas $\pi_{SC}^{R\ddagger} < \pi_{SC}^{T\ddagger}$ if $\gamma < \bar{\gamma}_1$.

We then compare the buyer's equilibrium expected profits under restricted and unrestricted data permission. By Assumptions 1 and 2, we have

$$\pi_B^{R\ddagger} > \pi_B^{T\ddagger} \Leftrightarrow (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l > (\gamma-2)2^{\frac{2}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma+3}{\gamma-2}} - l$$
$$\Leftrightarrow \gamma > 4.$$

Thus, the comparison between $\pi_B^{R\ddagger}$ and $\pi_B^{T\ddagger}$ can be characterized by threshold $\bar{\gamma}_2 = 4$ such that $\pi_B^{R\ddagger} < \pi_B^{T\ddagger}$ if $\gamma < \bar{\gamma}_2$; whereas $\pi_B^{R\ddagger} > \pi_B^{T\ddagger}$ if $\gamma > \bar{\gamma}_2$.

We then compare the downstream supplier's equilibrium expected profits under restricted and unrestricted data permission. By Assumptions 1 and 2, we have

$$\pi_{S_1}^{R\ddagger} < \pi_{S_1}^{T\ddagger} \Leftrightarrow (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} > (\gamma-2)2^{\frac{\gamma}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma+1}{\gamma-2}}$$
$$\Leftrightarrow \left(\frac{\gamma-1}{\gamma-2}\right)\gamma^{\frac{1}{\gamma-2}} \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-2}} < 1.$$

The last inequality always holds by Assumption 2. Thus, $\pi_{S_1}^{R\ddagger} < \pi_{S_1}^{T\ddagger}$ always holds.

Finally, we compare the upstream supplier's equilibrium expected profits under restricted and unrestricted data permission. By Assumptions 1 and 2, we have

$$\pi_{S_2}^{R\ddagger} > \pi_{S_2}^{T\ddagger} \Leftrightarrow (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} > (\gamma-1)2^{\frac{\gamma}{\gamma-2}}(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3\gamma-1}{\gamma-2}}$$
$$\Leftrightarrow \gamma^{\gamma-1} > 2^{\gamma}.$$

Thus, the comparison between $\pi_{S_2}^{R\ddagger}$ and $\pi_{S_2}^{T\ddagger}$ can be characterized by threshold $\bar{\gamma}_3$, which is the unique solution to $\gamma^{\gamma-1} = 2^{\gamma}$ in the range of $\gamma > 2$, such that $\pi_{S_2}^{R\ddagger} < \pi_{S_2}^{T\ddagger}$ if $\gamma < \bar{\gamma}_3$; whereas $\pi_{S_2}^{R\ddagger} > \pi_{S_2}^{T\ddagger}$ if $\gamma > \bar{\gamma}_3$. □

Proof of Theorem 6. The theorem can be proved by comparing Theorem 5 to Proposition 6. □

Proof of Proposition C.1. We first derive the suppliers' optimal quality and transfer payment decisions. Given w_i and t_i, supplier $i \in \{1,2\}$ chooses q_i and t_{-i} to maximize $\pi_{S_i}(q_i, t_{-i}|w_i, t_i, q_{-i})$. For supplier i, the first-order conditions of $\pi_{S_i}(q_i, t_{-i}|w_i, t_i, q_{-i})$ are $\frac{\partial \pi_{S_i}(q_i,t_{-i}|w_i,t_i,q_{-i})}{\partial q_i}\Big|_{q_i=\tilde{q}_i(w_i,w_{-i}),t_{-i}=\tilde{t}_{-i}(w_i,w_{-i})} = (w_i - t_i + \tilde{t}_{-i}(w_i,w_{-i}))q_{-i} + t_i - \theta\gamma(\tilde{q}_i(w_i,w_{-i}))^{\gamma-1} = 0$ and $\frac{\partial \pi_{S_i}(q_i,t_{-i}|w_i,t_i,q_{-i})}{\partial t_{-i}}\Big|_{q_i=\tilde{q}_i(w_i,w_{-i}),t_{-i}=\tilde{t}_{-i}(w_i,w_{-i})} = -q_{-i}(1-\tilde{q}_i(w_i,w_{-i})) < 0$. Thus, $\tilde{t}_{-i}(w_i,w_{-i}) = 0$ always holds, and the suppliers' optimal transfer payment is $t_i^{R\dagger} = t_{-i}^{R\dagger} = 0$. Taking the second-order derivative of $\pi_{S_i}(q_i, t_{-i}|w_i, t_i, q_{-i})$ w.r.t. q_i yields $\frac{\partial^2 \pi_{S_i}(q_i,t_{-i}|w_i,t_i,q_{-i})}{\partial q_i^2} = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, the solution of the first-order condition is supplier i's optimal quality in response to w_i and t_i. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i, w_{-i}) = \left(\frac{w_i^{\gamma-1}w_{-i}}{\theta^{\gamma}\gamma^{\gamma}}\right)^{\frac{1}{\gamma(\gamma-2)}}. \tag{SC.5}$$

Note that (SC.5) is the same as (SB.1), and the buyer's contracting problem (C.2) is the same as (3) due to $t_i^{R\dagger} = t_{-i}^{R\dagger} = 0$. Hence, the rest of the proof follows from that of Proposition 3. □

Proof of Theorem C.1 and Proposition C.2. By comparing Proposition C.1 to Proposition 3, we can see that the equilibrium contracts and quality levels under restricted data permission are the same as those in the case without traceability. Thus, the equilibrium profits under restricted and unrestricted data permission are characterized in the proof of Theorem 4. Hence, the proof of the theorem and the proposition follows from that of Theorem 4. □

Proof of Theorem C.2. The theorem can be proved by comparing Theorem C.1 to Proposition C.2. □

SD. Limited Liability of Downstream Supplier: Proofs

Proof of Proposition E.1. We use backward induction to solve the game. Recall that the game consists of three stages. First, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in previous stages. Specifically, for supplier 1, the first-order condition of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ is $\frac{d\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1}\Big|_{q_1=\tilde{q}_1(w_1,w_2,q_2)} = (w_1 - w_2)q_2 + \min\{w_2, b\}q_2 - \theta\gamma(\tilde{q}_1(w_1,w_2,q_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ w.r.t. q_1 yields $\frac{d^2\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1^2} = -\theta\gamma(\gamma-1)q_1^{\gamma-2} < 0$. On the other hand, for supplier 2, the first-order condition of $\pi_{S_2}(q_2|w_2, q_1)$ is $\frac{d\pi_{S_2}(q_2|w_2,q_1)}{dq_2}\Big|_{q_2=\tilde{q}_2(w_2,q_1)} = w_2q_1 + \min\{w_2, b\}(1 - q_1) - \theta\gamma(\tilde{q}_2(w_2,q_1))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_2}(q_2|w_2, q_1)$ w.r.t. q_2 yields $\frac{d^2\pi_{S_2}(q_2|w_2,q_1)}{dq_2^2} = -\theta\gamma(\gamma-1)q_2^{\gamma-2} < 0$. Solving the suppliers' best response functions yields their optimal quality decisions in stage 3 as follows: (i) if $w_2 \leqslant b$,

$$\tilde{q}_1(w_1,w_2) = \left(\frac{w_1^{\gamma-1}w_2}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{(\gamma-1)^2}}, \tag{SD.1}$$

$$\tilde{q}_2(w_1,w_2) = \left(\frac{w_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}; \tag{SD.2}$$

(ii) if $w_2 > b$,

$$\tilde{q}_1(w_1,w_2) = \frac{\theta\gamma(\tilde{q}_2(w_1,w_2))^{\gamma-1} - b}{w_2 - b}, \tag{SD.3}$$

$$\tilde{q}_2(w_1,w_2) = \frac{\theta\gamma(\tilde{q}_1(w_1,w_2))^{\gamma-1}}{w_1 - w_2 + b}. \tag{SD.4}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1,w_2)|w_1, \tilde{q}_2(w_1,w_2))$, given w_1 decided in stage 1. Denote

$$\bar{b}(w_1) \equiv \left[\frac{w_1^{\gamma(\gamma-1)}}{\theta^\gamma\gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}.$$

Then, following the proof of Proposition 2, we can show that there are two possible cases in stage 2 equilibrium, depending on b: (i) if $b \geqslant \bar{b}(w_1)$, we have $\tilde{w}_2(w_1) = \left[\frac{w_1^{\gamma(\gamma-1)}}{\theta^\gamma\gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}$, and the following proof is the same as that of Proposition 2; (ii) if $b < \bar{b}(w_1)$, we cannot obtain the equilibrium due to limited tractability, and thus we resort to numerical studies for this case (note that when $b = 0$, the proof is the same as that of Proposition 1). □

Proof of Theorem E.1. The proposition follows immediately by comparing Proposition E.1 to Propositions 1 and 2. □

SE. Downstream Supplier's Use of Traceability Information Upon Receiving the Product: Proofs

Proof of Proposition F.1. We use backward induction to solve the game. Note that the game consists of three stages and the last two stages remain the same as in (2). Specifically, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, $\pi_{S_1}(w_2, q_1|w_1, q_2)$ and $\pi_{S_2}(q_2|w_2)$, given w_1 and w_2 decided in previous stages. In similar fashion to the proof of Proposition 2, we obtain the suppliers' optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1, w_2) = \left(\frac{w_1^{\gamma-1} w_2}{\theta^\gamma \gamma^\gamma}\right)^{\frac{1}{(\gamma-1)^2}}, \tag{SE.1}$$

$$\tilde{q}_2(w_2) = \left(\frac{w_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SE.2}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1, w_2)|w_1, \tilde{q}_2(w_2))$, given w_1 decided in stage 1. In similar fashion to the proof of Proposition 2, we obtain supplier 1's optimal contracting decision as follows:

$$\tilde{w}_2(w_1) = \left[\frac{w_1^{\gamma(\gamma-1)}}{\theta^\gamma \gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SE.3}$$

Then, plugging (SE.3) into (SE.1) and (SE.2), we have

$$\tilde{q}_1(w_1) = \left[\frac{w_1^\gamma}{\theta^\gamma \gamma^{\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SE.4}$$

$$\tilde{q}_2(w_1) = \left[\frac{w_1^\gamma}{\theta^\gamma \gamma^{2\gamma-1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SE.5}$$

Comparing (SE.4) and (SE.5) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = \gamma^{\frac{1}{\gamma}} > 1$.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, to maximize $\pi_B(w_1|\tilde{q}_1(w_1, \tilde{w}_2(w_1)), \tilde{q}_2(\tilde{w}_2(w_1)))$. Plugging (SE.3), (SE.4) and (SE.5) into (F.1), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1}\ \pi_B(w_1) &= (p+l)\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l\tilde{q}_2(w_1) - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1)\\ &= (p+l-w_1) w_1^{\frac{2}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} - l w_1^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma-1}{\gamma(\gamma-2)}}.\end{aligned}$$

We now analyze the buyer's optimal contracting decision. Taking the first-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d\pi_B(w_1)}{dw_1} = \frac{1}{\gamma-2} w_1^{\frac{4-\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} \left[2(p+l) - \gamma w_1 - l\left(\frac{1}{w_1}\right)^{\frac{1}{\gamma-2}} \theta^{\frac{1}{\gamma-2}} \gamma^{\frac{\gamma+1}{\gamma(\gamma-2)}}\right]. \tag{SE.6}$$

Solving (SE.6) yields the solution of the buyer's first-order condition, $w_1^{T\ddagger}$, that satisfies

$$2(p+l) - \gamma w_1^{T\ddagger} = l\left[\frac{\theta^\gamma \gamma^{\gamma+1}}{\left(w_1^{T\ddagger}\right)^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SE.7}$$

Then, in similar fashion to the proof of Proposition 2, we can show that $\left.\frac{d^2\pi_B(w_1)}{dw_1^2}\right|_{w_1^{T\ddagger}} < 0$ by Assumption 2. Thus, $w_1^{T\ddagger}$ is the buyer's optimal contracting decision.

Finally, plugging $w_1^{T\ddagger}$ into (SE.3), (SE.4) and (SE.5), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{T\ddagger} = \left[\frac{\left(w_1^{T\ddagger}\right)^{\gamma(\gamma-1)}}{\theta^\gamma \gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}$, $q_1^{T\ddagger} = \left[\frac{\left(w_1^{T\ddagger}\right)^{\gamma}}{\theta^\gamma \gamma^{\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}$, $q_2^{T\ddagger} = \left[\frac{\left(w_1^{T\ddagger}\right)^{\gamma}}{\theta^\gamma \gamma^{2\gamma-1}}\right]^{\frac{1}{\gamma(\gamma-2)}}$. By Assumptions 1 and 2, we have $q_i^{T\ddagger} \in (0,1)$ for $i \in \{1,2\}$. Moreover, we can show that $q_1^{T\ddagger}/q_2^{T\ddagger} = \gamma^{\frac{1}{\gamma}} > 1$. □

SF. An Assembly Supply Chain: Proofs

Proof of Proposition G.1. We first derive the suppliers' optimal quality decisions. Given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i, q_{-i})$. The first-order condition of $\pi_{S_i}(q_i|w_i, q_{-i})$ is $\left.\frac{d\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i}\right|_{q_i=\tilde{q}_i(w_i,q_{-i})} = w_i q_{-i} - C'(\tilde{q}_i(w_i,q_{-i})) = w_i q_{-i} - \theta\gamma(\tilde{q}_i(w_i,q_{-i}))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_i}(q_i|w_i,q_{-i})$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i^2} = -C''(q_i) = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, supplier i's optimal quality in response of w_i is $\tilde{q}_i(w_i,q_{-i}) = \left(\frac{w_i q_{-i}}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i, w_{-i}) = \left(\frac{w_i^{\gamma-1} w_{-i}}{\theta^\gamma \gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}. \tag{SF.1}$$

Next, consider the buyer's problem. Plugging (SF.1) into $\pi_{S_i}(q_i|w_i,q_{-i})$, we have

$$\begin{aligned}
\pi_{S_i}(\tilde{q}_i(w_i,w_{-i})|w_i,\tilde{q}_{-i}(w_i,w_{-i})) &= w_i\tilde{q}_i(w_i,w_{-i})\tilde{q}_{-i}(w_i,w_{-i}) - \theta(\tilde{q}_i(w_i,w_{-i}))^\gamma \\
&= w_i\left(\frac{w_i^{\gamma-1}w_{-i}}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}\left(\frac{w_{-i}^{\gamma-1}w_i}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}} - \theta\left[\left(\frac{w_i^{\gamma-1}w_{-i}}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}\right]^\gamma \\
&= (\gamma-1)w_i^{\frac{\gamma-1}{\gamma-2}} w_{-i}^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} \geqslant 0,
\end{aligned}$$

for any $w_i \geqslant 0$ and $w_{-i} \geqslant 0$. Thus, IR_i is always satisfied. Then, plugging (SF.1) into (G.1), the buyer's problem becomes

$$\begin{aligned}
\max_{w_1,w_2}\ \pi_B(w_1,w_2) &= (p+l)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l - (w_1+w_2)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) \\
&= (p+l-w_1-w_2)\left(\frac{w_1^{\gamma-1}w_2}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}\left(\frac{w_2^{\gamma-1}w_1}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}} - l \\
&= (p+l-w_1-w_2)w_1^{\frac{1}{\gamma-2}}w_2^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} - l.
\end{aligned}$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1, w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial \pi_B(w_1,w_2)}{\partial w_1} = w_1^{\frac{3-\gamma}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l-w_1-w_2}{\gamma-2} - w_1\right], \tag{SF.2}$$

$$\frac{\partial \pi_B(w_1,w_2)}{\partial w_2} = w_1^{\frac{1}{\gamma-2}} w_2^{\frac{3-\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l-w_1-w_2}{\gamma-2} - w_2\right]. \tag{SF.3}$$

Solving (SF.2) and (SF.3) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{N\star} = w_2^{N\star} = \frac{p+l}{\gamma}. \tag{SF.4}$$

Then, we need to show that $(w_1^{N\star}, w_2^{N\star})$ are the buyer's optimal contracting decisions. Taking the second-order derivatives of $\pi_B(w_1, w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1^2} = \frac{1}{\gamma-2} w_1^{\frac{5-2\gamma}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2} - 2w_1\right],$$

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_2^2} = \frac{1}{\gamma-2} w_1^{\frac{1}{\gamma-2}} w_2^{\frac{5-2\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2} - 2w_2\right],$$

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1 \partial w_2} = \frac{1}{\gamma-2} w_1^{\frac{3-\gamma}{\gamma-2}} w_2^{\frac{3-\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l-w_1-w_2}{\gamma-2} - w_1 - w_2\right].$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1, w_2)$ is negative definite in the neighborhood of $(w_1^{N\star}, w_2^{N\star})$. Thus, $(w_1^{N\star}, w_2^{N\star})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{N\star}, w_2^{N\star})$ into (SF.1), we obtain the suppliers' optimal quality decisions: $q_1^{N\star} = q_2^{N\star} = \left(\frac{p+l}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{N\star} \in (0,1)$. □

Proof of Proposition G.2. By comparing (G.2) and (C.1), we can see that the model formulation for the assembly supply chain case with traceability is equivalent to that for the serial supply chain case under restricted data permission. Thus, the proof of this proposition follows from that of Proposition 5. □

Proof of Theorem G.1. Consider the case without traceability. Based on the equilibrium characterized in Proposition G.1, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{N\star} = (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l,$$

$$\pi_{S_i}^{N\star} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}},$$

$$\pi_{SC}^{N\star} = (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l.$$

Consider the case with traceability. Based on the equilibrium characterized in Proposition G.2, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{T\star} = (\gamma-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l,$$

$$\pi_{S_i}^{T\star} = (\gamma-1)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}},$$

$$\pi_{SC}^{T\star} = (\gamma^2-2)(p+l)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l.$$

The theorem can be proved by comparing the equilibrium wholesale prices, the equilibrium quality levels, and the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain for the cases with and without traceability. □

SG. Buyer's Product Inspection: Proofs

Proof of Proposition H.1. We use backward induction to solve the game. Note that the game consists of three stages and the last two stages remain the same as in (1). Specifically, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, $\pi_{S_1}(w_2, q_1|w_1, q_2)$ and $\pi_{S_2}(q_2|w_2, q_1)$, given w_1 and w_2 decided in previous stages. In similar fashion to the proof of Proposition 1, we obtain the suppliers' optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1, w_2) = \left[\frac{(w_1-w_2)^{\gamma-1}w_2}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SG.1}$$

$$\tilde{q}_2(w_1, w_2) = \left[\frac{(w_1-w_2)w_2^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SG.2}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1, w_2)|w_1, \tilde{q}_2(w_1, w_2))$, given w_1 decided in stage 1. In similar fashion to the proof of Proposition 1, we obtain supplier 1's optimal contracting decision as follows:

$$\tilde{w}_2(w_1) = \frac{w_1}{\gamma}. \tag{SG.3}$$

Then, plugging (SG.3) into (SG.1) and (SG.2), we have

$$\tilde{q}_1(w_1) = \left[\frac{(\gamma-1)^{\gamma-1}w_1^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SG.4}$$

$$\tilde{q}_2(w_1) = \left[\frac{(\gamma-1)w_1^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SG.5}$$

Comparing (SG.4) and (SG.5) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, and the inspection level β to maximize $\pi_B(w_1,\beta|\tilde{q}_1(w_1,\tilde{w}_2(w_1)),\tilde{q}_2(w_1,\tilde{w}_2(w_1)))$. Plugging (SG.3), (SG.4) and (SG.5) into (H.1), we have the buyer's problem as follows:

$$\begin{aligned}\max_{w_1,\beta}\ \pi_B(w_1,\beta) &= \big[p+l(1-\beta)\big]\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l(1-\beta) - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1) - I(\beta)\\ &= \big[p+l(1-\beta)-w_1\big]\left[\frac{(\gamma-1)^{\gamma-1}w_1^{\gamma}}{\theta^{\gamma}\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(\gamma-1)w_1^{\gamma}}{\theta^{\gamma}\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}} - l(1-\beta) - \frac{1}{2}\mu\beta^2\\ &= \big[p+l(1-\beta)-w_1\big](\gamma-1)^{\frac{1}{\gamma-2}}w_1^{\frac{2}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} - l(1-\beta) - \frac{1}{2}\mu\beta^2.\end{aligned}$$

We now analyze the buyer's optimal contracting decision and inspection level. Taking the first-order derivatives of $\pi_B(w_1,\beta)$ w.r.t. w_1 and β yields

$$\frac{\partial\pi_B(w_1,\beta)}{\partial w_1} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}}w_1^{\frac{4-\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}}\Big[2\big[p+l(1-\beta)\big]-\gamma w_1\Big], \tag{SG.6}$$

$$\frac{\partial\pi_B(w_1,\beta)}{\partial\beta} = -l(\gamma-1)^{\frac{1}{\gamma-2}}w_1^{\frac{2}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} + l - \mu\beta. \tag{SG.7}$$

Solving (SG.6) and (SG.7) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{N\ddagger} = \frac{2\left[p+l\left(1-\beta^{N\ddagger}\right)\right]}{\gamma}, \tag{SG.8}$$

and $\beta^{N\ddagger}$ satisfies

$$(\gamma-1)^{\frac{1}{\gamma-2}}\left[\frac{2\left[p+l\left(1-\beta^{N\ddagger}\right)\right]}{\theta\gamma^3}\right]^{\frac{2}{\gamma-2}} = 1-\frac{\mu\beta^{N\ddagger}}{l}. \tag{SG.9}$$

Then, we need to show that $w_1^{N\ddagger}$ and $\beta^{N\ddagger}$ are the buyer's optimal decisions. Taking the second-order derivative of $\pi_B(w_1,\beta)$ w.r.t. β yields $\frac{\partial^2\pi_B(w_1,\beta)}{\partial\beta^2} = -\mu < 0$. Thus, $\beta^{N\ddagger}$ is the buyer's optimal inspection level. Then, in similar fashion to the proof of Proposition 1, if the stationary point characterized in (SG.8) is a strict local maximum, then $w_1^{N\ddagger}$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_B(w_1,\beta)$ w.r.t. w_1 yields

$$\frac{\partial^2\pi_B(w_1,\beta)}{\partial w_1^2} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}}w_1^{\frac{6-2\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}}\left[\left(\frac{4-\gamma}{\gamma-2}\right)\Big[2\big[p+l(1-\beta)\big]-\gamma w_1\Big]-\gamma w_1\right].$$

By Assumption 2, we can show that $\left.\frac{\partial^2\pi_B(w_1,\beta)}{\partial w_1^2}\right|_{(w_1^{N\ddagger},\beta^{N\ddagger})} < 0$. Thus, $w_1^{N\ddagger}$ is the buyer's optimal contracting decision.

Finally, plugging $w_1^{N\ddagger}$ and $\beta^{N\ddagger}$ into (SG.3), (SG.4) and (SG.5), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{N\ddagger} = \frac{2\left[p+l\left(1-\beta^{N\ddagger}\right)\right]}{\gamma^2}$, $q_1^{N\ddagger} = \left[\frac{2\left[p+l\left(1-\beta^{N\ddagger}\right)\right](\gamma-1)^{\frac{\gamma-1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$, $q_2^{N\ddagger} = \left[\frac{2\left[p+l\left(1-\beta^{N\ddagger}\right)\right](\gamma-1)^{\frac{1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{N\ddagger}\in(0,1)$ for $i\in\{1,2\}$. Moreover, we can show that $w_1^{N\ddagger}/w_2^{N\ddagger} = \gamma \geqslant 2$ and $q_1^{N\ddagger}/q_2^{N\ddagger} = (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$. □

Proof of Proposition H.2. We use backward induction to solve the game. Note that the game consists of three stages and the last two stages remain the same as in (2). Specifically, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, $\pi_{S_1}(w_2, q_1|w_1, q_2)$ and $\pi_{S_2}(q_2|w_2)$, given w_1 and w_2 decided in previous stages. In similar fashion to the proof of Proposition 2, we obtain the suppliers' optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1, w_2) = \left(\frac{w_1^{\gamma-1} w_2}{\theta^\gamma \gamma^\gamma}\right)^{\frac{1}{(\gamma-1)^2}}, \tag{SG.10}$$

$$\tilde{q}_2(w_2) = \left(\frac{w_2}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SG.11}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1, w_2)|w_1, \tilde{q}_2(w_2))$, given w_1 decided in stage 1. In similar fashion to the proof of Proposition 2, we obtain supplier 1's optimal contracting decision as follows:

$$\tilde{w}_2(w_1) = \left[\frac{w_1^{\gamma(\gamma-1)}}{\theta^\gamma \gamma^{\gamma^2-\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SG.12}$$

Then, plugging (SG.12) into (SG.10) and (SG.11), we have

$$\tilde{q}_1(w_1) = \left[\frac{w_1^{\gamma}}{\theta^\gamma \gamma^{\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SG.13}$$

$$\tilde{q}_2(w_1) = \left[\frac{w_1^{\gamma}}{\theta^\gamma \gamma^{2\gamma-1}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SG.14}$$

Comparing (SG.13) and (SG.14) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = \gamma^{\frac{1}{\gamma}} > 1$.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, and the inspection level β to maximize $\pi_B(w_1, \beta|\tilde{q}_1(w_1, \tilde{w}_2(w_1)), \tilde{q}_2(\tilde{w}_2(w_1)))$. Plugging (SG.12), (SG.13) and (SG.14) into (H.2), we have the buyer's problem as follows:

$$\begin{aligned}
\max_{w_1,\beta}\ \pi_B(w_1,\beta) &= \big[p + l(1-\beta)\big]\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l(1-\beta) - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1) - I(\beta) \\
&= \big[p + l(1-\beta) - w_1\big]\left[\frac{w_1^{\gamma}}{\theta^\gamma \gamma^{\gamma+1}}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{w_1^{\gamma}}{\theta^\gamma \gamma^{2\gamma-1}}\right]^{\frac{1}{\gamma(\gamma-2)}} - l(1-\beta) - \frac{1}{2}\mu\beta^2 \\
&= \big[p + l(1-\beta) - w_1\big] w_1^{\frac{2}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} - l(1-\beta) - \frac{1}{2}\mu\beta^2.
\end{aligned}$$

We now analyze the buyer's optimal contracting decision and inspection level. Taking the first-order derivatives of $\pi_B(w_1, \beta)$ w.r.t. w_1 and β yields

$$\frac{\partial \pi_B(w_1,\beta)}{\partial w_1} = \frac{1}{\gamma-2} w_1^{\frac{4-\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}}\Big[2\big[p + l(1-\beta)\big] - \gamma w_1\Big], \tag{SG.15}$$

$$\frac{\partial \pi_B(w_1,\beta)}{\partial \beta} = -l w_1^{\frac{2}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} + l - \mu\beta. \tag{SG.16}$$

Solving (SG.15) and (SG.16) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{T\ddagger} = \frac{2\left[p + l\left(1-\beta^{T\ddagger}\right)\right]}{\gamma}, \tag{SG.17}$$

and $\beta^{T\ddagger}$ satisfies

$$\left[\frac{2\left[p + l\left(1-\beta^{T\ddagger}\right)\right]}{\theta\gamma^{\frac{5}{2}}}\right]^{\frac{2}{\gamma-2}} = 1 - \frac{\mu\beta^{T\ddagger}}{l}. \tag{SG.18}$$

Then, we need to show that $w_1^{T\ddagger}$ and $\beta^{T\ddagger}$ are the buyer's optimal decisions. Taking the second-order derivative of $\pi_B(w_1,\beta)$ w.r.t. β yields $\frac{\partial^2 \pi_B(w_1,\beta)}{\partial \beta^2} = -\mu < 0$. Thus, $\beta^{T\ddagger}$ is the buyer's optimal inspection level. Then, in similar fashion to the proof of Proposition 2, if the stationary point characterized in (SG.17) is a strict local maximum, then $w_1^{T\ddagger}$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_B(w_1,\beta)$ w.r.t. w_1 yields

$$\frac{\partial^2 \pi_B(w_1,\beta)}{\partial w_1^2} = \frac{1}{\gamma-2} w_1^{\frac{6-2\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{3}{\gamma-2}} \left[\left(\frac{4-\gamma}{\gamma-2}\right)\left[2\big[p + l(1-\beta)\big] - \gamma w_1\right] - \gamma w_1\right].$$

By Assumption 2, we can show that $\left.\frac{\partial^2 \pi_B(w_1,\beta)}{\partial w_1^2}\right|_{(w_1^{T\ddagger},\beta^{T\ddagger})} < 0$. Thus, $w_1^{T\ddagger}$ is the buyer's optimal contracting decision.

Finally, plugging $w_1^{T\ddagger}$ into (SG.12), (SG.13) and (SG.14), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{T\ddagger} = \left[2\left[p + l\left(1-\beta^{T\ddagger}\right)\right]\right]^{\frac{\gamma-1}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma^2-2\gamma+1}{\gamma(\gamma-2)}}$, $q_1^{T\ddagger} = \left[\frac{2\left[p+l\left(1-\beta^{T\ddagger}\right)\right]}{\theta\gamma^{2+\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}}$, $q_2^{T\ddagger} = \left[\frac{2\left[p+l\left(1-\beta^{T\ddagger}\right)\right]}{\theta\gamma^{3-\frac{1}{\gamma}}}\right]^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{T\ddagger} \in (0,1)$ for $i \in \{1,2\}$. Moreover, we can show that $w_1^{T\ddagger}/w_2^{T\ddagger} = \gamma/q_1^{T\ddagger} > \gamma$ and $q_1^{T\ddagger}/q_2^{T\ddagger} = \gamma^{\frac{1}{\gamma}} > 1$. □

Proof of Proposition H.3. We use backward induction to solve the game. Note that the game consists of two stages and the last stage remains the same as in (3). Specifically, given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i,q_{-i})$. In similar fashion to the proof of Proposition 3, we obtain the suppliers' optimal quality decisions:

$$\tilde{q}_i(w_i,w_{-i}) = \left(\frac{w_i^{\gamma-1} w_{-i}}{\theta^\gamma \gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}}. \tag{SG.19}$$

Next, consider the buyer's problem. Plugging (SG.19) into (H.3), we have the buyer's problem as follows:

$$\begin{aligned}
\max_{w_1,w_2,\beta} \ \pi_B(w_1,w_2,\beta) &= \big[p + l(1-\beta)\big]\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l(1-\beta) - (w_1+w_2)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - I(\beta) \\
&= \big[p + l(1-\beta) - w_1 - w_2\big]\left(\frac{w_1^{\gamma-1} w_2}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}} \left(\frac{w_2^{\gamma-1} w_1}{\theta^\gamma\gamma^\gamma}\right)^{\frac{1}{\gamma(\gamma-2)}} - l(1-\beta) - \frac{1}{2}\mu\beta^2 \\
&= \big[p + l(1-\beta) - w_1 - w_2\big] w_1^{\frac{1}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} - l(1-\beta) - \frac{1}{2}\mu\beta^2.
\end{aligned}$$

We now analyze the buyer's optimal contracting decisions and inspection level. Taking the first-order derivatives of $\pi_B(w_1,w_2,\beta)$ w.r.t. w_1, w_2 and β yields

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial w_1} = w_1^{\frac{3-\gamma}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l(1-\beta)-w_1-w_2}{\gamma-2} - w_1\right], \tag{SG.20}$$

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial w_2} = w_1^{\frac{1}{\gamma-2}} w_2^{\frac{3-\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l(1-\beta)-w_1-w_2}{\gamma-2} - w_2\right], \tag{SG.21}$$

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial \beta} = -l w_1^{\frac{1}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} + l - \mu\beta. \tag{SG.22}$$

Solving (SG.20), (SG.21) and (SG.22) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{N\dagger} = w_2^{N\dagger} = \frac{p + l\left(1-\beta^{N\dagger}\right)}{\gamma}, \tag{SG.23}$$

and $\beta^{N\dagger}$ satisfies

$$\left[\frac{p + l\left(1-\beta^{N\dagger}\right)}{\theta\gamma^2}\right]^{\frac{2}{\gamma-2}} = 1 - \frac{\mu\beta^{N\dagger}}{l}. \tag{SG.24}$$

Then, we need to show that $(w_1^{N\dagger}, w_2^{N\dagger})$ and $\beta^{N\dagger}$ are the buyer's optimal decisions. Taking the second-order derivative of $\pi_B(w_1,w_2,\beta)$ w.r.t. β yields $\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial \beta^2} = -\mu < 0$. Thus, $\beta^{N\dagger}$ is the buyer's optimal inspection level. Then, in similar fashion to the proof of Proposition 3, if the stationary point characterized in (SG.23) is a strict local maximum, then $(w_1^{N\dagger}, w_2^{N\dagger})$ must be the unique global maximum, proved by contradiction. Taking the second-order derivatives of $\pi_B(w_1,w_2,\beta)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_1^2} = \frac{1}{\gamma-2} w_1^{\frac{5-2\gamma}{\gamma-2}} w_2^{\frac{1}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{(3-\gamma)\left[p+l(1-\beta)-w_1-w_2\right]}{\gamma-2} - 2w_1\right],$$

$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_2^2} = \frac{1}{\gamma-2} w_1^{\frac{1}{\gamma-2}} w_2^{\frac{5-2\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{(3-\gamma)\left[p+l(1-\beta)-w_1-w_2\right]}{\gamma-2} - 2w_2\right],$$

$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_1 \partial w_2} = \frac{1}{\gamma-2} w_1^{\frac{3-\gamma}{\gamma-2}} w_2^{\frac{3-\gamma}{\gamma-2}} \left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} \left[\frac{p+l(1-\beta)-w_1-w_2}{\gamma-2} - w_1 - w_2\right].$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1,w_2,\beta)$ is negative definite in the neighborhood of $(w_1^{N\dagger}, w_2^{N\dagger})$. Thus, $(w_1^{N\dagger}, w_2^{N\dagger})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{N\dagger}, w_2^{N\dagger})$ and $\beta^{N\dagger}$ into (SG.19), we obtain the suppliers' optimal quality decisions: $q_1^{N\dagger} = q_2^{N\dagger} = \left[\frac{p+l\left(1-\beta^{N\dagger}\right)}{\theta\gamma^2}\right]^{\frac{1}{\gamma-2}}$. By Assumptions 1 and 2, we have $q_i^{N\dagger} \in (0,1)$ for $i \in \{1,2\}$. □

Proof of Proposition H.4. We use backward induction to solve the game. Note that the game consists of two stages and the last stage remains the same as in (4). Specifically, given w_i, supplier

$i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i)$. In similar fashion to the proof of Proposition 4, we obtain the suppliers' optimal quality decisions:

$$\tilde{q}_i(w_i) = \left(\frac{w_i}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SG.25}$$

Next, consider the buyer's problem. Plugging (SG.25) into (H.4), we have the buyer's problem as follows:

$$\begin{aligned}
\max_{w_1,w_2}\ \pi_B(w_1,w_2,\beta) &= \frac{1}{2}p\Big[\tilde{q}_1(w_1)+\tilde{q}_2(w_2)\Big] - \frac{1}{2}l\Big[1-\tilde{q}_1(w_1)\Big](1-\beta) - \frac{1}{2}l\Big[1-\tilde{q}_2(w_2)\Big](1-\beta) \\
&\quad - w_1\tilde{q}_1(w_1) - w_2\tilde{q}_2(w_2) - I(\beta) \\
&= \frac{1}{2}\big[p+l(1-\beta)\big]\Big[\tilde{q}_1(w_1)+\tilde{q}_2(w_2)\Big] - l(1-\beta) - w_1\tilde{q}_1(w_1) - w_2\tilde{q}_2(w_2) - I(\beta) \\
&= \frac{1}{2}\big[p+l(1-\beta)\big]\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[w_1^{\frac{1}{\gamma-1}} + w_2^{\frac{1}{\gamma-1}}\right] - l(1-\beta) - w_1^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} \\
&\quad - w_2^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - \frac{1}{2}\mu\beta^2.
\end{aligned}$$

We now analyze the buyer's optimal contracting decisions and inspection level. Taking the first-order derivatives of $\pi_B(w_1,w_2,\beta)$ w.r.t. w_1, w_2 and β yields

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial w_1} = \frac{1}{\gamma-1} w_1^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{p+l(1-\beta)}{2} - \gamma w_1\right], \tag{SG.26}$$

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial w_2} = \frac{1}{\gamma-1} w_2^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{p+l(1-\beta)}{2} - \gamma w_2\right], \tag{SG.27}$$

$$\frac{\partial \pi_B(w_1,w_2,\beta)}{\partial \beta} = -\frac{1}{2}l\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[w_1^{\frac{1}{\gamma-1}} + w_2^{\frac{1}{\gamma-1}}\right] + l - \mu\beta. \tag{SG.28}$$

Solving (SG.26), (SG.27) and (SG.28) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{T\dagger} = w_2^{T\dagger} = \frac{p + l\,(1-\beta^{T\dagger})}{2\gamma}, \tag{SG.29}$$

and $\beta^{T\dagger}$ satisfies

$$\left[\frac{p + l\,(1-\beta^{T\dagger})}{2\theta\gamma^2}\right]^{\frac{1}{\gamma-1}} = 1 - \frac{\mu\beta^{T\dagger}}{l}. \tag{SG.30}$$

Then, we need to show that $(w_1^{T\dagger}, w_2^{T\dagger})$ and $\beta^{T\dagger}$ are the buyer's optimal decisions. Taking the second-order derivative of $\pi_B(w_1,w_2,\beta)$ w.r.t. β yields $\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial \beta^2} = -\mu < 0$. Thus, $\beta^{T\dagger}$ is the buyer's optimal inspection level. Then, in similar fashion to the proof of Proposition 4, if the stationary point characterized in (SG.29) is a strict local maximum, then $(w_1^{T\dagger}, w_2^{T\dagger})$ must be the unique global maximum, proved by contradiction. Taking the second-order derivatives of $\pi_B(w_1,w_2,\beta)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_1^2} = \frac{1}{\gamma-1} w_1^{\frac{3-2\gamma}{\gamma-1}} \left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} \left[\frac{(2-\gamma)\big[p+l(1-\beta)-2\gamma w_1\big]}{2(\gamma-1)} - \gamma w_1\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_2^2} = \frac{1}{\gamma-1} w_2^{\frac{3-2\gamma}{\gamma-1}} \left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} \left[\frac{(2-\gamma)\big[p+l(1-\beta)-2\gamma w_2\big]}{2(\gamma-1)} - \gamma w_2\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2,\beta)}{\partial w_1 \partial w_2} = 0.$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1,w_2,\beta)$ is negative definite in the neighborhood of $(w_1^{T\dagger}, w_2^{T\dagger})$. Thus, $(w_1^{T\dagger}, w_2^{T\dagger})$ are the buyer's optimal contracting decisions.

Finally, plugging $(w_1^{T\dagger}, w_2^{T\dagger})$ into (SG.25), we obtain the suppliers' optimal quality decisions: $q_1^{T\dagger} = q_2^{T\dagger} = \left[\frac{p+l\left(1-\beta^{T\dagger}\right)}{2\theta\gamma^2}\right]^{\frac{1}{\gamma-1}}$. By Assumptions 1 and 2, we have $q_i^{T\dagger} \in (0,1)$ for $i \in \{1,2\}$. $\square$

SH. Suppliers' Exogenous Loss: Proofs

Proof of Proposition I.1. We use backward induction to solve the game. Recall that the game consists of three stages. First, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in previous stages. Specifically, for supplier 1, the first-order condition of $\pi_{S_1}(w_2,q_1|w_1,q_2)$ is $\frac{d\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1}\Big|_{q_1=\tilde{q}_1(w_1,w_2,q_2)} = (w_1-w_2+l_s)q_2 - C'(\tilde{q}_1(w_1,w_2,q_2)) = (w_1-w_2+l_s)q_2 - \theta\gamma(\tilde{q}_1(w_1,w_2,q_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_1}(w_2,q_1|w_1,q_2)$ w.r.t. q_1 yields $\frac{d^2\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1^2} = -C''(q_1) = -\theta\gamma(\gamma-1)q_1^{\gamma-2} < 0$. Thereby, supplier 1's optimal quality in response to w_1 and w_2 is $\tilde{q}_1(w_1,w_2,q_2) = \left[\frac{(w_1-w_2+l_s)q_2}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. On the other hand, for supplier 2, the first-order condition of $\pi_{S_2}(q_2|w_2,q_1)$ is $\frac{d\pi_{S_2}(q_2|w_2,q_1)}{dq_2}\Big|_{q_2=\tilde{q}_2(w_2,q_1)} = (w_2+l_s)q_1 - C'(\tilde{q}_2(w_2,q_1)) = (w_2+l_s)q_1 - \theta\gamma(\tilde{q}_2(w_2,q_1))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_2}(q_2|w_2,q_1)$ w.r.t. q_2 yields $\frac{d^2\pi_{S_2}(q_2|w_2,q_1)}{dq_2^2} = -C''(q_2) = -\theta\gamma(\gamma-1)q_2^{\gamma-2} < 0$. Thereby, supplier 2's optimal quality in response to w_2 is $\tilde{q}_2(w_2,q_1) = \left[\frac{(w_2+l_s)q_1}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1,w_2) = \left[\frac{(w_1-w_2+l_s)^{\gamma-1}(w_2+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}, \tag{SH.1}$$
$$\tilde{q}_2(w_1,w_2) = \left[\frac{(w_1-w_2+l_s)(w_2+l_s)^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SH.2}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2,\tilde{q}_1(w_1,w_2)|w_1,\tilde{q}_2(w_1,w_2))$, given w_1 decided in stage 1. Plugging (SH.1) and (SH.2) into $\pi_{S_2}(q_2|w_2,q_1)$, we have

$$\pi_{S_2}(\tilde{q}_2(w_1,w_2)|w_2,\tilde{q}_1(w_1,w_2)) = (w_2+l_s)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l_s - \theta(\tilde{q}_2(w_1,w_2))^\gamma$$
$$= (w_2+l_s)\left[\frac{(w_1-w_2+l_s)^{\gamma-1}(w_2+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(w_1-w_2+l_s)(w_2+l_s)^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}$$
$$- l_s - \theta\left[\left[\frac{(w_1-w_2+l_s)(w_2+l_s)^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\right]^\gamma$$
$$= (\gamma-1)(w_1-w_2+l_s)^{\frac{1}{\gamma-2}}(w_2+l_s)^{\frac{\gamma-1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} - l_s.$$

Then, plugging (SH.1) and (SH.2) into (I.1), we have supplier 1's problem as follows:

$$\max_{w_2}\ \pi_{S_1}(w_2|w_1) = (w_1+l_s)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l_s - \theta(\tilde{q}_1(w_1,w_2))^\gamma - w_2\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2)$$
$$= (w_1-w_2+l_s)\left[\frac{(w_1-w_2+l_s)^{\gamma-1}(w_2+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(w_1-w_2+l_s)(w_2+l_s)^{\gamma-1}}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}$$
$$- l_s - \theta\left[\left[\frac{(w_1-w_2+l_s)^{\gamma-1}(w_2+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\right]^\gamma$$
$$= (\gamma-1)(w_1-w_2+l_s)^{\frac{\gamma-1}{\gamma-2}}(w_2+l_s)^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} - l_s.$$

We now analyze supplier 1's optimal contracting decision. Taking the first-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\frac{d\pi_{S_1}(w_2|w_1)}{dw_2} = \frac{\gamma-1}{\gamma-2}(w_1-w_2+l_s)^{\frac{1}{\gamma-2}}(w_2+l_s)^{\frac{3-\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}}\Big[w_1-\gamma w_2-(\gamma-2)l_s\Big]. \quad \text{(SH.3)}$$

Solving (SH.3) yields the solution of supplier 1's first-order condition as follows:

$$\tilde{w}_2(w_1) = \frac{w_1-(\gamma-2)l_s}{\gamma}. \quad \text{(SH.4)}$$

Then, we need to show that $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. In similar fashion to the proof of Proposition 1, if the stationary point characterized in (SH.4) is a strict local maximum, then $\tilde{w}_2(w_1)$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2} = \frac{\gamma-1}{\gamma-2}(w_1-w_2+l_s)^{\frac{3-\gamma}{\gamma-2}}(w_2+l_s)^{\frac{5-2\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}}\Big[-\gamma(w_1-w_2+l_s)(w_2+l_s)$$
$$+\frac{3-\gamma}{\gamma-2}\big[w_1-\gamma w_2-(\gamma-2)l_s\big](w_1-w_2+l_s) - \frac{1}{\gamma-2}\big[w_1-\gamma w_2-(\gamma-2)l_s\big](w_2+l_s)\Big].$$

By Assumption 2 and $\tilde{w}_2(w_1) < w_1$, we can show that $\frac{d^2\pi_{S_1}(w_2|w_1)}{dw_2^2}\Big|_{\tilde{w}_2(w_1)} < 0$. Thus, $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. Then, plugging (SH.4) into (SH.1) and (SH.2), we have

$$\tilde{q}_1(w_1) = \left[\frac{(\gamma-1)^{\gamma-1}(w_1+2l_s)^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}, \quad \text{(SH.5)}$$

$$\tilde{q}_2(w_1) = \left[\frac{(\gamma-1)(w_1+2l_s)^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SH.6}$$

Comparing (SH.5) and (SH.6) yields $\frac{\tilde{q}_1(w_1)}{\tilde{q}_2(w_1)} = (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$. Besides, we can show that IR_2 is satisfied when l_s is sufficiently small.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, to maximize $\pi_B(w_1|\tilde{q}_1(w_1,\tilde{w}_2(w_1)),\tilde{q}_2(w_1,\tilde{w}_2(w_1)))$. Plugging (SH.4) into $\pi_{S_1}(w_2|w_1)$, we have

$$\begin{aligned}
\pi_{S_1}(\tilde{w}_2(w_1)|w_1) &= (\gamma-1)\Big[w_1-\tilde{w}_2(w_1)+l_s\Big]^{\frac{\gamma-1}{\gamma-2}}\Big[\tilde{w}_2(w_1)+l_s\Big]^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} - l_s \\
&= (\gamma-1)\left[w_1 - \frac{w_1-(\gamma-2)l_s}{\gamma}+l_s\right]^{\frac{\gamma-1}{\gamma-2}}\left[\frac{w_1-(\gamma-2)l_s}{\gamma}+l_s\right]^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} - l_s \\
&= (\gamma-1)^{\frac{2\gamma-3}{\gamma-2}}(w_1+2l_s)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l_s.
\end{aligned}$$

Then, plugging (SH.4), (SH.5) and (SH.6) into (I.1), we have the buyer's problem as follows:

$$\begin{aligned}
\max_{w_1}\ \pi_B(w_1) &= (p+l)\tilde{q}_1(w_1)\tilde{q}_2(w_1) - l - w_1\tilde{q}_1(w_1)\tilde{q}_2(w_1) \\
&= (p+l-w_1)\left[\frac{(\gamma-1)^{\gamma-1}(w_1+2l_s)^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(\gamma-1)(w_1+2l_s)^\gamma}{\theta^\gamma\gamma^{2\gamma}}\right]^{\frac{1}{\gamma(\gamma-2)}} - l \\
&= (p+l-w_1)(\gamma-1)^{\frac{1}{\gamma-2}}(w_1+2l_s)^{\frac{2}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}} - l.
\end{aligned}$$

We now analyze the buyer's optimal contracting decision. Taking the first-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d\pi_B(w_1)}{dw_1} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}}(w_1+2l_s)^{\frac{4-\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}}\Big[2(p+l)-\gamma w_1-2(\gamma-2)l_s\Big]. \tag{SH.7}$$

Solving (SH.7) yields the solution of the buyer's first-order condition as follows:

$$w_1^{N\ddagger} = \frac{2\big[p+l-(\gamma-2)l_s\big]}{\gamma}. \tag{SH.8}$$

Then, we need to show that $w_1^{N\ddagger}$ is the buyer's optimal contracting decision. Similar to the previous proof, if the stationary point characterized in (SH.8) is a strict local maximum, then $w_1^{N\ddagger}$ must be the unique global maximum, proved by contradiction. Taking the second-order derivative of $\pi_B(w_1)$ w.r.t. w_1 yields

$$\frac{d^2\pi_B(w_1)}{dw_1^2} = \frac{1}{\gamma-2}(\gamma-1)^{\frac{1}{\gamma-2}}(w_1+2l_s)^{\frac{6-2\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{4}{\gamma-2}}\left[\left(\frac{4-\gamma}{\gamma-2}\right)\Big[2(p+l)-\gamma w_1-2(\gamma-2)l_s\Big]-\gamma w_1\right].$$

By Assumption 2, we can show that $\frac{d^2\pi_B(w_1)}{dw_1^2}\Big|_{w_1^{N\ddagger}} < 0$. Thus, $w_1^{N\ddagger}$ is the buyer's optimal contracting decision. Besides, we can show that IR_1 is satisfied when l_s is sufficiently small.

Finally, plugging $w_1^{N\ddagger}$ into (SH.4), (SH.5) and (SH.6), we obtain the suppliers' equilibrium quality and contracting decisions: $w_2^{N\ddagger} = \frac{2(p+l)-(\gamma-2)(\gamma+2)l_s}{\gamma^2}$, $q_1^{N\ddagger} = \left[\frac{2(p+l+2l_s)(\gamma-1)^{\frac{\gamma-1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$, $q_2^{N\ddagger} = \left[\frac{2(p+l+2l_s)(\gamma-1)^{\frac{1}{\gamma}}}{\theta\gamma^3}\right]^{\frac{1}{\gamma-2}}$. By Assumptions I.1 and 2, we have $q_i^{N\ddagger} \in (0,1)$ for $i \in \{1,2\}$. □

Proof of Proposition I.2. We use backward induction to solve the game. Recall that the game consists of three stages. First, in stage 3, suppliers 1 and 2 simultaneously choose q_1 and q_2 to maximize their own expected profits, given w_1 and w_2 decided in previous stages. Specifically, for supplier 1, the first-order condition of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ is $\frac{d\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1}\Big|_{q_1=\tilde{q}_1(w_1,q_2)} = (w_1 + l_s)q_2 - C'(\tilde{q}_1(w_1,q_2)) = (w_1 + l_s)q_2 - \theta\gamma(\tilde{q}_1(w_1,q_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_1}(w_2, q_1|w_1, q_2)$ w.r.t. q_1 yields $\frac{d^2\pi_{S_1}(w_2,q_1|w_1,q_2)}{dq_1^2} = -C''(q_1) = -\theta\gamma(\gamma-1)q_1^{\gamma-2} < 0$. Thereby, supplier 1's optimal quality in response to w_1 is $\tilde{q}_1(w_1, q_2) = \left[\frac{(w_1+l_s)q_2}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. On the other hand, for supplier 2, the first-order condition of $\pi_{S_2}(q_2|w_2)$ is $\frac{d\pi_{S_2}(q_2|w_2)}{dq_2}\Big|_{q_2=\tilde{q}_2(w_2)} = (w_2 + l_s) - C'(\tilde{q}_2(w_2)) = (w_2 + l_s) - \theta\gamma(\tilde{q}_2(w_2))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_2}(q_2|w_2)$ w.r.t. q_2 yields $\frac{d^2\pi_{S_2}(q_2|w_2)}{dq_2^2} = -C''(q_2) = -\theta\gamma(\gamma-1)q_2^{\gamma-2} < 0$. Thereby, supplier 2's optimal quality in response to w_2 is $\tilde{q}_2(w_2) = \left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions in stage 3 as follows:

$$\tilde{q}_1(w_1, w_2) = \left[\frac{(w_1 + l_s)^{\gamma-1}(w_2 + l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{(\gamma-1)^2}}, \tag{SH.9}$$

$$\tilde{q}_2(w_2) = \left(\frac{w_2 + l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SH.10}$$

Next, consider stage 2, where supplier 1 chooses the contract to offer to supplier 2, w_2, to maximize $\pi_{S_1}(w_2, \tilde{q}_1(w_1, w_2)|w_1, \tilde{q}_2(w_2))$, given w_1 decided in stage 1. Plugging (SH.10) into $\pi_{S_2}(q_2|w_2)$, we have

$$\begin{aligned}
\pi_{S_2}(\tilde{q}_2(w_2)|w_2) &= (w_2 + l_s)\tilde{q}_2(w_2) - l_s - \theta(\tilde{q}_2(w_2))^\gamma \\
&= (w_2 + l_s)\left(\frac{w_2 + l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - \theta\left[\left(\frac{w_2 + l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\right]^\gamma - l_s \\
&= (\gamma-1)(w_2 + l_s)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - l_s.
\end{aligned}$$

Then, plugging (SH.9) and (SH.10) into (I.2), we have supplier 1's problem as follows:

$$\begin{aligned}\max_{w_2}\ \pi_{S_1}(w_2|w_1) &= (w_1+l_s)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_2) - l_s - \theta(\tilde{q}_1(w_1,w_2))^{\gamma} - w_2\tilde{q}_2(w_2)\\ &= (w_1+l_s)\left[\frac{(w_1+l_s)^{\gamma-1}(w_2+l_s)}{\theta^{\gamma}\gamma^{\gamma}}\right]^{\frac{1}{(\gamma-1)^2}}\left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s\\ &\quad -\theta\left[\left[\frac{(w_1+l_s)^{\gamma-1}(w_2+l_s)}{\theta^{\gamma}\gamma^{\gamma}}\right]^{\frac{1}{(\gamma-1)^2}}\right]^{\gamma} - w_2\left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\\ &= (\gamma-1)(w_1+l_s)^{\frac{\gamma}{\gamma-1}}(w_2+l_s)^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{2\gamma-1}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2}{(\gamma-1)^2}} - w_2(w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s.\end{aligned}$$

We now analyze supplier 1's optimal contracting decision. Taking the first-order derivative of $\pi_{S_1}(w_2|w_1)$ w.r.t. w_2 yields

$$\frac{d\pi_{S_1}(w_2|w_1)}{dw_2} = \frac{1}{\gamma-1}(w_2+l_s)^{\frac{3\gamma-\gamma^2-1}{(\gamma-1)^2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[(w_1+l_s)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-1)^2}} - (w_2+l_s)^{\frac{-1}{(\gamma-1)^2}}\left[\gamma w_2+(\gamma-1)l_s\right]\right]. \tag{SH.11}$$

Solving (SH.11) yields the solution of supplier 1's first-order condition, $\tilde{w}_2(w_1)$, that satisfies

$$(w_1+l_s)^{\gamma(\gamma-1)}\left(\frac{1}{\theta\gamma}\right)^{\gamma} = \left[\tilde{w}_2(w_1)+l_s\right]^{-1}\left[\gamma\tilde{w}_2(w_1)+(\gamma-1)l_s\right]^{(\gamma-1)^2}. \tag{SH.12}$$

Note that $\tilde{w}_2(w_1)$ is supplier 1's optimal contracting decision. However, due to limited tractability, we cannot obtain the closed-form $\tilde{w}_2(w_1)$ from solving (SH.12). Hence, we derive the inverse function of $\tilde{w}_2(w_1)$ from (SH.12), i.e., $\tilde{w}_1(w_2)$, as follows:

$$\tilde{w}_1(w_2) = \left[\frac{\theta^{\gamma}\gamma^{\gamma}\left[\gamma w_2+(\gamma-1)l_s\right]^{(\gamma-1)^2}}{w_2+l_s}\right]^{\frac{1}{\gamma(\gamma-1)}} - l_s. \tag{SH.13}$$

Then, plugging (SH.13) into (SH.9) and (SH.10), we have

$$\tilde{q}_1(w_2) = \left[\gamma w_2+(\gamma-1)l_s\right]^{\frac{1}{\gamma}}(w_2+l_s)^{\frac{1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}, \tag{SH.14}$$

$$\tilde{q}_2(w_2) = \left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SH.15}$$

Comparing (SH.14) and (SH.15) yields $\frac{\tilde{q}_1(w_2)}{\tilde{q}_2(w_2)} = \left[\frac{\gamma w_2+(\gamma-1)l_s}{w_2+l_s}\right]^{\frac{1}{\gamma}} > (\gamma-1)^{\frac{1}{\gamma}} \geqslant 1$. Besides, we can show that IR_2 is satisfied when l_s is sufficiently small.

Next, consider stage 1, where the buyer chooses the contract to offer to supplier 1, w_1, to maximize $\pi_B(w_1|\tilde{q}_1(w_1,\tilde{w}_2(w_1)),\tilde{q}_2(\tilde{w}_2(w_1)))$. Note that it is equivalent for the buyer to choose the optimal w_2 and offer the corresponding contract $\tilde{w}_1(w_2)$ to supplier 1. Plugging (SH.13) into $\pi_{S_1}(w_2|w_1)$, we have

$$\pi_{S_1}(w_2|\tilde{w}_1(w_2)) = (\gamma-1)\Big[\tilde{w}_1(w_2)+l_s\Big]^{\frac{\gamma}{\gamma-1}}(w_2+l_s)^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{2\gamma-1}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2}{(\gamma-1)^2}} - w_2(w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s$$

$$= (\gamma-1)\left[\left[\frac{\theta^\gamma\gamma^\gamma\big[\gamma w_2+(\gamma-1)l_s\big]^{(\gamma-1)^2}}{w_2+l_s}\right]^{\frac{1}{\gamma(\gamma-1)}}\right]^{\frac{\gamma}{\gamma-1}}(w_2+l_s)^{\frac{\gamma}{(\gamma-1)^2}}\left(\frac{1}{\theta}\right)^{\frac{2\gamma-1}{(\gamma-1)^2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma^2}{(\gamma-1)^2}}$$

$$- w_2(w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s$$

$$= (\gamma-1)\Big[\gamma w_2+(\gamma-1)l_s\Big](w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - w_2(w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s.$$

Then, plugging (SH.13), (SH.14) and (SH.15) into (I.2), we have the buyer's problem as follows:

$$\max_{w_1}\ \pi_B(w_1) = \max_{w_2}\ \pi_B(\tilde{w}_1(w_2))$$

$$= (p+l)\tilde{q}_1(\tilde{w}_1(w_2))\tilde{q}_2(\tilde{w}_1(w_2)) - l - \tilde{w}_1(w_2)\tilde{q}_1(\tilde{w}_1(w_2))\tilde{q}_2(\tilde{w}_1(w_2))$$

$$= (p+l)\Big[\gamma w_2+(\gamma-1)l_s\Big]^{\frac{1}{\gamma}}(w_2+l_s)^{\frac{1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l$$

$$- \left[\left[\frac{\theta^\gamma\gamma^\gamma\big[\gamma w_2+(\gamma-1)l_s\big]^{(\gamma-1)^2}}{w_2+l_s}\right]^{\frac{1}{\gamma(\gamma-1)}} - l_s\right]\Big[\gamma w_2+(\gamma-1)l_s\Big]^{\frac{1}{\gamma}}\cdot(w_2+l_s)^{\frac{1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$= (p+l+l_s)\Big[\gamma w_2+(\gamma-1)l_s\Big]^{\frac{1}{\gamma}}(w_2+l_s)^{\frac{\gamma+1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-1}} - l - \Big[\gamma w_2+(\gamma-1)l_s\Big](w_2+l_s)^{\frac{1}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}.$$

We now analyze the buyer's optimal contracting decision. Taking the first-order derivative of $\pi_B(\tilde{w}_1(w_2))$ w.r.t. w_2 yields

$$\frac{d\pi_B(\tilde{w}_1(w_2))}{dw_2} = \frac{1}{\gamma-1}(w_2+l_s)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\Bigg[(p+l+l_s)\Big[\gamma w_2+(\gamma-1)l_s\Big]^{\frac{1-\gamma}{\gamma}}$$
$$\cdot\Big[2\gamma^2 w_2+(\gamma-1)(2\gamma+1)l_s\Big](w_2+l_s)^{\frac{1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - \gamma^2 w_2 - (\gamma-1)(\gamma+1)l_s\Bigg]. \tag{SH.16}$$

Solving (SH.16) yields the solution of the buyer's first-order condition, $w_2^{T\ddagger}$, that satisfies

$$\begin{aligned}(p+l+l_s)\left(w_2^{T\dagger}+l_s\right)^{\frac{1}{\gamma(\gamma-1)}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\Big[2\gamma^2 w_2^{T\dagger}+(\gamma-1)(2\gamma+1)l_s\Big]\\ = \Big[\gamma^2 w_2^{T\dagger}+(\gamma-1)(\gamma+1)l_s\Big]\Big[\gamma w_2^{T\dagger}+(\gamma-1)l_s\Big]^{\frac{\gamma-1}{\gamma}}\end{aligned} \tag{SH.17}$$

Then, plugging $w_2^{T\ddagger}$ into (SH.13), we obtain the buyer's optimal contracting decisions: $w_1^{T\ddagger} = \tilde{w}_1(w_2^{T\ddagger}) = \left[\gamma w_2^{T\ddagger}+(\gamma-1)l_s\right]^{\frac{\gamma-1}{\gamma}}(w_2^{T\ddagger}+l_s)^{-\frac{1}{\gamma(\gamma-1)}}(\theta\gamma)^{\frac{1}{\gamma-1}} - l_s$. Besides, we can show that IR_1 is satisfied when l_s is sufficiently small.

Finally, plugging $w_2^{T\ddagger}$ into (SH.14) and (SH.15), we obtain the suppliers' equilibrium quality decisions: $q_1^{T\ddagger} = \left[\gamma w_2^{T\ddagger} + (\gamma-1)l_s\right]^{\frac{1}{\gamma}} (w_2^{T\ddagger} + l_s)^{\frac{1}{\gamma(\gamma-1)}} \left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}$, $q_2^{T\ddagger} = \left[\frac{w_2^{T\ddagger}+l_s}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. By Assumptions I.1 and 2, we have $q_i^{T\ddagger} \in (0,1)$ for $i \in \{1,2\}$. $\square$

Proof of Proposition I.3. We first derive the suppliers' optimal quality decisions. Given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i,q_{-i})$. For supplier i, the first-order condition of $\pi_{S_i}(q_i|w_i,q_{-i})$ is $\frac{d\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i}\Big|_{q_i=\tilde{q}_i(w_i,q_{-i})} = (w_i+l_s)q_{-i} - C'(\tilde{q}_i(w_i,q_{-i})) = (w_i+l_s)q_{-i} - \theta\gamma(\tilde{q}_i(w_i,q_{-i}))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_i}(q_i|w_i,q_{-i})$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i,q_{-i})}{dq_i^2} = -C''(q_i) = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, supplier i's optimal quality in response to w_i is $\tilde{q}_i(w_i,q_{-i}) = \left[\frac{(w_i+l_s)q_{-i}}{\theta\gamma}\right]^{\frac{1}{\gamma-1}}$. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i,w_{-i}) = \left[\frac{(w_i+l_s)^{\gamma-1}(w_{-i}+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}. \tag{SH.18}$$

Next, consider the buyer's problem. Plugging (SH.18) into $\pi_{S_i}(q_i|w_i,q_{-i})$, we have

$$\begin{aligned}
\pi_{S_i}(\tilde{q}_i(w_i,w_{-i})|w_i,\tilde{q}_{-i}(w_i,w_{-i})) &= (w_i+l_s)\tilde{q}_i(w_i,w_{-i})\tilde{q}_{-i}(w_i,w_{-i}) - l_s - \theta(\tilde{q}_i(w_i,w_{-i}))^\gamma \\
&= (w_i+l_s)\left[\frac{(w_i+l_s)^{\gamma-1}(w_{-i}+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(w_{-i}+l_s)^{\gamma-1}(w_i+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}} \\
&\quad - l_s - \theta\left[\left[\frac{(w_i+l_s)^{\gamma-1}(w_{-i}+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\right]^\gamma \\
&= (\gamma-1)(w_i+l_s)^{\frac{\gamma-1}{\gamma-2}}(w_{-i}+l_s)^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-2}} - l_s.
\end{aligned}$$

Then, plugging (SH.18) into (I.3), we have the buyer's problem as follows:

$$\begin{aligned}
\max_{w_1,w_2}\ \pi_B(w_1,w_2) &= (p+l)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) - l - (w_1+w_2)\tilde{q}_1(w_1,w_2)\tilde{q}_2(w_1,w_2) \\
&= (p+l-w_1-w_2)\left[\frac{(w_1+l_s)^{\gamma-1}(w_2+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}}\left[\frac{(w_2+l_s)^{\gamma-1}(w_1+l_s)}{\theta^\gamma\gamma^\gamma}\right]^{\frac{1}{\gamma(\gamma-2)}} - l \\
&= (p+l-w_1-w_2)(w_1+l_s)^{\frac{1}{\gamma-2}}(w_2+l_s)^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}} - l.
\end{aligned}$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_1} = (w_1+l_s)^{\frac{3-\gamma}{\gamma-2}}(w_2+l_s)^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2} - w_1 - l_s\right], \tag{SH.19}$$

$$\frac{\partial\pi_B(w_1,w_2)}{\partial w_2} = (w_1+l_s)^{\frac{1}{\gamma-2}}(w_2+l_s)^{\frac{3-\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2} - w_2 - l_s\right]. \tag{SH.20}$$

Solving (SH.19) and (SH.20) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{N\dagger} = w_2^{N\dagger} = \frac{p+l-(\gamma-2)l_s}{\gamma}. \tag{SH.21}$$

Then, we need to show that $(w_1^{N\dagger}, w_2^{N\dagger})$ are the buyer's optimal contracting decisions. In similar fashion to the proof of Proposition 3, if the stationary point characterized in (SH.25) is a strict local maximum, then $(w_1^{N\dagger}, w_2^{N\dagger})$ must be the unique global maximum, proved by contradiction. Taking the second-order derivatives of $\pi_B(w_1, w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1^2} = \frac{1}{\gamma-2}(w_1+l_s)^{\frac{5-2\gamma}{\gamma-2}}(w_2+l_s)^{\frac{1}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2}-2(w_1+l_s)\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_2^2} = \frac{1}{\gamma-2}(w_1+l_s)^{\frac{1}{\gamma-2}}(w_2+l_s)^{\frac{5-2\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{(3-\gamma)(p+l-w_1-w_2)}{\gamma-2}-2(w_2+l_s)\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1 \partial w_2} = \frac{1}{\gamma-2}(w_1+l_s)^{\frac{3-\gamma}{\gamma-2}}(w_2+l_s)^{\frac{3-\gamma}{\gamma-2}}\left(\frac{1}{\theta\gamma}\right)^{\frac{2}{\gamma-2}}\left[\frac{p+l-w_1-w_2}{\gamma-2}-w_1-w_2-2l_s\right].$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1, w_2)$ is negative definite in the neighborhood of $(w_1^{N\dagger}, w_2^{N\dagger})$. Thus, $(w_1^{N\dagger}, w_2^{N\dagger})$ are the buyer's optimal contracting decisions. Besides, we can show that IR_i is satisfied when l_s is sufficiently small.

Finally, plugging $(w_1^{N\dagger}, w_2^{N\dagger})$ into (SH.18), we obtain the suppliers' optimal quality decisions: $q_1^{N\dagger} = q_2^{N\dagger} = \left(\frac{p+l+2l_s}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}}$. By Assumptions I.1 and 2, we have $q_i^{N\dagger} \in (0,1)$ for $i \in \{1,2\}$. □

Proof of Proposition I.4. We first derive the suppliers' optimal quality decisions. Given w_i, supplier $i \in \{1,2\}$ chooses q_i to maximize $\pi_{S_i}(q_i|w_i)$. For supplier i, the first-order condition of $\pi_{S_i}(q_i|w_i)$ is $\left.\frac{d\pi_{S_i}(q_i|w_i)}{dq_i}\right|_{q_i=\tilde{q}_i(w_i)} = w_i + l_s - C'(\tilde{q}_i(w_i)) = w_i + l_s - \theta\gamma(\tilde{q}_i(w_i))^{\gamma-1} = 0$. Taking the second-order derivative of $\pi_{S_i}(q_i|w_i)$ w.r.t. q_i yields $\frac{d^2\pi_{S_i}(q_i|w_i)}{dq_i^2} = -C''(q_i) = -\theta\gamma(\gamma-1)q_i^{\gamma-2} < 0$. Thereby, the solution of the first-order condition is supplier i's optimal quality in response to w_i. Solving the suppliers' best response functions yields their optimal quality decisions:

$$\tilde{q}_i(w_i) = \left(\frac{w_i+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{SH.22}$$

Next, consider the buyer's problem. Plugging (SH.22) into $\pi_{S_i}(q_i|w_i)$, we have

$$\begin{aligned}\pi_{S_i}(\tilde{q}_i(w_i)|w_i) &= (w_i+l_s)\tilde{q}_i(w_i) - l_s - \theta(\tilde{q}_i(w_i))^\gamma \\ &= (w_i+l_s)\left(\frac{w_i+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - l_s - \theta\left[\left(\frac{w_i+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\right]^\gamma \\ &= (\gamma-1)(w_i+l_s)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - l_s.\end{aligned}$$

Then, plugging (SH.22) into (I.4), we have the buyer's problem as follows:

$$\max_{w_1,w_2} \ \pi_B(w_1,w_2) = p\tilde{q}_1(w_1)\tilde{q}_2(w_2) + \frac{1}{2}(p-l)\tilde{q}_1(w_1)\Big[1-\tilde{q}_2(w_2)\Big] + \frac{1}{2}(p-l)\tilde{q}_2(w_2)\Big[1-\tilde{q}_1(w_1)\Big]$$
$$- l\Big[1-\tilde{q}_1(w_1)\Big]\Big[1-\tilde{q}_2(w_2)\Big] - w_1\tilde{q}_1(w_1) - w_2\tilde{q}_2(w_2)$$
$$= \frac{1}{2}(p+l)\Big[\tilde{q}_1(w_1)+\tilde{q}_2(w_2)\Big] - l - w_1\tilde{q}_1(w_1) - w_2\tilde{q}_2(w_2)$$
$$= \frac{1}{2}(p+l)\left[\left(\frac{w_1+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} + \left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\right] - l - w_1\left(\frac{w_1+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}} - w_2\left(\frac{w_2+l_s}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}.$$

We now analyze the buyer's optimal contracting decisions. Taking the first-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial \pi_B(w_1,w_2)}{\partial w_1} = \frac{1}{\gamma-1}(w_1+l_s)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{1}{2}(p+l)-\gamma w_1-(\gamma-1)l_s\right], \tag{SH.23}$$
$$\frac{\partial \pi_B(w_1,w_2)}{\partial w_2} = \frac{1}{\gamma-1}(w_2+l_s)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{1}{2}(p+l)-\gamma w_2-(\gamma-1)l_s\right]. \tag{SH.24}$$

Solving (SH.23) and (SH.24) yields the solution of the buyer's first-order conditions as follows:

$$w_1^{T\dagger} = w_2^{T\dagger} = \frac{p+l-2(\gamma-1)l_s}{2\gamma}. \tag{SH.25}$$

Then, we need to show that $(w_1^{T\dagger}, w_2^{T\dagger})$ are the buyer's optimal contracting decisions. In similar fashion to the proof of Proposition 4, if the stationary point characterized in (SH.25) is a strict local maximum, then $(w_1^{T\dagger}, w_2^{T\dagger})$ must be the unique global maximum, proved by contradiction. Taking the second-order derivatives of $\pi_B(w_1,w_2)$ w.r.t. w_1 and w_2 yields

$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1^2} = \frac{1}{\gamma-1}(w_1+l_s)^{\frac{3-2\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{(2-\gamma)(p+l-2\gamma w_1)}{2(\gamma-1)}-\gamma w_1-2l_s\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_2^2} = \frac{1}{\gamma-1}(w_2+l_s)^{\frac{3-2\gamma}{\gamma-1}}\left(\frac{1}{\theta\gamma}\right)^{\frac{1}{\gamma-1}}\left[\frac{(2-\gamma)(p+l-2\gamma w_2)}{2(\gamma-1)}-\gamma w_2-2l_s\right],$$
$$\frac{\partial^2 \pi_B(w_1,w_2)}{\partial w_1 \partial w_2} = 0.$$

By Assumption 2, we can show that the Hessian of $\pi_B(w_1,w_2)$ is negative definite in the neighborhood of $(w_1^{T\dagger}, w_2^{T\dagger})$. Thus, $(w_1^{T\dagger}, w_2^{T\dagger})$ are the buyer's optimal contracting decisions. Besides, we can show that IR_i is satisfied when l_s is sufficiently small.

Finally, plugging $(w_1^{T\dagger}, w_2^{T\dagger})$ into (SH.22), we obtain the suppliers' optimal quality decisions: $q_1^{T\dagger} = q_2^{T\dagger} = \left(\frac{p+l+2l_s}{2\theta\gamma^2}\right)^{\frac{1}{\gamma-1}}$. By Assumptions I.1 and 2, we have $q_i^{T\dagger} \in (0,1)$ for $i \in \{1,2\}$. □

Proof of Theorem I.1. The theorem follows from comparing the equilibrium contracts and suppliers' quality decisions characterized in Propositions I.3 and I.4. First, it is easy to see that $w_i^{T\dagger} < w_i^{N\dagger}$ always holds. Second, by Assumptions I.1 and 2, we have

$$q_i^{T\dagger} > q_i^{N\dagger} \Leftrightarrow \left(\frac{p+l+2l_s}{2\theta\gamma^2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l+2l_s}{\theta\gamma^2}\right)^{\frac{1}{\gamma-2}}$$
$$\Leftrightarrow \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l+2l_s}{\theta\gamma^2}\right)^{\frac{1}{(\gamma-2)(\gamma-1)}} \Leftrightarrow \frac{\gamma^2}{2^{\gamma-2}} > \frac{p+l+2l_s}{\theta}.$$

Thus, the comparison between $q_i^{T\dagger}$ and $q_i^{N\dagger}$ can be characterized by thresholds $\tilde{l}$, or $\tilde{p}$, or $\tilde{\theta}$, or $\tilde{\gamma}$ such that $q_i^{T\dagger} > q_i^{N\dagger}$ if $l < \tilde{l}$, or $p < \tilde{p}$, or $\theta > \tilde{\theta}$, or $\gamma < \tilde{\gamma}$; whereas $q_i^{T\dagger} < q_i^{N\dagger}$ if $l > \tilde{l}$, or $p > \tilde{p}$, or $\theta < \tilde{\theta}$, or $\gamma > \tilde{\gamma}$, where

$$\tilde{l} \equiv \frac{\theta\gamma^2}{2^{\gamma-2}} - p - 2l_s,\ \tilde{p} \equiv \frac{\theta\gamma^2}{2^{\gamma-2}} - l - 2l_s,\ \tilde{\theta} \equiv \frac{2^{\gamma-2}(p+l+2l_s)}{\gamma^2},\ \tilde{\gamma} \equiv \begin{cases} \tilde{\gamma}_0 & \text{if } \frac{p+l+2l_s}{\theta} \leqslant 4, \\ 2 & \text{if } \frac{p+l+2l_s}{\theta} > 4, \end{cases}$$

and $\tilde{\gamma}_0$ is the unique solution to $\frac{\gamma^2}{2^{\gamma-2}} = \frac{p+l+2l_s}{\theta}$ in the range of $\gamma > 2$. Besides, $\tilde{\gamma}_0 > 4$, and it is decreasing in p and l, while increasing in θ. Hence, the theorem is proved. □

Proof of Theorem I.2. Consider the case without traceability. Based on the equilibrium characterized in Proposition I.3, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{N\dagger} = (\gamma-2)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l,$$
$$\pi_{S_i}^{N\dagger} = (\gamma-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l_s,$$
$$\pi_{SC}^{N\dagger} = (\gamma^2-2)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l - 2l_s.$$

Consider the case with traceability. Based on the equilibrium characterized in Proposition I.4, we obtain the equilibrium expected profits for the buyer, the suppliers, and the entire supply chain as follows:

$$\pi_B^{T\dagger} = (\gamma-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+1}{\gamma-1}} - l,$$
$$\pi_{S_i}^{T\dagger} = (\gamma-1)\left(\frac{p+l+2l_s}{2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l_s,$$
$$\pi_{SC}^{T\dagger} = (\gamma^2-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l - 2l_s.$$

We first compare the buyer's equilibrium expected profits with and without traceability. By Assumptions I.1 and 2, we have

$$\pi_B^{T\dagger} > \pi_B^{N\dagger} \Leftrightarrow (\gamma-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+1}{\gamma-1}} - l > (\gamma-2)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}} \left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}} \left(\frac{1}{\gamma}\right)^{\frac{\gamma+2}{\gamma-2}} - l$$
$$\Leftrightarrow \left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} \gamma^{\frac{2}{(\gamma-2)(\gamma-1)}} > \left(\frac{\gamma-2}{\gamma-1}\right)\left(\frac{p+l+2l_s}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}.$$

The last inequality always holds since $\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} \geqslant 1$, $\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}} > 1$, $\frac{\gamma-2}{\gamma-1} < 1$, and $\left(\frac{p+l+2l_s}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}} < 1$. Hence, we can see that $\pi_B^{T\dagger} > \pi_B^{N\dagger}$ always holds.

We then compare the suppliers' equilibrium expected profits with and without traceability. By Assumptions I.1 and 2, we have

$$\pi_{S_i}^{T\dagger} > \pi_{S_i}^{N\dagger} \Leftrightarrow (\gamma-1)\left(\frac{p+l+2l_s}{2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l_s > (\gamma-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l_s$$
$$\Leftrightarrow \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} > \left(\frac{p+l+2l_s}{\theta\gamma^2}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}} \Leftrightarrow \frac{\gamma^2}{2^{\gamma-2}} > \frac{p+l+2l_s}{\theta}.$$

Therefore, the comparison between $\pi_{S_i}^{T\dagger}$ and $\pi_{S_i}^{N\dagger}$ can be characterized by thresholds $\tilde{l}$, or $\tilde{p}$, or $\tilde{\theta}$, or $\tilde{\gamma}$ such that $\pi_{S_i}^{T\dagger} > \pi_{S_i}^{N\dagger}$ if $l < \tilde{l}$, or $p < \tilde{p}$, or $\theta > \tilde{\theta}$, or $\gamma < \tilde{\gamma}$; whereas $\pi_{S_i}^{T\dagger} < \pi_{S_i}^{N\dagger}$ if $l > \tilde{l}$, or $p > \tilde{p}$, or $\theta < \tilde{\theta}$, or $\gamma > \tilde{\gamma}$, where the thresholds $\tilde{l}$, $\tilde{p}$, $\tilde{\theta}$, and $\tilde{\gamma}$ have been characterized in the proof of Theorem I.1.

Finally, we compare the equilibrium total supply chain profits with and without traceability. By Assumptions I.1 and 2, we have

$$\pi_{SC}^{T\dagger} > \pi_{SC}^{N\dagger} \Leftrightarrow (\gamma^2-1)(p+l+2l_s)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{2\theta}\right)^{\frac{1}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-1}} - l - 2l_s > (\gamma^2-2)(p+l+2l_s)^{\frac{\gamma}{\gamma-2}}\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma-2}}\left(\frac{1}{\gamma}\right)^{\frac{2\gamma}{\gamma-2}} - l - 2l_s$$
$$\Leftrightarrow \left(\frac{\gamma^2-1}{\gamma^2-2}\right)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l+2l_s}{\theta\gamma^2}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}$$
$$\Leftrightarrow \left(\frac{\gamma^2-1}{\gamma^2-2}\right)\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}}\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} > \left(\frac{p+l+2l_s}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}}.$$

The last inequality always holds since $\frac{\gamma^2-1}{\gamma^2-2} > 1$, $\gamma^{\frac{2}{(\gamma-2)(\gamma-1)}} > 1$, $\left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}} \geqslant 1$, and $\left(\frac{p+l+2l_s}{\theta\gamma}\right)^{\frac{\gamma}{(\gamma-2)(\gamma-1)}} < 1$. Hence, we can see that $\pi_{SC}^{T\dagger} > \pi_{SC}^{N\dagger}$ always holds. □

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