E-Companion to
“Ups and Downs in Experience Design”

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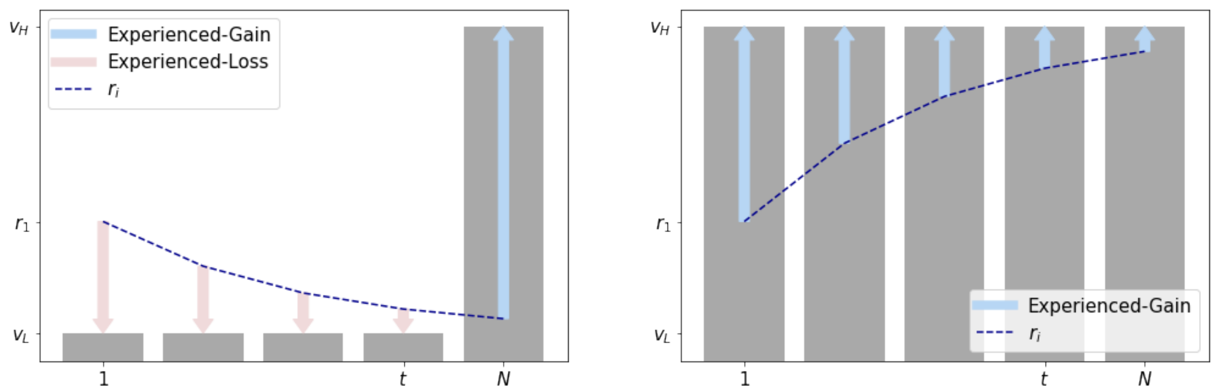
A. Additional Figures

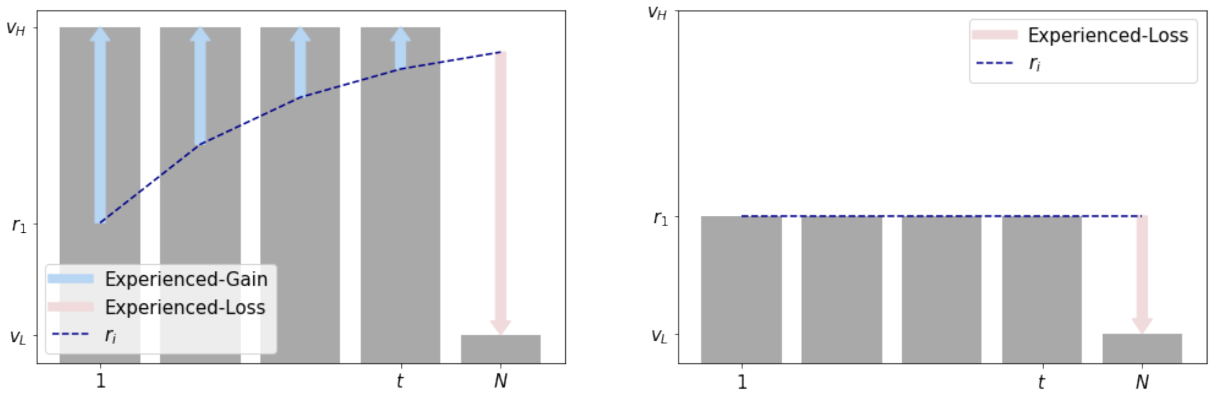
Figure EC.1 How to Release Good News: Illustration of Proposition 1



(a) $\lambda \leq \lambda^g$: When the public is sufficiently gain-seeking, it is optimal to first misguide the public and lower their expectations for the first t periods. To illustrate this scenario, we let $t = N - 1$ in the figure above.

(b) $\lambda > \lambda^g$: When the public is sufficiently loss-averse, it is suboptimal to initially lower expectations. Instead, it is optimal to immediately reveal the good news such that the public never experiences any losses.

Figure EC.2 How to Release Bad News: Illustration of Proposition 2



(a) $\lambda \leq 1/(1-\theta)$: When the public is sufficiently gain-seeking, it is optimal first to misguide the public and provide them with $N-1$ experienced gains. The bad news should not be revealed until the last period.

(b) $\lambda > 1/(1-\theta)$: When the public is sufficiently loss-averse, it is optimal not to reveal any information about the bad news and keep the public's expectation constant until the last period.

Figure EC.3 Optimal Performance Sequencing for an Event ($N_H = 2$)

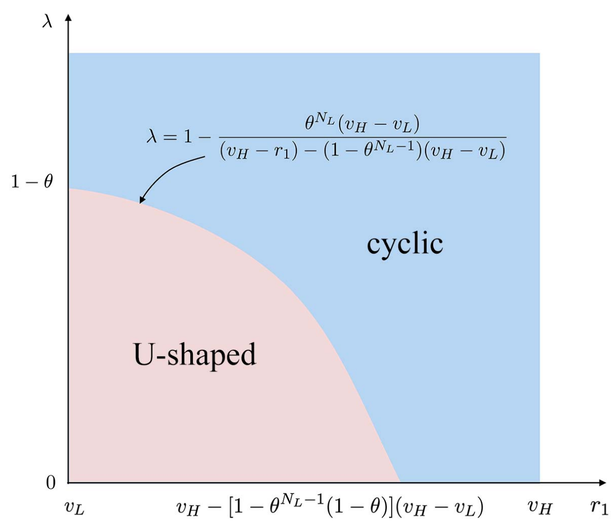
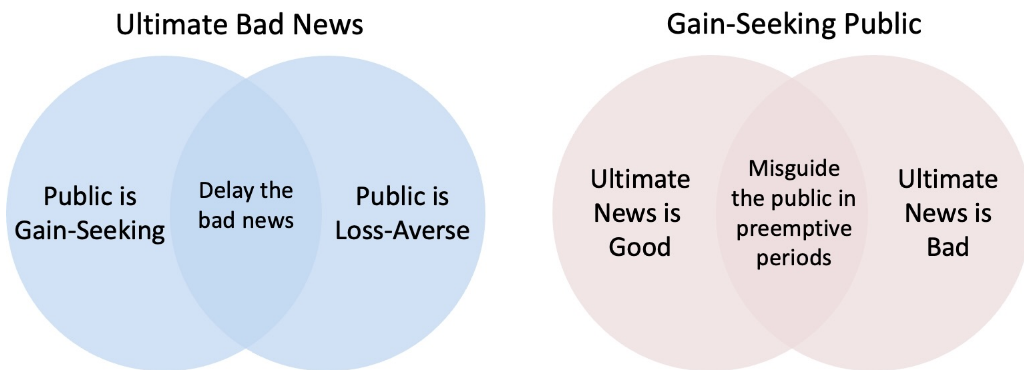


Figure EC.4 How to Release News: Illustration of the Findings in Section 3



(a) Proposition 2: It is never optimal to reveal the bad news earlier than the last period when it is ultimately required.

(b) Propositions 1(a) and 2(a): It is always optimal to misguide a gain-seeking public before revealing the ultimate news, irrespective of its sentiment.

Figure EC.5 Possible Patterns of the Optimal Performance Sequence: Illustration of the Findings in Section 4

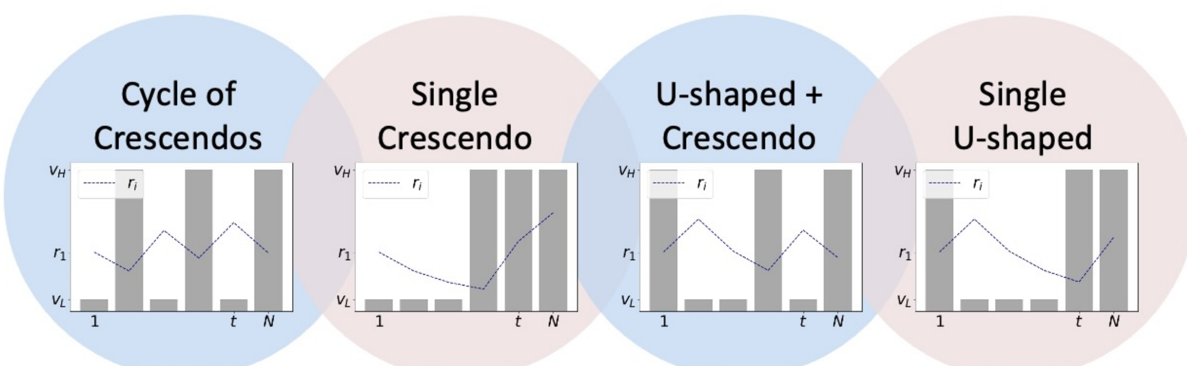
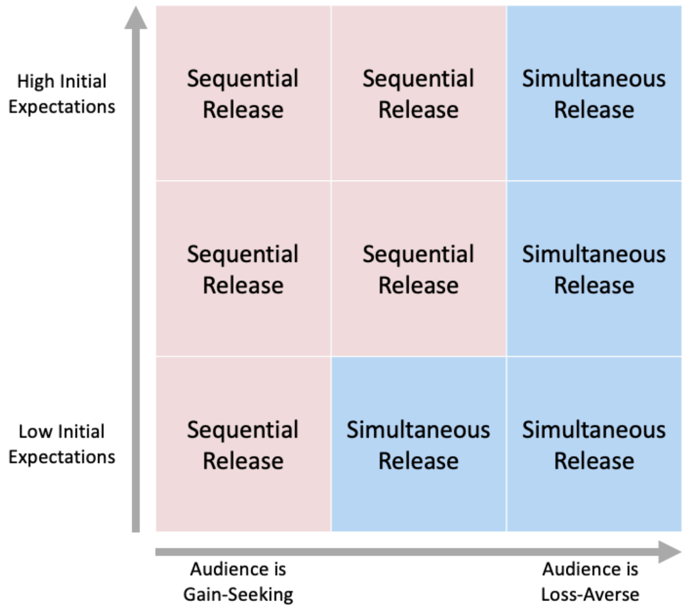


Figure EC.6 Optimal Series Release Strategy: Illustration of the Findings in Section 5



B. Auxiliary Lemmas

B.1. Auxiliary Lemmas for Proofs of Section 3

LEMMA EC.1. *Denote $x_i = v_i - r_i \geq 0$ for $i \in \{1, 2, \dots, N-1\}$, problem P_g is equivalent to the following problem:*

$$P_g': \quad \max_{x_i} \quad v_H + \theta\alpha \sum_{i=1}^{N-1} x_i + \alpha(v_H - r_1)$$
$$s.t. \quad 0 \leq \theta x_i \leq x_{i+1},\ i \in \{1, 2, \dots, N-2\},\ (1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \leq v_H - r_1.$$

Moreover, problem P_g' is maximized by $x_i = \theta^{i-1}(v_H - r_1)$, i.e., $v_i = v_H$ for any $i \in \{1, 2, \dots, N\}$, and the corresponding objective value is $v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1)$.

LEMMA EC.2. *Denote $x_i = r_i - v_i \geq 0$ for $i \in \{1, 2, \dots, t\}$, and $x_i = v_i - r_i \geq 0$ for $i \in \{t+1, t+2, \dots, N-1\}$, where $t \in \{1, \dots, N-1\}$, problem P_g is equivalent to the following problem:*

$$P_g^t: \quad \max_{x_i} \quad v_H - (\lambda - (1-\theta))\alpha \sum_{i=1}^{t} x_i + \theta\alpha \sum_{i=t+1}^{N-1} x_i + \alpha(v_H - r_1)$$
$$s.t. \quad 0 \leq x_{i+1} \leq \theta x_i,\ i \in \{1, 2, \dots, t-1\},\ 0 \leq \theta x_i \leq x_{i+1},\ i \in \{t+1, t+2, \dots, N-2\},$$
$$(1-\theta) \sum_{i=t+1}^{N-2} x_i + x_{N-1} - (1-\theta) \sum_{i=1}^{t} x_i \leq v_H - r_1,\ 0 \leq x_1 \leq r_1 - v_L.$$

Moreover, if $\lambda \leq 1 - \theta^{N-t}$, the optimal solution of problem P_g^t is $x_i = \theta^{i-1}(r_1 - v_L)$ for $i \in \{1, 2, \dots, t\}$ and $x_i = \theta^{i-t-1}[(1-\theta^t)(r_1 - v_L) + v_H - r_1]$ for $i \in \{t+1, t+2, \dots, N-1\}$, i.e., $v_i = v_L$ for $i \in \{1, 2, \dots, t\}$ and $v_i = v_H$ for $i \in \{t+1, t+2, \dots, N\}$, the corresponding objective value is

$$v_H - \lambda \frac{1-\theta^t}{1-\theta}\alpha(r_1 - v_L) + \frac{1-\theta^{N-t}}{1-\theta}\alpha[v_H - \theta^t r_1 - (1-\theta^t)v_L] \equiv F(t);$$

Otherwise, the optimal solution of problem P_g^t is $x_i = 0$ for $i \in \{1, 2, \dots, t\}$ and $x_i = \theta^{i-t-1}(v_H - r_1)$ for $i \in \{t+1, t+2, \dots, N-1\}$, i.e., $v_i = r_1$ for $i \in \{1, 2, \dots, t\}$ and $v_i = v_H$ for $i \in \{t+1, t+2, \dots, N\}$, the corresponding objective value is $v_H + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$.

LEMMA EC.3. *Denote $x_i = r_i - v_i \geq 0$ for $i \in \{1, 2, \dots, N-1\}$, problem P_b is equivalent to the following problem:*

$$P_b': \quad \max_{x_i} \quad v_L - \lambda\theta\alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$$
$$s.t. \quad 0 \leq \theta x_i \leq x_{i+1},\ i \in \{1, 2, \dots, N-2\},\ (1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \leq r_1 - v_L.$$

Moreover, problem P_b' is maximized by $x_i = 0$ for $i \in \{1, 2, \dots, N-1\}$, i.e., $v_i = r_1$ for $i \in \{1, 2, \dots, N-1\}$, and the corresponding objective value is $v_L - \lambda\alpha(r_1 - v_L)$.

Lemma EC.4. *Denote* $x_i = v_i - r_i \geq 0$ *for* $i \in \{1,2,\ldots,N-1\}$*, problem* P_b *is equivalent to the following problem:*

$$P_b'': \quad \max_{x_i} \quad v_L + (1-\lambda(1-\theta))\alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$$
$$s.t. \quad 0 \leq x_1 \leq v_H - r_1,\ 0 \leq x_{i+1} \leq \theta x_i,\ i \in \{1,2,\ldots,N-2\}.$$

Moreover, if $\lambda \leq 1/(1-\theta)$*, the optimal solution of problem* P_b'' *is* $x_i = \theta^{i-1}(v_H - r_1)$ *for* $i \in \{1,2,\ldots,N-1\}$*, i.e.,* $v_i = v_H$ *for* $i \in \{1,2,\ldots,N-1\}$*, and the corresponding objective value is* $v_L + \frac{1-\lambda(1-\theta)}{1-\theta}\alpha(v_H - r_1) - \lambda\alpha(r_1 - v_L)$*. Otherwise, the optimal solution of problem* P_b'' *is* $x_i = 0$ *for* $i \in \{1,2,\ldots,N-1\}$*, i.e.,* $v_i = r_1$ *for* $i \in \{1,2,\ldots,N-1\}$*, and the corresponding objective value is* $v_L - \lambda\alpha(r_1 - v_L)$*.*

B.2. Auxiliary Lemmas for Proofs of Section 4

Lemma EC.5. *There exists a threshold* λ' *such that if* $\lambda > \lambda'$*, a crescendo pattern is better than a U-shaped pattern; otherwise, a U-shaped pattern is better, where*

$$\lambda' = \begin{cases} 1 - \theta^{N_H-1}(v_H - v_L)/(v_H - r_1), & \text{if } r_1 \leq v_H - \theta^{N_H-1}(v_H - v_L), \\ 0, & \text{if } r_1 > v_H - \theta^{N_H-1}(v_H - v_L). \end{cases} \tag{EC.1}$$

Lemma EC.6 (Ending in Peak). *For any* $N_H \geq 1$ *and* $N_L \geq 1$*, it is always optimal to end the event with a high-type performance. Moreover,*

(a) when $N_H = 1$*, it is always optimal to arrange the unique high-type performance in the last place, which corresponds to a crescendo pattern;*

(b) when $N_L = 1$*, it is always optimal to arrange the unique low-type performance earlier as* λ *becomes larger,*

- *(i) if* $\lambda \leq \lambda'$*, it is optimal to arrange the performances in a U-shaped pattern, with the unique low-type performance arranged from the second last place to the second place as* λ *increases from 0 to* λ'*;*
- *(ii) if* $\lambda > \lambda'$*, it is optimal to arrange the performances in a crescendo pattern, with the unique low-type performance arranged in the first place.*

B.3. Auxiliary Lemmas for Proofs of Section 5

Lemma EC.7. *Denote* $\Theta_i = \{1, \theta, \ldots, \theta^{i-2}\}$ *and* δ_i *is a random variable with equal probability* $\frac{1}{2^{i-1}}$ *such that* $\delta_i = \sum_{a \in A} a$*, where* A *is a subset of* Θ_i *and* $\delta_i = 0$ *if* $A = \emptyset$*. Thus, we have* $r_i = \theta^{i-1} r_1 + \delta_i(1-\theta)v_L + (1 - \delta_i(1-\theta) - \theta^{i-1})v_H$ *and* $\mathbb{E}[r_i] = \theta^{i-1} r_1 + \frac{1}{2}(1-\theta^{i-1})v_L + \frac{1}{2}(1-\theta^{i-1})v_H$*.*

Lemma EC.8. *If* $r_1 = \frac{1}{2}(v_H + v_L)$*,* $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1)$ *is decreasing in* λ*. Moreover,* $u_{se}(r_1, r_2, \ldots, r_N) = u_{si}(r_1)$ *if and only if* $\lambda = 1$*,* $u_{se}(r_1, r_2, \ldots, r_N) > u_{si}(r_1)$ *for* $\lambda < 1$*, and* $u_{se}(r_1, r_2, \ldots, r_N) < u_{si}(r_1)$ *for* $\lambda > 1$*.*

LEMMA EC.9. *For* $r_1 \in [v_L, v_H]$,

(a) there exists $\frac{1}{2}(v_H + v_L) < \bar{r} < v_H$*, if* $r_1 \le \bar{r}$*,* $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1)$ *is decreasing in* λ*, otherwise, it is increasing in* λ*;*

(b) there exists $v_L < \underline{r} < \frac{1}{2}(v_H + v_L)$*, when* $\lambda = 0$*, if* $r_1 < \underline{r}$*,* $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1) < 0$*, otherwise,* $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1) \ge 0$*;*

(c) if $r_1 \le \frac{1}{2}(v_H + v_L)$ *and* $\lambda \in [0, 1]$*, or* $r_1 \ge \frac{1}{2}(v_H + v_L)$ *and* $\lambda \in [1, +\infty)$*,* $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1)$ *is increasing in* r_1*.*

C. Main Proofs

C.1. Proofs of Section 3

Proof of Proposition 1. From the proof of Lemma EC.1, problem P_g is equivalent to problem P_g'', which means $v_{i+1} \ge v_i$ for $i \in \{1, 2, \ldots, N-1\}$. Now we consider problem P_g''. If $v_i \ge r_i$, then $v_{i+1} \ge r_{i+1} = \theta r_i + (1-\theta) v_i$ due to $v_{i+1} \ge v_i \ge r_i$. Thus any policy of P_g'' has the structure: there exists a threshold $\bar{i} \in \{1, 2, \ldots, N\}$, such that if $i < \bar{i}$, $v_i \le r_i$; otherwise, $v_i \ge r_i$. From Lemmas EC.1 and EC.2, we know the optimal policy is given by problem P_g' or problem P_g^t. We next show the optimal policy is given by problem P_g^t if $\lambda \le \lambda^g$ and problem P_g' otherwise, where

$$\lambda^g = \begin{cases} 0, & \text{if } r_1 \le \theta^{N-1} v_H + (1-\theta^{N-1}) v_L, \\ 1 - \dfrac{\theta^{N-1}(v_H - v_L)}{r_1 - v_L}, & \text{if } r_1 > \theta^{N-1} v_H + (1-\theta^{N-1}) v_L. \end{cases}$$

Note $F(0) = v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1)$, which is the optimal objective value of problem P_g'. We have

$$\begin{aligned} F(t) - F(0) &= -\lambda \frac{1-\theta^t}{1-\theta}\alpha(r_1 - v_L) + \frac{1-\theta^{N-t}}{1-\theta}\alpha[v_H - \theta^t r_1 - (1-\theta^t) v_L] - \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1) \\ &= \frac{1-\theta^t}{1-\theta}\alpha[-\lambda(r_1 - v_L) - \theta^{N-t}(v_H - v_L) + r_1 - v_L]. \end{aligned}$$

Let

$$\lambda_t^g = \begin{cases} 0, & \text{if } r_1 \le \theta^{N-t} v_H + (1-\theta^{N-t}) v_L, \\ 1 - \dfrac{\theta^{N-t}(v_H - v_L)}{r_1 - v_L}, & \text{if } r_1 > \theta^{N-t} v_H + (1-\theta^{N-t}) v_L. \end{cases}$$

Thus, $F(t) \ge F(0)$ if $\lambda \le \lambda_t^g$, and $F(t) < F(0)$ otherwise. Note $F(0) = v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1) \ge v_H + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$. Therefore, by Lemma EC.2, the optimal objective value of problem P_g' is greater than that of problem P_g^t if and only if $\lambda > \lambda_t^g$. Moreover, $\lambda^g = \lambda_1^g > \lambda_2^g > \cdots > \lambda_{N-1}^g$, thus the optimal policy is given by problem P_g^t if $\lambda \le \lambda^g$ and problem P_g' otherwise.

Now we show the optimal policy of problem P_g is described as Proposition 1. If $\lambda \le \lambda^g$, the optimal policy is given by problem P_g^t. By Lemma EC.2, the optimal objective value of problem P_g is given by $F(t)$ or $v_H + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$. However, $F(0) = v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1) \ge v_H + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$. Then, the optimal objective value of problem P_g is given by $F(t)$, and otherwise, the optimal policy cannot be given by problem P_g^t. Consequently, $v_i = v_L$ for $i \in \{1, 2, \ldots, t\}$ and $v_i = v_H$ for $i \in \{t+1, t+2, \ldots, N\}$ by Lemma EC.2. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2, \ldots, N\}$, we have $\delta_1 = v_L$, $\delta_{t+1} = v_H - v_L$, and $\delta_i = 0$ for any $i \in \{2 \ldots, t, t+2, \ldots, N\}$.

If $\lambda > \lambda^g$, the optimal policy is given by problem P_g', then $v_i = v_H$ for any $i \in \{1, 2, \ldots, N\}$ by Lemma EC.1. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2, \ldots, N\}$, we have $\delta_1 = v_H$ and $\delta_i = 0$ for $i \in \{2, \ldots, N\}$. □

Proof of Corollary 1. Note Proposition 1(a) shows the style of optimal releasing policy for $\lambda \le \lambda^g$, but not gives the characterization of optimal t. We now show the optimal t in Proposition 1(a) should satisfy $\bar{\lambda}^{t+1} \le \lambda \le \bar{\lambda}^t$, where

$$\bar{\lambda}^t = \begin{cases} 0, & \text{if } r_1 \le \theta^{N-2t+1} v_H + (1 - \theta^{N-2t+1}) v_L, \\ 1 - \dfrac{\theta^{N-2t+1}(v_H - v_L)}{r_1 - v_L}, & \text{if } r_1 > \theta^{N-2t-1} v_H + (1 - \theta^{N-2t+1}) v_L. \end{cases}$$

And $\bar{\lambda}^t$ is decreasing in t. Denoted the optimal objective value of problem P_g^t by u_g^t, then

$$u_g^t = \begin{cases} F(t), & \text{if } \lambda \le 1 - \theta^{N-t}, \\ v_H + \dfrac{1 - \theta^{N-t}}{1 - \theta} \alpha (v_H - r_1), & \text{if } \lambda > 1 - \theta^{N-t}, \end{cases}$$

by Lemma EC.2. Then it is equivalent to show $u_g^t \ge u_g^i$ for any $i \ne t$ if and only if $\bar{\lambda}^{t+1} \le \lambda \le \bar{\lambda}^t$.

First, we show $u_g^t \ge u_g^{t-1}$ if and only if $\lambda \le \bar{\lambda}^t$. Note

$$\begin{aligned} F(t) - F(t-1) &= -\lambda \alpha \theta^{t-1}(r_1 - v_L) - \theta^{N-t} \alpha (v_H - v_L) + \theta^{t-1} \alpha (r_1 - v_L) \\ &= \alpha [\theta^{t-1}(r_1 - v_L)(1 - \lambda) - \theta^{N-t}(v_H - v_L)], \end{aligned}$$

which implies $F(t) \ge F(t-1)$ if and only if $\lambda \le \bar{\lambda}^t$. Furthermore, $\bar{\lambda}^t \le 1 - \theta^{N-t}$ and $v_H + \frac{1-\theta^{N-t}}{1-\theta} \alpha (v_H - r_1)$ is decreasing in t. Hence, we get $u_g^t \ge u_g^{t-1}$ if and only if $\lambda \le \bar{\lambda}^t$.

Second, we show $u_g^t \ge u_g^i$ for any $i \ne t$ if and only if $\bar{\lambda}^{t+1} \le \lambda \le \bar{\lambda}^t$. On the one hand, $\lambda \le \bar{\lambda}^t$ implies $\lambda \le \bar{\lambda}^t \le \bar{\lambda}^{t-1} \le \cdots \le \bar{\lambda}^2$, then $u_g^t \ge u_g^{t-1} \ge u_g^{t-2} \ge \cdots \ge u_g^1$ by the former paragraph. On the other hand, $\lambda \ge \bar{\lambda}^{t+1}$ implies $\lambda \le \bar{\lambda}^{t+1} \ge \bar{\lambda}^{t+2} \le \cdots \le \bar{\lambda}^{N-2}$, then $u_g^t \ge u_g^{t+1} \ge u_g^{t+2} \ge \cdots \ge u_g^{N-1}$ by the former paragraph.

Finally, we verify Corollary 1. Note $\lambda_g = \bar{\lambda}^1$, then $[0, \lambda_g]$ can be divided into $N-1$ intervals by $\bar{\lambda}^{N-1}$, $\bar{\lambda}^{N-2}$, …, $\bar{\lambda}^1$. Thus, $\bar{\lambda}^{t+1} \le \lambda \le \bar{\lambda}^t$ means λ is located in $(N-t)$-th interval $[\bar{\lambda}^{t+1}, \bar{\lambda}^t]$. Hence, when λ increases, it would tend to be located in a latter interval, which leads to a smaller t. We get t is decreasing in λ. Note $\bar{\lambda}^t$ is increasing in r_1 and N, but decreasing in θ. Therefore, we get that t is increasing in r_1 and N, but decreasing in θ. □

Proof of Corollary 2. We show $v_i \ge r_i$ for $i \in \{1, 2, \ldots, N\}$ by induction. Note $\delta_1 = v_1 \ge r_1$, now we suppose $v_i \ge r_i$ and prove $v_{i+1} \ge r_{i+1}$. That is $v_{i+1} \ge r_{i+1} = \theta r_i + (1-\theta) v_i$ due to $v_{i+1} \ge v_i \ge r_i$.

From Lemma EC.1, we know the optimal policy is given by problem P_g'. That is $v_i = v_H$ for any $i \in \{1,2,\ldots,N\}$. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2,\ldots,N\}$, we have $\delta_1 = v_H$ and $\delta_i = 0$ for $i \in \{2,\ldots,N\}$. $\square$

Proof of Proposition 2. From the proof of Lemma EC.3, problem P_b is equivalent to problem $P_b^{'''}$, which means $v_i \geq v_{i+1}$ for $i \in \{1,2,\ldots,N-1\}$. Now we consider problem $P_b^{'''}$. If $v_i \leq r_i$, then $v_{i+1} \leq r_{i+1} = \theta r_i + (1-\theta)v_i$ due to $v_{i+1} \leq v_i \leq r_i$. Thus any policy of $P_b^{'''}$ has the structure: there exists a threshold $\bar{i} \in \{1,2,\ldots,N\}$, such that if $i < \bar{i}$, $v_i \geq r_i$; otherwise, $v_i \leq r_i$.

When v_i is fixed for $i \in \{1,2,\ldots,\bar{i}-1\}$, to decide optimal v_i for $i \in \{\bar{i},\bar{i}+1,\ldots,N-1\}$ is equivalent to solve problem P_b' by replacing r_1 and N into $r_{\bar{i}}$ and $N-\bar{i}+1$. By Lemma EC.3, it is optimal to let $v_i = r_i$ for $i \in \{\bar{i},\bar{i}+1,\ldots,N-1\}$. Hence, any optimal policy of problem P_b should satisfy $v_i \geq r_i$ for $i \in \{1,2,\ldots,N-1\}$, which is indeed problem $P_b^{''}$. Therefore, solving problem $P_b^{'''}$ is equivalent to solve problem $P_b^{''}$.

By Lemma EC.4, if $\lambda \leq 1/(1-\theta)$, it is optimal to let $v_i = v_H$ for $i \in \{1,2,\ldots,N-1\}$ and $v_N = v_L$. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2,\ldots,N\}$, we have $\delta_1 = v_H$, $\delta_i = 0$ for any $i \in \{2,\ldots,N-1\}$, and $\delta_N = v_L - v_H$. Otherwise, $v_i = r_1$ for $i \in \{1,2,\ldots,N-1\}$ and $v_N = v_L$. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2,\ldots,N\}$, we have $\delta_1 = r_1$, $\delta_i = 0$ for any $i \in \{2,\ldots,N-1\}$, and $\delta_N = v_L - r_1$. $\square$

Proof of Corollary 3. We show $v_i \leq r_i$ for $i \in \{1,2,\ldots,N\}$ by induction. Note $\delta_1 = v_1 \leq r_1$, now we suppose $v_i \leq r_i$ and prove $v_{i+1} \leq r_{i+1}$. That is $v_{i+1} \leq r_{i+1} = \theta r_i + (1-\theta)v_i$ due to $v_{l+1} \leq v_i \leq r_i$. From Lemma EC.3, we know the optimal policy is given by problem P_b'. That is $v_i = r_i$ for any $i \in \{1,2,\ldots,N-1\}$ and $v_N = v_L$. Due to $\delta_1 = v_1$ and $\delta_i = v_i - v_{i-1}$ for $i \in \{2,\ldots,N\}$, we have $\delta_1 = r_1$, $\delta_i = 0$ for any $i \in \{2,\ldots,N-1\}$, and $\delta_N = v_L - r_1$. $\square$

C.2. Proofs of Section 4

We first prove Proposition 4, then verify Proposition 3 based on the proof of Proposition 4.

Proof of Proposition 4. When $N_H = 2$, by Lemma EC.6(a), any possible optimal policy can be: the first x performances are low-type, then one high-type performance and y low-type performances, the last performance is high-type, where $x + y = N_L$ and $0 \leq x \leq N_L$. Note it is a crescendo pattern if $x = N_L$, a U-shaped pattern if $x = 0$, and two crescendo patterns otherwise. We will show the utility function is concave in x when $\lambda \leq 1-\theta$, decreasing in x when $1-\theta \leq \lambda \leq 1$, convex in x when $\lambda \geq 1$. By (2) and (4), we have the utility function

$$u(r_1,r_2,\ldots,r_N|x) = 2v_H + N_L v_L + \alpha(2v_H + \lambda N_L v_L) - \alpha\left[\lambda\sum_{i=1}^{x} r_i + r_{x+1} + \lambda\sum_{i=x+2}^{N_L+1} r_i + r_{N_L+2}\right]. \quad \text{(EC.2)}$$

If $i \le x+1$, $r_i = \theta^{i-1} r_1 + (1-\theta^{i-1}) v_L$, $r_{x+2} = \theta r_{x+1} + (1-\theta) v_H$, if $x+2 \le i \le N_L+2$, $r_i = \theta^{i-x-2} r_{x+2} + (1-\theta^{i-x-2}) v_L$. Then,

$$\begin{aligned}&\lambda \sum_{i=1}^{x} r_i + r_{x+1} + \lambda \sum_{i=x+2}^{N_L+1} r_i + r_{N_L+2} \\ =&\lambda \frac{1-\theta^x}{1-\theta} r_1 + \lambda\left(x - \frac{1-\theta^x}{1-\theta}\right) v_L + r_{x+1} + \lambda \frac{1-\theta^y}{1-\theta} r_{x+2} + \lambda\left(y - \frac{1-\theta^y}{1-\theta}\right) v_L + r_{N_L+2},\end{aligned} \tag{EC.3}$$

where $r_{x+1} = \theta^x r_1 + (1-\theta^x) v_L$, $r_{x+2} = \theta^{x+1} r_1 + \theta(1-\theta^x) v_L + (1-\theta) v_H$, $r_{N_L+2} = \theta^y r_{x+2} + (1-\theta^y) v_L = \theta^{N_L+1}(r_1 - v_L) + \theta^y (1-\theta)(v_H - v_L) + v_L$, and

$$\begin{aligned}&\frac{\partial\left(\lambda \sum_{i=1}^{x} r_i + r_{x+1} + \lambda \sum_{i=x+2}^{N_L+1} r_i + r_{N_L+2}\right)}{\partial x} \\ =&-\lambda \frac{\theta^x ln\theta}{1-\theta}(r_1 - v_L) + \theta^x ln\theta (r_1 - v_L) + \lambda \frac{\theta^y ln\theta}{1-\theta}(r_{x+2} - v_L) + \lambda \frac{1-\theta^y}{1-\theta} \theta^{x+1} ln\theta (r_1 - v_L) - \theta^y ln\theta (1-\theta)(v_H - v_L) \\ =&-\lambda \frac{\theta^x ln\theta}{1-\theta}(r_1 - v_L) + \theta^x ln\theta (r_1 - v_L) + \lambda \frac{\theta^y ln\theta}{1-\theta}[\theta^{x+1}(r_1 - v_L) + (1-\theta)(v_H - v_L)] \\ &+\lambda \frac{1-\theta^y}{1-\theta} \theta^{x+1} ln\theta (r_1 - v_L) - \theta^y ln\theta (1-\theta)(v_H - v_L) = (1-\lambda)\theta^x ln\theta (r_1 - v_L) + (\lambda - 1 + \theta)\theta^y ln\theta (v_H - v_L),\end{aligned} \tag{EC.4}$$

which is increasing in x when $\lambda \le 1-\theta$, less than 0 when $1-\theta \le \lambda \le 1$, and decreasing in x when $\lambda \ge 1$ due to $ln\theta < 0$. Thus $\lambda \sum_{i=1}^{x} r_i + r_{x+1} + \lambda \sum_{i=x+2}^{N_L+1} r_i + r_{N_L+2}$ is convex in x when $\lambda \le 1-\theta$, decreasing in x when $1-\theta \le \lambda \le 1$, and concave in x when $\lambda \ge 1$. Consequently, $u(r_1, r_2, \ldots, r_N | x)$ is concave in x when $\lambda \le 1-\theta$, increasing in x when $1-\theta \le \lambda \le 1$, and convex in x when $\lambda \ge 1$.

We first show that the sufficient and necessary condition for a crescendo pattern to be optimal is: $r_1 > v_L + (1-\theta)(v_H - v_L)/\theta^{N_L-1}$ or $\lambda > 1 - \theta(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)]$, i.e., $\lambda > \bar{\lambda}'$, where

$$\bar{\lambda}' = \begin{cases} 1 - \theta(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)], & \text{if } r_1 \le v_L + (1-\theta)(v_H - v_L)/\theta^{N_L-1}, \\ 0, & \text{if } r_1 > v_L + (1-\theta)(v_H - v_L)/\theta^{N_L-1}. \end{cases}$$

When $1-\theta \le \lambda \le 1$, note $u(r_1, r_2, \ldots, r_N | x)$ is increasing in x, then it is maximized at $x = N_L$, i.e., a crescendo pattern is optimal. When $\lambda \ge 1$, note $u(r_1, r_2, \ldots, r_N | x)$ is convex in x, then it is maximized at $x = 0$ or $x = N_L$. In other words, a crescendo pattern or a U-shaped pattern is optimal. By Lemma EC.5, $r_1 > v_H - \theta^{N_H-1}(v_H - v_L)$ or $\lambda > 1 - \theta^{N_H-1}(v_H - v_L)/(v_H - r_1)$, a crescendo pattern is better than a U-shaped pattern. Thus a crescendo pattern is optimal when $\lambda \ge 1$. Consider $\lambda \le 1-\theta$, note $u(\lambda, \theta, r_1, N_H = 2, N_L | x)$ is concave in x, thus a crescendo pattern is optimal if and only if $u(r_1, r_2, \ldots, r_N | x = N_L) > u(r_1, r_2, \ldots, r_N | x = N_L - 1)$. By (EC.2) and (EC.3),

$$u(r_1,r_2,\dots,r_N|x=N_L)=2v_H+N_Lv_L+\alpha(2v_H+\lambda N_Lv_L)$$
$$-\alpha\left[\lambda\frac{1-\theta^{N_L}}{1-\theta}r_1+\lambda\left(N_L-\frac{1-\theta^{N_L}}{1-\theta}\right)v_L+\theta^{N_L}r_1+(1-\theta^{N_L})v_L+\theta^{N_L+1}(r_1-v_L)+(1-\theta)(v_H-v_L)+v_L\right],$$

$$u(r_1,r_2,\dots,r_N|x=N_L-1)=2v_H+N_Lv_L+\alpha(2v_H+\lambda N_Lv_L)$$
$$-\alpha\left[\lambda\frac{1-\theta^{N_L-1}}{1-\theta}r_1+\lambda\left(N_L-1-\frac{1-\theta^{N_L-1}}{1-\theta}\right)v_L+\theta^{N_L-1}r_1+(1-\theta^{N_L-1})v_L\right.$$
$$\left.+\lambda[\theta(\theta^{N_L-1}r_1+(1-\theta^{N_L-1})v_L)+(1-\theta)v_H]+\theta^{N_L+1}(r_1-v_L)+\theta(1-\theta)(v_H-v_L)+v_L\right].$$

Thus, we have

$$u(r_1,r_2,\dots,r_N|x=N_L)-u(r_1,r_2,\dots,r_N|x=N_L-1)$$
$$=\alpha\big[-\lambda[\theta^{N_L-1}(1-\theta)r_1+(1-\theta^{N_L-1})(1-\theta)v_L-(1-\theta)v_H]+\theta^{N_L-1}(1-\theta)(r_1-v_L)-(1-\theta)^2(v_H-v_L)\big]$$
$$=\alpha(1-\theta)\big[\theta(v_H-v_L)-(1-\lambda)[(v_H-v_L)-\theta^{N_L-1}(r_1-v_L)]\big].$$

Note $0\le\lambda\le1-\theta$ and $v_L\le r_1\le v_H$, then $\theta(v_H-v_L)-(1-\lambda)[(v_H-v_L)-\theta^{N_L-1}(r_1-v_L)]$ is increasing in r_1 and λ. Let $\lambda=0$, $\theta(v_H-v_L)-(1-\lambda)[(v_H-v_L)-\theta^{N_L-1}(r_1-v_L)]=\theta^{N_L-1}(r_1-v_L)-(1-\theta)(v_H-v_L)$, which equals to 0 if $r_1=v_L+(1-\theta)(v_H-v_L)/\theta^{N_L-1}$. $u(r_1,r_2,\dots,r_N|x=N_L)-u(r_1,r_2,\dots,r_N|x=N_L-1)>0$ implies $r_1>v_L+(1-\theta)(v_H-v_L)/\theta^{N_L-1}$ or $\lambda>1-\theta(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)]$.

We next show that the sufficient and necessary condition of a U-shaped pattern is optimal is: $r_1\le v_H-[1-\theta^{N_L-1}(1-\theta)](v_H-v_L)$ and $\lambda\le1-\theta^{N_L}(v_H-v_L)/[v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)]$, i.e., $\lambda\le\underline{\lambda}$, where

$$\underline{\lambda}=\begin{cases}1-\frac{\theta^{N_L}(v_H-v_L)}{(v_H-r_1)-(1-\theta^{N_L-1})(v_H-v_L)}, & \text{if } r_1\le v_H-[1-\theta^{N_L-1}(1-\theta)](v_H-v_L),\\ 0, & \text{if } r_1>v_H-[1-\theta^{N_L-1}(1-\theta)](v_H-v_L).\end{cases}$$

From the analysis of the paragraph above, we just need to consider $\lambda\le1-\theta(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)]\le1-\theta$. Recall $u(r_1,r_2,\dots,r_N|x)$ is concave in x when $\lambda\le1-\theta$, thus a crescendo pattern is optimal if and only if $u(r_1,r_2,\dots,r_N|x=0)\ge u(r_1,r_2,\dots,r_N|x=1)$. By (EC.2) and (EC.3),

$$u(r_1,r_2,\dots,r_N|x=0)=2v_H+N_Lv_L+\alpha(2v_H+\lambda N_Lv_L)$$
$$-\alpha\left[r_1+\lambda\frac{1-\theta^{N_L}}{1-\theta}[\theta r_1+(1-\theta)v_H]+\lambda\left(N_L-\frac{1-\theta^{N_L}}{1-\theta}\right)v_L+\theta^{N_L+1}(r_1-v_L)+\theta^{N_L}(1-\theta)(v_H-v_L)+v_L\right],$$

$$u(r_1,r_2,\dots,r_N|x=1)=2v_H+N_Lv_L+\alpha(2v_H+\lambda N_Lv_L)$$
$$-\alpha\Bigg[\lambda r_1+\theta r_1+(1-\theta)v_L+\lambda\frac{1-\theta^{N_L-1}}{1-\theta}[\theta(\theta r_1+(1-\theta)v_L)+(1-\theta)v_H]+\lambda\bigg(N_L-1-\frac{1-\theta^{N_L-1}}{1-\theta}\bigg)v_L$$
$$+\theta^{N_L+1}(r_1-v_L)+\theta^{N_L-1}(1-\theta)(v_H-v_L)+v_L\Bigg].$$

Thus, we have

$$u(r_1,r_2,\dots,r_N|x=0)-u(r_1,r_2,\dots,r_N|x=1)$$
$$=\alpha\big[\lambda(1-\theta)(r_1-v_L)-(1-\theta)(r_1-v_L)+\theta^{N_l-1}(1-\theta)^2(v_H-v_L)-\lambda\theta^{N_L-1}(1-\theta)(v_H-v_L)\big]$$
$$=\alpha(1-\theta)\big[(1-\lambda)[v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)]-\theta^{N_L}(v_H-v_L)\big].$$

Note $0\le\lambda\le 1-\theta$ and $v_L\le r_1\le v_H$, consider $v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)>0$, otherwise, $u(r_1,r_2,\dots,r_N|x=0)-u(r_1,r_2,\dots,r_N|x=1)<0$. Then $(1-\lambda)[v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)]-\theta^{N_L}(v_H-v_L)$ is decreasing in r_1 and λ. Let $\lambda=0$, $(1-\lambda)[v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)]-\theta^{N_L}(v_H-v_L)=v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)-\theta^{N_L}(v_H-v_L)$, which equals to 0 if $r_1=v_H-[1-\theta^{N_L-1}(1-\theta)](v_H-v_L)$. $u(r_1,r_2,\dots,r_N|x=0)-u(r_1,r_2,\dots,r_N|x=1)\ge 0$ implies $r_1\le v_H-[1-\theta^{N_L-1}(1-\theta)](v_H-v_L)$ and $\lambda\le 1-\theta^{N_L}(v_H-v_L)/[v_H-r_1-(1-\theta^{N_L-1})(v_H-v_L)]$.

Last, we show the number of low-type performances arranged before the first high-type performance increases with λ when an optimal policy is to arrange the performances in one or two crescendo patterns. Note $\lambda\le\underline{\lambda}$ is the condition that U-shaped pattern is optimal and a crescendo pattern is optimal when $\lambda>\bar{\lambda}'$ and $\bar{\lambda}'<1-\theta$. Thus, two crescendo patterns is optimal if $\underline{\lambda}<\lambda\le\bar{\lambda}'<1-\theta$. Recall $u(r_1,r_2,\dots,r_N|x)$ is concave in x when $\lambda\le 1-\theta$, then we show the optimal x for two crescendo patterns is increasing in λ. Let (EC.4) equal to 0, we have $\theta^{2x^*}=(1-\frac{\theta}{1-\lambda})\frac{\theta^{N_L}(v_H-v_L)}{r_1-v_L}$, and it is optimal to let $x=[x^*]$ or $x=[x^*]+1$, where $[x^*]$ is the maximum integer that is less than x^*. Note x^* is increasing in λ. Hence, the optimal x is also increasing in λ.

Therefore, we get the desired results. □

Proof of Proposition 3. (a) First, we show that if $r_1>v_L+(1-\theta^{N_H-1})(v_H-v_L)/\theta^{N_L-1}$ or $\lambda>1-\theta^{N_H-1}(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)]$, i.e., $\lambda>\bar{\lambda}''$, a crescendo pattern is optimal, where

$$\bar{\lambda}''=\begin{cases}1-\theta^{N_H-1}(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)], & \text{if } r_1\le v_L+(1-\theta^{N_H-1})(v_H-v_L)/\theta^{N_L-1},\\ 0, & \text{if } r_1>v_L+(1-\theta^{N_H-1})(v_H-v_L)/\theta^{N_L-1}.\end{cases}$$

Note $N_L\ge 1$, then $v_L+(1-\theta^{N_H-1})(v_H-v_L)/\theta^{N_L-1}\ge v_L+(1-\theta^{N_H-1})(v_H-v_L)=v_H-\theta^{N_H-1}(v_H-v_L)$, and $1-\theta^{N_H-1}(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)]\ge 1-\theta^{N_H-1}(v_H-v_L)/[v_H-v_L-\theta^{N_L-1}(r_1-v_L)]\ge 1-\theta^{N_H-1}(v_H-v_L)/(v_H-r_1)$. Hence, we have $\bar{\lambda}''\ge\bar{\lambda}'$. By Lemma EC.5, a crescendo pattern is better than a U-shaped pattern. Now we prove the optimal policy cannot contain a U-shaped pattern by contradiction. Suppose the optimal policy contains at least one U-shaped pattern and the last U-shaped pattern has j performances. Consider a new policy that is as same as the optimal policy except for the last j performances. Specially, we can change the last U-shaped pattern into a crescendo structure. Note that the first $N_H + N_L - j$ performances are the same under the two policies, and we just need to compare the last j performances. In other words, we can compare two policies with j performances with the first reference point is $r_{N_H+N_L-j}$, one is a U-shaped pattern, and the other is a crescendo pattern. $r_{N_H+N_L-j}$ is no less than $\theta^{N_L-1}r_1 + (1-\theta^{N_L-1})v_L$ due to that there is at least one low-type performance in the last j performances. On the one hand, if $r_1 > v_L + (1-\theta^{N_H-1})(v_H - v_L)/\theta^{N_L-1}$, $r_{N_H+N_L-j} \geq \theta^{N_L-1}r_1 + (1-\theta^{N_L-1})v_L > v_H - \theta^{N_H-1}(v_H - v_L)$. On the other hand, if $\lambda > 1-\theta^{N_H-1}(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)]$, $\lambda > 1-\theta^{N_H-1}(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)] \geq 1-\theta^{N_H-1}(v_H - v_L)/(v_H - r_{N_H+N_L-j})$ due to $1-\theta^{N_H-1}(v_H - v_L)/(v_H - r_{N_H+N_L-j})$ is decreasing in $r_{N_H+N_L-j}$ and $r_{N_H+N_L-j} \geq \theta^{N_L-1}r_1 + (1-\theta^{N_L-1})v_L$. Therefore, we can conclude that the crescendo pattern is better than the U-shaped pattern with j performances by Lemma EC.5. Hence, we find a better policy, which is a contradiction.

Next, we show that if $r_1 \leq v_L + (1-\theta^{N_H-1})(v_H - v_L)/\theta^{N_L-1}$ or $\lambda \leq 1-\theta^{N_H-1}(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)]$, i.e., $\lambda \leq \bar{\lambda}''$, a crescendo pattern is not optimal. We prove it by contradiction. Suppose a crescendo pattern is optimal. We consider the first $N_L + 2$ performances, note $r_1 < v_L + (1-\theta^{N_H-1})(v_H - v_L)/\theta^{N_L-1} < v_L + (1-\theta)(v_H - v_L)/\theta^{N_L-1}$ and $\lambda < 1-\theta^{N_H-1}(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)] < 1-\theta(v_H - v_L)/[v_H - v_L - \theta^{N_L-1}(r_1 - v_L)]$, then $\lambda < \bar{\lambda}'$. By the proof of Proposition 4, a crescendo pattern is not optimal for the first $N_L + 2$ performances. Note that the reference point will be highest after a crescendo structure, if the first $N_L + 2$ performances are replaced by other structures, the total utility after it will increase. This is a contradiction.

Last, we show that if $\lambda \leq \underline{\lambda}$, the optimal policy starts with high-type performance(s). In other words, it is optimal to arrange the performances in some combination of a U-shaped pattern followed by several crescendo patterns. It is easy to see $\underline{\lambda} < \bar{\lambda}''$, and thus, the optimal design is not a crescendo pattern by the former analysis. We prove this by contradiction, suppose the first performance is low-type, i.e., it is optimal to arrange the performances in some combination of more than two crescendo patterns. There are two cases: the first crescendo pattern has only one high-type performance or at least two high-type performances. For the first case, consider the first crescendo pattern and a part of the second crescendo pattern that only contains one high-type performance, i.e., two crescendo patterns with two high-type performances. Suppose there are x low-type performances, note $\lambda \leq \underline{\lambda} \leq \underline{\lambda}'$, where

$$\underline{\lambda}' = \begin{cases} 1 - \frac{\theta^x (v_H - v_L)}{(v_H - r_1) - (1-\theta^{x-1})(v_H - v_L)}, & \text{if } r_1 \le v_H - [1-\theta^{x-1}(1-\theta)](v_H - v_L), \\ 0, & \text{if } r_1 > v_H - [1-\theta^{x-1}(1-\theta)](v_H - v_L). \end{cases}$$

By Proposition 4, it is better to replace this structure with a U-shaped pattern. Moreover, the total utility after it will also be better since the reference point is lower when this structure is replaced by a U-shaped structure. For the second case, a part of the first crescendo pattern only contains two high-type performances, i.e., it is a crescendo pattern with two high-type performances. With a similar analysis of the first case, it is better to replace the crescendo pattern with a U-shaped pattern. In sum, we can find a better policy for the two cases, which is a contradiction. Hence, we get the announced result.

(b) First, we prove a policy with the first place a high-type performance, and the rest performances arranged in a crescendo pattern are better than all policies with the first performance being a high-type performance when $\lambda > \lambda_1$, where

$$\lambda_1 = \begin{cases} 1 - \frac{\theta^{N_H-2}(v_H - v_L)}{[1-(1-\theta)\theta^{N_L-1}](v_H - v_L) - \theta^{N_L}(r_1 - v_L)}, & \text{if } r_1 \le v_H - \frac{(\theta^{N_L-1}+\theta^{N_H-2}-1)(v_H-v_L)}{\theta^{N_L}}, \\ 0, & \text{if } r_1 > v_H - \frac{(\theta^{N_L-1}+\theta^{N_H-2}-1)(v_H-v_L)}{\theta^{N_L}}. \end{cases}$$

Moreover, we denote this policy as policy A. We arrange the high-type performance in the first place and consider the rest performances. We replace r_1 and N_H by $\theta r_1 + (1-\theta)v_H$ and $N_H - 1$, respectively. Thus, $\bar{\lambda}''$ is equal to λ_1. By the analysis in the first paragraph of (a), it is optimal to arrange the rest performances in a crescendo pattern.

Second, we prove policy B is better than A when $\lambda > \lambda_2$, where

$$\lambda_2 = \begin{cases} 1 - \frac{\theta^{N_L+N_H-2}(v_H - v_L)}{\theta^{N_L-1}(v_H - v_L) - (r_1 - v_L)}, & \text{if } r_1 \le v_L + (1-\theta^{N_H-1})\theta^{N_L-1}(v_H - v_L), \\ 0, & \text{if } r_1 > v_L + (1-\theta^{N_H-1})\theta^{N_L-1}(v_H - v_L), \end{cases}$$

the policy B is such that the first and second places are low-type and high-type performances, and the rest performances are arranged in a crescendo pattern. Then the total utility for the two policies is:

$$\begin{aligned} & u_A(r_1, r_2, \ldots, r_N) \\ =& N_L v_L + N_H v_H - \lambda\alpha \frac{1-\theta^{N_L}}{1-\theta}[\theta r_1 + (1-\theta)v_H - v_L] \\ &+ \alpha(v_H - r_1) + \alpha \frac{1-\theta^{N_H-1}}{1-\theta}[(1-\theta^{N_L}+\theta^{N_L+1})v_H - \theta^{N_L+1}r_1 - (1-\theta^{N_L})v_L], \end{aligned}$$

$$\begin{aligned} & u_B(r_1, r_2, \ldots, r_N) \\ =& N_L v_L + N_H v_H - \lambda\alpha(r_1 - v_L) - \lambda\alpha \frac{1-\theta^{N_L-1}}{1-\theta}[\theta^2 r_1 + (1-\theta)v_H - (1-\theta+\theta^2)v_L] \\ &+ \alpha[v_H - \theta r_1 - (1-\theta)v_L] + \alpha \frac{1-\theta^{N_H-1}}{1-\theta}[(1-\theta^{N_L-1}+\theta^{N_L})v_H - \theta^{N_L+1}r_1 - (1-\theta^{N_L-1}+\theta^{N_L}-\theta^{N_L+1})v_L]. \end{aligned}$$

Thus,

$$
\begin{aligned}
&u_B(r_1,r_2,\ldots,r_N)-u_A(r_1,r_2,\ldots,r_N)\\
=&\lambda\alpha(1-\theta)[\theta^{N_L-1}v_H+(1-\theta^{N_L-1})v_L-r_1]-\alpha\theta^{N_L-1}(1-\theta)(1-\theta^{N_H-1})(v_H-v_L)+\alpha(1-\theta)(r_1-v_L)\\
=&\alpha(1-\theta)[\lambda[\theta^{N_L-1}v_H+(1-\theta^{N_L-1})v_L-r_1]-\theta^{N_L-1}(1-\theta^{N_H-1})(v_H-v_L)+r_1-v_L].
\end{aligned}
$$

Therefore, $u_B(r_1,r_2,\ldots,r_N)-u_A(r_1,r_2,\ldots,r_N)>0$ if $r_1>v_L+(1-\theta^{N_H-1})\theta^{N_L-1}(v_H-v_L)$ or $\lambda>1-\frac{\theta^{N_L+N_H-2}(v_H-v_L)}{\theta^{N_L-1}(v_H-v_L)-(r_1-v_L)}$, i.e., $\lambda>\lambda_2$.

Last, let $\bar{\lambda}=\max\{\lambda_1,\lambda_2\}$, it is easy to see $0\le\underline{\lambda}\le\lambda_2\le\bar{\lambda}<1$. From the analysis above, we know it is optimal to arrange a low-type performance in the first place when $\lambda>\bar{\lambda}$. In other words, it is optimal to arrange the performances in some combination of crescendo patterns for $\lambda>\bar{\lambda}$. □

C.3. Proofs of Section 5

Proof of Proposition 5. First, we give the specific formulation of $u_{si}(r_1)$ and $u_{se}(r_1,r_2,\ldots,r_N)$. For $u_{si}(r_1)$, we let m satisfy $(m-1)v_H+(N-m+1)v_L\le Nr_1\le mv_H+(N-m)v_L$. Note $\sum_{i=1}^{N}v_i=jv_H+(N-j)v_L$ with probability $C_N^j/2^N$, and $jv_H+(N-j)v_L\ge Nr_1$ for $j\ge m$, $jv_H+(N-j)v_L\le Nr_1$ for $j<m$. Hence, by (2), we have

$$
\begin{aligned}
u_{si}(r_1)=&\frac{N}{2}(v_H+v_L)+\mathbb{E}\big[s\big(\sum_{i=1}^{N}v_i-Nr_1\big)\big]\\
=&\frac{N}{2}(v_H+v_L)+\frac{\alpha}{2^N}\big\{\sum_{j=m}^{N}C_N^j[jv_H+(N-j)v_L-Nr_1]+\lambda\sum_{j=0}^{m-1}C_N^j[jv_H+(N-j)v_L-Nr_1]\big\}.
\end{aligned}
\tag{EC.5}
$$

We now verify

$$
u_{se}(r_1,r_2,\ldots,r_N)=\frac{N}{2}(v_H+v_L)+\alpha\big[\frac{N}{4}(1-\lambda)(v_H-v_L)+\frac{1}{4}(1+\lambda)\frac{1-\theta^N}{1-\theta}(v_H+v_L-2r_1)\big].\tag{EC.6}
$$

Note $1-\delta_i(1-\theta)-\theta^{i-1}=(1+\theta+\theta^2+\cdots+\theta^{i_1}-\delta_i)(1-\theta)=\sum_{a\in\Theta_i-A}a$. Thus $\forall\delta_i$, by Lemma EC.7, $\theta^{i-1}r_1+\delta_i(1-\theta)v_L+(1-\delta_i(1-\theta)-\theta^{i-1})v_H$ and $\theta^{i-1}r_1+(1-\delta_i(1-\theta)-\theta^{i-1})v_L+\delta_i(1-\theta)v_H$ are a pair of reference points in period i. It is easy to see $\theta^{i-1}r_1+\delta_i(1-\theta)v_L+(1-\delta_i(1-\theta)-\theta^{i-1})v_H+\theta^{i-1}r_1+(1-\delta_i(1-\theta)-\theta^{i-1})v_L+\delta_i(1-\theta)v_H=v_H+v_L$. Then we have

$$
\begin{aligned}
&u_{se}(r_1,r_2,\ldots,r_N)\\
=&\frac{N}{2}(v_H+v_L)+\sum_{i=1}^{N}\frac{1}{2^i}\mathbb{E}[s(v_i-r_i)]=\frac{N}{2}(v_H+v_L)+\frac{\alpha}{2}\sum_{i=1}^{N}\{\mathbb{E}[v_H-r_i]+\lambda\mathbb{E}[v_L-r_i]\}\\
=&\frac{N}{2}(v_H+v_L)+\frac{\alpha}{2}\big[N(v_H+\lambda v_L)-(1+\lambda)\sum_{i=1}^{N}\mathbb{E}[r_i]\big]\\
=&\frac{N}{2}(v_H+v_L)+\frac{\alpha}{2}\big[N(v_H+\lambda v_L)-\frac{N}{2}(1+\lambda)(v_H+v_L)+\frac{1}{2}(1+\lambda)\frac{1-\theta^N}{1-\theta}(v_H+v_L-2r_1)\big]
\end{aligned}
$$

$$=\frac{N}{2}(v_H+v_L)+\alpha\Big[\frac{N}{4}(1-\lambda)(v_H-v_L)+\frac{1}{4}(1+\lambda)\frac{1-\theta^N}{1-\theta}(v_H+v_L-2r_1)\Big].$$

Then we prove this proposition. By Lemma EC.9(a) and (b), if $r_1<\underline{r}$, $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)$ is decreasing in λ and $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)<0$ when $\lambda=0$. Then $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)<0$ for $\lambda\geq 0$. If $\underline{r}\leq r_1\leq\bar{r}$, $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)$ is decreasing in λ and there exists $\hat{\lambda}'$ such that $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)=0$ for $\lambda=\hat{\lambda}'$. Then $u_{se}(\lambda,\theta,r_1)-u_{si}(\lambda,r_1)\geq 0$ for $\lambda\leq\hat{\lambda}'$ and $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)\leq 0$ for $\lambda\geq\hat{\lambda}'$. If $r_1>\bar{r}$, $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)$ is increasing in λ and $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)>0$ when $\lambda=0$. Then $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)>0$ for $\lambda\geq 0$. Let

$$\hat{\lambda}=\begin{cases}0, & \text{if } r_1<\underline{r},\\ \hat{\lambda}', & \text{if } \underline{r}\leq r_1\leq\bar{r},\\ \infty, & \text{if } r_1>\bar{r}.\end{cases}$$

Thus, Proposition 5 holds. □

Proof of Corollary 4. Lemma EC.8 implies that $\hat{\lambda}=1$ when $r_1=\frac{1}{2}(v_H+v_L)$, i.e., $u_{se}(r_1=\frac{1}{2}(v_H+v_L),r_2,\ldots,r_N)-u_{si}(r_1=\frac{1}{2}(v_H+v_L))=0$ when $\lambda=1$. By Lemma EC.9(c), when $\lambda=1$, $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)<u_{se}(r_1=\frac{1}{2}(v_H+v_L),r_2,\ldots,r_N)-u_{si}(r_1=\frac{1}{2}(v_H+v_L))=0$ when $r_1<\frac{1}{2}(v_H+v_L)$ and $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)>u_{se}(r_1=\frac{1}{2}(v_H+v_L),r_2,\ldots,r_N)-u_{si}(r_1=\frac{1}{2}(v_H+v_L))=0$ when $r_1>\frac{1}{2}(v_H+v_L)$. Based on Proposition 5, we have $\hat{\lambda}\leq 1$ if $r_1\leq\frac{1}{2}(v_H+v_L)$, $\hat{\lambda}>1$ otherwise.

We next prove $\hat{\lambda}$ is increasing in r_1. Recall $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)$ is decreasing in λ for $\underline{r}\leq r_1\leq\bar{r}$ due to Lemma EC.9(a), and $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)=0$ when $\lambda=\hat{\lambda}'$ for $\underline{r}\leq r_1\leq\bar{r}$. Based on Lemma EC.9(c) and Corollary 4, we have $\hat{\lambda}'$ is increasing in $\underline{r}\leq r_1\leq\bar{r}$. From the definition of $\hat{\lambda}$ in the proof of Proposition 5, we have $\hat{\lambda}$ is increasing in r_1. □

Proof of Corollary 5. Recall the definition of $\underline{r}$ and $\bar{r}$ in the proof of Lemma EC.9, which is the solution of $u(\lambda=0,r_1)=0$ and $\frac{\partial u(\lambda,r_1)}{\partial\lambda}=0$, respectively. Note $u(\lambda=0,r_1)$ is increasing in θ and r_1 when $r_1\leq\frac{1}{2}(v_H+v_L)$ and $\frac{\partial u(\lambda,r_1)}{\partial\lambda}$ is decreasing in θ and increasing in r_1 when $r_1\geq\frac{1}{2}(v_H+v_L)$. Therefore, we have $\underline{r}$ is decreasing in θ and $\bar{r}$ is increasing in θ. Recall $\hat{\lambda}'$ is the solution of $u(\lambda,r_1)=0$ for $\underline{r}\leq r_1\leq\bar{r}$. By (EC.11), it is easy to see $u(\lambda,r_1)$ is increasing in θ when $r_1\leq\frac{1}{2}(v_H+v_L)$ and decreasing in θ when $r_1>\frac{1}{2}(v_H+v_L)$. Recall $u_{se}(r_1,r_2,\ldots,r_N)-u_{si}(r_1)$ is decreasing in λ for $\underline{r}\leq r_1\leq\bar{r}$ due to Lemma EC.9(a), i.e., $u(\lambda,r_1)$ is decreasing in λ for $\underline{r}\leq r_1\leq\bar{r}$. Thus, $\hat{\lambda}'$ is increasing in θ when $r_1\leq\frac{1}{2}(v_H+v_L)$ and decreasing in θ when $r_1>\frac{1}{2}(v_H+v_L)$. From the definition of $\hat{\lambda}$ in the proof of Proposition 5, we get the announced results. □

D. Proofs of Auxiliary Lemmas

Proof of Lemma EC.1. First, we show problem P_g is equivalent to the following problem:

$$\begin{aligned} P_g'':\quad & \max_{v_1,\ldots,v_N} \quad u_g(r_1,r_2,\ldots,r_N)=v_H+\sum_{i=1}^{N}s(v_i-r_i)\\ & \text{s.t.} \quad v_L\leq v_1\leq v_2\leq\cdots\leq v_N=v_H.\end{aligned}$$

Note $v_i = \sum_{j=1}^{i} \delta_j$, then $\delta_1 = v_1$, $\delta_i = v_i - v_{i-1}$ for $i \in \{2, \dots, N\}$, and $\delta_1 + \delta_2 + \dots + \delta_N = v_N = v_H$. It is easy to see the objective function of problem P_g is equivalent to that of problem $P_g^{''}$ due to $v_i = \sum_{j=1}^{i} \delta_j$. Moreover, $\delta_i \geq 0$ for $i \in \{2, \dots, N\}$ implies $v_1 \leq v_2 \leq \dots \leq v_N$. Hence, problem P_g is equivalent to problem $P_g^{''}$.

Second, we show problem $P_g^{''}$ is equivalent to problem P_g' when $x_i = v_i - r_i \geq 0$ for $i \in \{1, 2, \dots, N-1\}$ by the following three steps. Consequently, problem P_g is equivalent to problem P_g' by the former paragraph.

Step 1: We prove $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$ by induction. If $N = 2$, $(1-\theta)\sum_{i=1}^{N-1} x_i = (1-\theta)x_1 = (1-\theta)(v_1 - r_1) = \theta r_1 + (1-\theta)v_1 - r_1 = r_2 - r_1$. If $N = k$, we suppose $(1-\theta)\sum_{i=1}^{k-1} x_i = r_k - r_1$, then $(1-\theta)\sum_{i=1}^{k} x_i = (1-\theta)\sum_{i=1}^{k-1} x_i + (1-\theta)x_k = r_k - r_1 + (1-\theta)(v_k - r_k) = \theta r_k + (1-\theta)v_k - r_1 = r_{k+1} - r_1$. Hence, $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$ holds for $N = k+1$.

Step 2: We prove $u_g(r_1, r_2, \dots, r_N) = v_H + \sum_{i=1}^{N} s(v_i - r_i) = v_H + \theta\alpha \sum_{i=1}^{N-1} x_i + \alpha(v_H - r_1)$. That is $v_H + \sum_{i=1}^{N} s(v_i - r_i) = v_H + \alpha \sum_{i=1}^{N-1} x_i + \alpha(v_H - r_N) = v_H + \theta\alpha \sum_{i=1}^{N-1} x_i + \alpha(r_N - r_1) + \alpha(v_H - r_N) = v_H + \theta\alpha \sum_{i=1}^{N-1} x_i + \alpha(v_H - r_1)$, where the second equality is due to $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$.

Step 3: We show $\theta x_i \leq x_{i+1}$ is equivalent to $v_i \leq v_{i+1}$ for $i \in \{1, 2, \dots, N-2\}$, and $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \leq v_H - r_1$ is equivalent to $v_{N-1} \leq v_N$. Note $\theta x_i - x_{i+1} = \theta(v_i - r_i) - (v_{i+1} - r_{i+1}) = \theta(v_i - r_i) - (v_{i+1} - \theta r_i - (1-\theta)v_i) = v_i - v_{i+1}$, then $\theta x_i \leq x_{i+1}$ is equivalent to $v_i \leq v_{i+1}$. Recall $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$ from step 1, then $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} - (v_H - r_1) = r_N - r_1 + \theta x_{N-1} - (v_H - r_1) = \theta r_{N-1} + (1-\theta)v_{N-1} + \theta(v_{N-1} - r_{N-1}) - v_H = v_{N-1} - v_H = v_{N-1} - v_N$, which implies $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \leq v_H - r_1$ is equivalent to $v_{N-1} \leq v_N$.

Last, we show problem P_g' is maximized by $x_i = \theta^{i-1}(v_H - r_1)$, i.e., $v_i = v_H$ for any $i \in \{1, 2, \dots, N\}$, and the corresponding objective value is $v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1)$. The objective function of problem P_g' is increasing in x_i, then it is optimal to let $\theta x_i = x_{i+1}$ for $i \in \{1, 2, \dots, N-2\}$, and $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} = v_H - r_1$. Note $\theta x_i = x_{i+1}$ for $i \in \{1, 2, \dots, N-2\}$ implies $x_i = \theta^{i-1}x_1$ for $i \in \{1, 2, \dots, N-2\}$. Thus, $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} = (1-\theta^{N-2})x_1 + \theta^{N-2}x_1 = x_1$, which implies $x_1 = v_H - r_1$. Consequently, $x_i = \theta^{i-1}x_1 = \theta^{i-1}(v_H - r_1)$, and the corresponding objective value is $v_H + \frac{1-\theta^N}{1-\theta}\alpha(v_H - r_1)$. Note $x_1 = v_H - r_1 = v_1 - r_1$, then $v_1 = v_H$. Based on $v_L \leq v_1 \leq v_2 \leq \dots \leq v_N = v_H$, we get $v_i = v_H$ for any $i \in \{1, 2, \dots, N\}$. $\square$

Proof of Lemma EC.2. From the proof of Lemma EC.1, we know problem P_g is equivalent to problem $P_g^{''}$. To show problem P_g is equivalent to problem P_g^t reduces to show problem $P_g^{''}$ is equivalent to problem P_g^t. Now we show $P_g^{''}$ is equivalent to problem P_g^t when $x_i = r_i - v_i \geq 0$ for $i \in \{1, 2, \ldots, t\}$, and $x_i = v_i - r_i \geq 0$ for $i \in \{t+1, t+2, \ldots, N-1\}$ by the following five steps.

Step 1: We prove $(1-\theta)\sum_{i=1}^{t} x_i = r_1 - r_{t+1}$ by induction. If $t = 1$, $(1-\theta)\sum_{i=1}^{t} x_i = (1-\theta)x_1 = (1-\theta)(r_1 - v_1) = r_1 - [\theta r_1 + (1-\theta)v_1] = r_1 - r_2$. If $t = k$, we suppose $(1-\theta)\sum_{i=1}^{k} x_i = r_1 - r_{k+1}$, then $(1-\theta)\sum_{i=1}^{k+1} x_i = (1-\theta)\sum_{i=1}^{k} x_i + (1-\theta)x_{k+1} = r_1 - r_{k+1} + (1-\theta)(r_{k+1} - v_{k+1}) = r_1 - [\theta r_{k+1} + (1-\theta)v_{k+1}] = r_1 - r_{k+2}$. Hence, $(1-\theta)\sum_{i=1}^{t} x_i = r_1 - r_{t+1}$ holds for $t = k+1$.

Step 2: We prove $(1-\theta)\sum_{i=t+1}^{N-1} x_i = r_N - r_{t+1}$ by induction. If $N = t+2$, $(1-\theta)\sum_{i=t+1}^{N-1} x_i = (1-\theta)(v_{t+1} - r_{t+1}) = \theta r_{t+1} + (1-\theta)v_{t+1} - r_{t+1} = r_{t+2} - r_{t+1}$. If $N = t+k$, we suppose $(1-\theta)\sum_{i=t+1}^{t+k-1} x_i = r_{t+k} - r_{t+1}$, then $(1-\theta)\sum_{i=t+1}^{t+k} x_i = (1-\theta)\sum_{i=t+1}^{t+k-1} x_i + (1-\theta)x_{t+k} = r_{t+k} - r_{t+1} + (1-\theta)(v_{t+k} - r_{t+k}) = \theta r_{t+k} + (1-\theta)v_{t+k} - r_{t+1} = r_{t+k+1} - r_{t+1}$. Hence, $(1-\theta)\sum_{i=t+1}^{N-1} x_i = r_N - r_{t+1}$ holds for $N = t+k+1$.

Step 3: We prove $u_g(r_1, r_2, \ldots, r_N) = v_H + \sum_{i=1}^{N} s(v_i - r_i) = v_H - (\lambda - (1-\theta))\alpha\sum_{i=1}^{t} x_i + \theta\alpha\sum_{i=t+1}^{N-1} x_i + \alpha(v_H - r_1)$. Based on steps 1 and 2, we have $(1-\theta)\sum_{i=1}^{t} x_i = r_1 - r_{t+1}$ and $(1-\theta)\sum_{i=t+1}^{N-1} x_i = r_N - r_{t+1}$. Then $r_N = (1-\theta)\sum_{i=t+1}^{N-1} x_i - (1-\theta)\sum_{i=1}^{t} x_i + r_1$. Hence, $v_H + \sum_{i=1}^{N} s(v_i - r_i) = v_H - \lambda\alpha\sum_{i=1}^{t} x_i + \alpha\sum_{i=t+1}^{N-1} x_i + \alpha(v_H - r_N) = v_H - \lambda\alpha\sum_{i=1}^{t} x_i + \alpha\sum_{i=t+1}^{N-1} x_i - \alpha(1-\theta)\sum_{i=t+1}^{N-1} x_i + \alpha(1-\theta)\sum_{i=1}^{t} x_i + \alpha(v_H - r_1) = v_H - (\lambda - (1-\theta))\alpha\sum_{i=1}^{t} x_i + \theta\alpha\sum_{i=t+1}^{N-1} x_i + \alpha(v_H - r_1)$.

Step 4: We now show $x_{i+1} \leq \theta x_i$ is equivalent to $v_i \leq v_{i+1}$ for $i \in \{1, 2, \ldots, t-1\}$, $\theta x_i \leq x_{i+1}$ is equivalent to $v_i \leq v_{i+1}$ for $i \in \{t+1, t+2, \ldots, N-2\}$, and $v_t \leq v_{t+1}$. For $i \in \{1, 2, \ldots, t-1\}$, $\theta x_i - x_{i+1} = \theta(r_i - v_i) - (r_{i+1} - v_{i+1}) = \theta(r_i - v_i) - (\theta r_i + (1-\theta)v_i - v_{i+1}) = v_{i+1} - v_i$, then $x_{i+1} \leq \theta x_i$ is equivalent to $v_i \leq v_{i+1}$. For $i \in \{t+1, t+2, \ldots, N-2\}$, $\theta x_i - x_{i+1} = \theta(v_i - r_i) - (v_{i+1} - r_{i+1}) = \theta(v_i - r_i) + \theta r_i + (1-\theta)v_i - v_{i+1} = v_i - v_{i+1}$, then $\theta x_i \leq x_{i+1}$ is equivalent to $v_i \leq v_{i+1}$. Note $x_{t+1} = v_{t+1} - r_{t+1} = v_{t+1} - \theta r_t - (1-\theta)v_t = v_{t+1} - v_t - \theta x_t \geq 0$, which implies $v_{t+1} - v_t \geq \theta x_t \geq 0$.

Step 5: We show $(1-\theta)\sum_{i=t+1}^{N-2} x_i + x_{N-1} - (1-\theta)\sum_{i=1}^{t} x_i \leq v_H - r_1$ is equivalent to $v_{N-1} \leq v_N = v_H$, and $x_1 \leq r_1 - v_L$ is equivalent to $v_1 \geq v_L$. Recall $r_N = (1-\theta)\sum_{i=t+1}^{N-1} x_i - (1-\theta)\sum_{i=1}^{t} x_i + r_1$, then $(1-\theta)\sum_{i=t+1}^{N-2} x_i + x_{N-1} - (1-\theta)\sum_{i=1}^{t} x_i - (v_H - r_1) = r_N - r_1 + \theta x_{N-1} - (v_H - r_1) = \theta r_{N-1} + (1-\theta)v_{N-1} + \theta(v_{N-1} - r_{N-1}) - v_H = v_{N-1} - v_H$, where the second equality is due to $r_N = \theta r_{N-1} + (1-\theta)v_{N-1}$ and $x_{N-1} = v_{N-1} - r_{N-1}$. It is easy to see $x_1 \leq r_1 - v_L$ is equivalent to $v_1 \geq v_L$ due to $x_1 = r_1 - v_1$.

Last, we solve problem P_g^t. Note the objective function of problem P_g^t is increasing in x_i for

$i > t$. Then it is optimal to let $(1-\theta)\sum_{i=t+1}^{N-2} x_i + x_{N-1} - (1-\theta)\sum_{i=1}^{t} x_i = v_H - r_1$ and $\theta x_i = x_{i+1}$ for $i \in \{t+1, t+2, \ldots, N-2\}$. Otherwise, we can always increase x_i for $i > t$ to increase the objective value. Solving $(1-\theta)\sum_{i=t+1}^{N-2} x_i + x_{N-1} - (1-\theta)\sum_{i=1}^{t} x_i = v_H - r_1$ and $\theta x_i = x_{i+1}$, we get $x_{t+1} = (1-\theta)\sum_{i=1}^{t} x_i + v_H - r_1$ and $x_i = \theta^{i-t-1} x_{t+1}$ for $i \in \{t+1, t+2, \ldots, N-1\}$. Thus, problem P_g is reduced to the following problem

$$P_g^{t'}: \quad \max_{x_i} \quad v_H + (1-\theta^{N-t} - \lambda)\alpha \sum_{i=1}^{t} x_i + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$$
$$s.t. \quad x_1 \le r_1 - v_L,\ 0 \le x_{i+1} \le \theta x_i,\ i \in \{1, 2, \ldots, t-1\}.$$

Hence, if $\lambda \le 1-\theta^{N-t}$, the objective function is increasing in x_i; otherwise, it is decreasing in x_i. We next show the optimal solution of problem P_g^t, and the corresponding objective value is described as Lemma EC.2.

If $\lambda \le 1-\theta^{N-t}$, it is optimal to let $x_1 = r_1 - v_L$ and $x_{i+1} = \theta x_i$ for $i \in \{1, 2, \ldots, t-1\}$ for problem $P_g^{t'}$. That is $x_i = \theta^{i-1}(r_1 - v_L)$ for $i \in \{1, 2, \ldots, t\}$, and the corresponding objective value is $v_H - \lambda\frac{1-\theta^t}{1-\theta}\alpha(r_1 - v_L) + \frac{1-\theta^{N-t}}{1-\theta}\alpha[v_H - \theta^t r_1 - (1-\theta^t)v_L]$. Recall $x_1 = r_1 - v_1$, and $x_{i+1} = \theta x_i$ is equivalent to $v_{i+1} = v_i$, then $v_i = v_L$ for $i \in \{1, 2, \ldots, t\}$. On the one hand, from the previous paragraph, $x_{t+1} = (1-\theta)\sum_{i=1}^{t} x_i + v_H - r_1 = (1-\theta^t)(r_1 - v_L) + v_H - r_1 = v_H - \theta^t r_1 - (1-\theta^t)v_1$. On the other hand, $x_{t+1} = v_{t+1} - r_{t+1} = v_{t+1} - \theta^t r_1 - (1-\theta^t)v_1$ due to $r_{i+1} = \theta r_i + (1-\theta)v_i$ and $v_i = v_L$ for $i \in \{1, 2, \ldots, t\}$. Hence, we get $v_{t+1} = v_H$. Based on $v_L \le v_1 \le v_2 \le \cdots \le v_N = v_H$, then $v_i = v_H$ for $i \in \{t+1, t+2, \ldots, N\}$. Moreover, $x_i = \theta^{i-t-1}x_{t+1} = \theta^{i-t-1}[(1-\theta^t)(r_1 - v_L) + v_H - r_1]$ for $i \in \{t+1, t+2, \ldots, N-1\}$.

If $\lambda > 1-\theta^{N-t}$, it is optimal to let $x_i = 0$ for $i \in \{1, 2, \ldots, t\}$ for problem $P_g^{t'}$, and the corresponding objective value is $v_H + \frac{1-\theta^{N-t}}{1-\theta}\alpha(v_H - r_1)$. Recall $x_1 = r_1 - v_1 = 0$, then $v_1 = r_1$. Then $x_i = r_i - v_i = 0$ implies $v_i = r_i = r_1$ for $i \in \{1, 2, \ldots, t\}$. On the one hand, from the previous paragraph, $x_{t+1} = (1-\theta)\sum_{i=1}^{t} x_i + v_H - r_1 = v_H - r_1$. On the other hand, $x_{t+1} = v_{t+1} - r_{t+1} = v_{t+1} - r_1$ due to $r_{i+1} = \theta r_i + (1-\theta)v_i$ and $v_i = r_1$ for $i \in \{1, 2, \ldots, t\}$. Hence, we get $v_{t+1} = v_H$. Based on $v_L \le v_1 \le v_2 \le \cdots \le v_N = v_H$, then $v_i = v_H$ for $i \in \{t+1, t+2, \ldots, N\}$. Moreover, $x_i = \theta^{i-t-1}x_{t+1} = \theta^{i-t-1}(v_H - r_1)$ for $i \in \{t+1, t+2, \ldots, N-1\}$. □

Proof of Lemma EC.3. First, we show problem P_b is equivalent to the following problem:

$$P_b^{'''}: \quad \max_{v_1, \ldots, v_N} \quad u_b(r_1, r_2, \ldots, r_N) = v_L + \sum_{i=1}^{N} s(v_i - r_i)$$
$$\text{s.t.} \quad v_H \ge v_1 \ge v_2 \ge \cdots \ge v_N = v_L.$$

Note $v_i = \sum_{j=1}^{i} \delta_j$, then $\delta_1 = v_1$, $\delta_i = v_i - v_{i-1}$ for $i \in \{2, \ldots, N\}$, and $\delta_1 + \delta_2 + \cdots + \delta_N = v_N = v_L$. It is easy to see the objective function of problem P_b is equivalent to that of problem $P_b^{'''}$ due to $v_i = \sum_{j=1}^{i} \delta_j$. Moreover, $\delta_i \le 0$ for $i \in \{2, \dots, N\}$ implies $v_1 \ge v_2 \ge \dots \ge v_N$. Hence, problem P_b is equivalent to problem $P_b^{'''}$.

Second, we show problem $P_b^{'''}$ is equivalent to problem P_g' when $x_i = r_i - v_i \ge 0$ for $i \in \{1, 2, \dots, N-1\}$ by the following three steps. Consequently, problem P_b is equivalent to problem P_b' by the former paragraph.

Step 1: We prove $(1-\theta)\sum_{i=1}^{N-1} x_i = r_1 - r_N$ by induction. If $N = 2$, $(1-\theta)\sum_{i=1}^{N-1} x_i = (1-\theta)x_1 = (1-\theta)(r_1 - v_1) = r_1 - [\theta r_1 + (1-\theta)v_1] = r_1 - r_2$. If $N = k$, we suppose $(1-\theta)\sum_{i=1}^{k-1} x_i = r_1 - r_k$, then $(1-\theta)\sum_{i=1}^{k} x_i = (1-\theta)\sum_{i=1}^{k-1} x_i + (1-\theta)x_k = r_1 - r_k + (1-\theta)(r_k - v_k) = r_1 - [\theta r_k + (1-\theta)v_k] = r_1 - r_{k+1}$. Hence, $(1-\theta)\sum_{i=1}^{N-1} x_i = r_1 - r_N$ holds for $N = k+1$.

Step 2: We prove $u_b(r_1, r_2, \dots, r_N) = v_L + \sum_{i=1}^{N} s(v_i - r_i) = v_L - \lambda\theta\alpha\sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$. That is $v_L + \sum_{i=1}^{N} s(v_i - r_i) = v_L - \lambda\alpha\sum_{i=1}^{N-1} x_i - \lambda\alpha(r_N - v_L) = v_L - \lambda\theta\alpha\sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - r_N) - \lambda\alpha(r_N - v_L) = v_L - \lambda\theta\alpha\sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$, where the second equality is due to $(1-\theta)\sum_{i=1}^{N-1} x_i = r_1 - r_N$.

Step 3: We show $\theta x_i \le x_{i+1}$ is equivalent to $v_i \ge v_{i+1}$ for $i \in \{1, 2, \dots, N-2\}$, and $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \le r_1 - v_L$ is equivalent to $v_{N-1} \ge v_N$. Note $\theta x_i - x_{i+1} = \theta(r_i - v_i) - (r_{i+1} - v_{i+1}) = \theta(r_i - v_i) - (\theta r_i + (1-\theta)v_i - v_{i+1}) = v_{i+1} - v_i$, then $\theta x_i \le x_{i+1}$ is equivalent to $v_i \ge v_{i+1}$. Recall $(1-\theta)\sum_{i=1}^{N-1} x_i = r_1 - r_N$ from step 1, then $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} - (r_1 - v_L) = r_1 - r_N + \theta x_{N-1} - (r_1 - v_L) = r_1 - r_N + \theta(r_{N-1} - v_{N-1}) - (r_1 - v_L) = r_1 - [\theta r_{N-1} + (1-\theta)v_{N-1}] + \theta(r_{N-1} - v_{N-1}) - (r_1 - v_L) = v_L - v_{N-1} = v_N - v_{N-1}$, which implies $(1-\theta)(x_1 + x_2 + \dots + x_{N-2}) + x_{N-1} \le r_1 - v_L$ is equivalent to $v_{N-1} \ge v_N$.

Last, we solve problem P_b'. It is easy to see the objective function of problem P_b' is decreasing in x_i for $i \in \{1, 2, \dots, N-1\}$. Then it is optimal to let $x_i = 0$ for $i \in \{1, 2, \dots, N-1\}$, and the the corresponding objective value is $v_L - \lambda\alpha(r_1 - v_L)$. Recall $x_1 = r_1 - v_1 = 0$, then $v_1 = r_1$. Then $x_i = r_i - v_i = 0$ implies $v_i = r_i = r_1$ for $i \in \{1, 2, \dots, N-1\}$. □

Proof of Lemma EC.4. From the proof of Lemma EC.3, we know problem P_b is equivalent to problem $P_b^{'''}$. To show problem P_b is equivalent to problem $P_b^{''}$ reduces to show problem $P_b^{'''}$ is equivalent to problem $P_b^{''}$. Now we show $P_b^{'''}$ is equivalent to problem $P_b^{''}$ when $x_i = v_i - r_i \ge 0$ for $i \in \{1, 2, \dots, N-1\}$ by the following three steps.

Step 1: We prove $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$ by induction. If $N = 2$, $(1-\theta)\sum_{i=1}^{N-1} x_i = (1-\theta)x_1 = (1-\theta)(v_1 - r_1) = \theta r_1 + (1-\theta)v_1 - r_1 = r_2 - r_1$. If $N = k$, we suppose $(1-\theta)\sum_{i=1}^{k-1} x_i = r_k - r_1$, then

$(1-\theta)\sum_{i=1}^{k} x_i = (1-\theta)\sum_{i=1}^{k-1} x_i + (1-\theta)x_k = r_k - r_1 + (1-\theta)(v_k - r_k) = \theta r_k + (1-\theta)v_k - r_1 = r_{k+1} - r_1$. Hence, $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$ holds for $N = k+1$.

Step 2: We prove $u_b(r_1, r_2, \ldots, r_N) = v_L + \sum_{i=1}^{N} s(v_i - r_i) = v_L + (1-\lambda(1-\theta))\alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$. That is $v_L + \sum_{i=1}^{N} s(v_i - r_i) = v_L + \alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha(r_N - v_L) = v_L + \alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha((1-\theta)\sum_{i=1}^{N-1} x_i + r_1 - v_L) = v_L + (1-\lambda(1-\theta))\alpha \sum_{i=1}^{N-1} x_i - \lambda\alpha(r_1 - v_L)$, where the second equality is due to $(1-\theta)\sum_{i=1}^{N-1} x_i = r_N - r_1$.

Step 3: We show $\theta x_i \geq x_{i+1}$ is equivalent to $v_i \geq v_{i+1}$ for $i \in \{1, 2, \ldots, N-2\}$, and $x_1 \leq v_H - r_1$. Note $\theta x_i - x_{i+1} = \theta(v_i - r_i) - (v_{i+1} - r_{i+1}) = \theta(v_i - r_i) - (v_{i+1} - \theta r_i - (1-\theta)v_i) = v_i - v_{i+1}$, then $\theta x_i \geq x_{i+1}$ is equivalent to $v_i \geq v_{i+1}$. It is easy to see $x_1 = v_1 - r_1 \leq v_H - r_1$.

Last, we solve problem P_b''. Note the objective function of problem P_b'' is increasing in x_i if $\lambda \leq 1/(1-\theta)$ and decreasing in x_i otherwise. If $\lambda \leq 1/(1-\theta)$, it is optimal to let $x_1 = v_H - r_1$ and $\theta x_i = x_{i+1}$ for $i \in \{1, 2, \ldots, N-2\}$, which implies $x_i = \theta^{i-1}(v_H - r_1)$ for $i \in \{1, 2, \ldots, N-1\}$. Consequently, the corresponding objective value is $v_L + \frac{1-\lambda(1-\theta)}{1-\theta}\alpha(v_H - r_1) - \lambda\alpha(r_1 - v_L)$. Note $\theta x_i = x_{i+1}$ is equivalent to $v_i = v_{i+1}$ for $i \in \{1, 2, \ldots, N-2\}$ from step 2. Based on $x_1 = v_1 - r_1 = v_H - r_1$ and $\theta x_i = x_{i+1}$ for $i \in \{1, 2, \ldots, N-2\}$, we get $v_i = v_H$ for $i \in \{1, 2, \ldots, N-1\}$. Otherwise, the optimal solution of problem P_b'' is $x_i = 0$ for $i \in \{1, 2, \ldots, N-1\}$, i.e., $v_i = r_1$ for $i \in \{1, 2, \ldots, N-1\}$, and the corresponding objective value is $v_L - \lambda\alpha(r_1 - v_L)$. Recall $x_1 = r_1 - v_1 = 0$, then $v_1 = r_1$. Then $x_i = r_i - v_i = 0$ implies $v_i = r_i = r_1$ for $i \in \{1, 2, \ldots, N-1\}$. □

Proof of Lemma EC.5. Consider a policy: the first x performances are high-type, then N_L performances are low-type, and the last y performances are high-type, where $x + y = N_H$ and $0 \leq x \leq N_H - 1$. Note it is a crescendo pattern if $x = 0$ and a U-shaped pattern if $x > 0$. We will show the utility function is concave in x when $\lambda \leq 1$ and decreasing in x when $\lambda > 1$. By (2) and (4), we have the utility function

$$u(r_1, r_2, \ldots, r_N|x) = N_H v_H + N_L v_L + \alpha(N_H v_H + \lambda N_L v_L) - \alpha\left[\sum_{i=1}^{x} r_i + \lambda \sum_{i=x+1}^{x+N_L} r_i + \sum_{i=x+N_L+1}^{N_H+N_L} r_i\right]. \tag{EC.7}$$

If $i \leq x+1$, $r_i = \theta^{i-1} r_1 + (1-\theta^{i-1})v_H$, if $x+1 \leq i \leq x+N_L+1$, $r_i = \theta^{i-x-1} r_{x+1} + (1-\theta^{i-x-1})v_L$, if $x+N_L+1 \leq i \leq N_H+N_L$, $r_i = \theta^{i-x-N_L-1} r_{x+N_L+1} + (1-\theta^{i-x-N_L-1})v_H$. Then,

$$\begin{aligned}
&\sum_{i=1}^{x} r_i + \lambda \sum_{i=x+1}^{x+N_L} r_i + \sum_{i=x+N_L+1}^{N_H+N_L} r_i \\
=&\frac{1-\theta^x}{1-\theta} r_1 + \left(x - \frac{1-\theta^x}{1-\theta}\right)v_H + \lambda\frac{1-\theta^{N_L}}{1-\theta} r_{x+1} + \lambda\left(N_L - \frac{1-\theta^{N_L}}{1-\theta}\right)v_L \\
&+ \frac{1-\theta^y}{1-\theta} r_{x+N_L+1} + \left(y - \frac{1-\theta^y}{1-\theta}\right)v_H,
\end{aligned} \tag{EC.8}$$

where $r_{x+1} = \theta^x r_1 + (1-\theta^x)v_H$, $r_{x+N_L+1} = \theta^{N_L}(\theta^x r_1 + (1-\theta^x)v_H) + (1-\theta^{N_L})v_L$, and

$$\begin{aligned}
&\frac{\partial\left(\sum_{i=1}^{x} r_i + \lambda \sum_{i=x+1}^{x+N_L} r_i + \sum_{i=x+N_L+1}^{N_H+N_L} r_i\right)}{\partial x} \\
=&\frac{\theta^x ln\theta}{1-\theta}(v_H - r_1) - \lambda\frac{1-\theta^{N_L}}{1-\theta}\theta^x ln\theta(v_H - r_1) + \frac{\theta^y ln\theta}{1-\theta}[\theta^{N_L}(\theta^x r_1 + (1-\theta^x)v_H) + (1-\theta^{N_L})v_L - v_H] \\
&-\frac{1-\theta^y}{1-\theta}\theta^{x+N_L} ln\theta(v_H - r_1) \\
=&\frac{\theta^x ln\theta}{1-\theta}(v_H - r_1) - \lambda\frac{1-\theta^{N_L}}{1-\theta}\theta^x ln\theta(v_H - r_1) - \frac{\theta^y}{1-\theta}\theta^{x+N_L} ln\theta(v_H - r_1) - \frac{\theta^y}{1-\theta}(1-\theta^{N_L}) ln\theta(v_H - v_L) \\
&-\frac{1-\theta^y}{1-\theta}\theta^{x+N_L} ln\theta(v_H - r_1) \\
=&\frac{(1-\lambda)(1-\theta^{N_L})}{1-\theta}\theta^x ln\theta(v_H - r_1) - \frac{\theta^y}{1-\theta}(1-\theta^{N_L}) ln\theta(v_H - v_L),
\end{aligned}$$

which is increasing in x when $\lambda \le 1$ and greater than 0 when $\lambda > 1$ due to $ln\theta < 0$. Thus $\sum_{i=1}^{x} r_i + \lambda \sum_{i=x+1}^{x+N_L} r_i + \sum_{i=x+N_L+1}^{N_H+N_L} r_i$ is convex in x when $\lambda \le 1$ and increasing in x when $\lambda > 1$. Consequently, $u(r_1, r_2, \ldots, r_N|x)$ is concave in x when $\lambda \le 1$ and decreasing in x when $\lambda > 1$. Hence, $u(r_1, r_2, \ldots, r_N|x)$ is maximized at $x = 0$ when $\lambda > 1$. In other words, a crescendo pattern can generate more utility than a U-shaped pattern when $\lambda > 1$.

Consider $\lambda \le 1$, since $u(r_1, r_2, \ldots, r_N|x)$ is concave in x, the sufficient and necessary condition of a crescendo pattern is better than a U-shaped pattern is that $u(r_1, r_2, \ldots, r_N|x = 0) \ge u(r_1, r_2, \ldots, r_N|x = 1)$. This is because if and only if $u(r_1, r_2, \ldots, r_N|x)$ is decreasing in $x \ge 0$ or $u(r_1, r_2, \ldots, r_N|x)$ is maximized at $0 < x^* < 1$ and $u(r_1, r_2, \ldots, r_N|x = 0) \ge u(r_1, r_2, \ldots, r_N|x = 1)$, a crescendo pattern is better than a U-shaped pattern. No matter in which case, it implies $u(r_1, r_2, \ldots, r_N|x = 0) \ge u(r_1, r_2, \ldots, r_N|x = 1)$. By (EC.7) and (EC.8),

$$\begin{aligned}
&u(r_1, r_2, \ldots, r_N|x = 0) = N_H v_H + N_L v_L + \alpha(N_H v_H + \lambda N_L v_L) - \alpha\lambda\left(N_L - \frac{1-\theta^{N_L}}{1-\theta}\right)v_L \\
&-\alpha\left[\lambda\frac{1-\theta^{N_L}}{1-\theta}r_1 + \frac{1-\theta^{N_H}}{1-\theta}[\theta^{N_L} r_1 + (1-\theta^{N_L})v_L] + \left(N_H - \frac{1-\theta^{N_H}}{1-\theta}\right)v_H\right], \\
&u(r_1, r_2, \ldots, r_N|x = 1) = N_H v_H + N_L v_L + \alpha(N_H v_H + \lambda N_L v_L) - \alpha\lambda\left(N_L - \frac{1-\theta^{N_L}}{1-\theta}\right)v_L \\
&-\alpha\left[r_1 + \lambda\frac{1-\theta^{N_L}}{1-\theta}(\theta r_1 + (1-\theta)v_H) + \frac{1-\theta^{N_H-1}}{1-\theta}[\theta^{N_L}(\theta r_1 + (1-\theta)v_H) + (1-\theta^{N_L})v_L]\right] \\
&+\alpha\left(N_H - 1 - \frac{1-\theta^{N_H-1}}{1-\theta}\right)v_H.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&u(r_1, r_2, \ldots, r_N|x = 0) - u(r_1, r_2, \ldots, r_N|x = 1) \\
=&\alpha\left[(\lambda - 1)(1-\theta^{N_L})(v_H - r_1) + \theta^{N_H-1}(1-\theta^{N_L})(v_H - v_L)\right].
\end{aligned}$$

Therefore, $u(r_1, r_2, \dots, r_N|x=0) - u(r_1, r_2, \dots, r_N|x=1) > 0$ equals to $(\lambda - 1)(1-\theta^{N_L})(v_H - r_1) + \theta^{N_H-1}(1-\theta^{N_L})(v_H - v_L) > 0$. In other words, $r_1 > v_H - \theta^{N_H-1}(v_H - v_L)$ or $\lambda > 1 - \theta^{N_H-1}(v_H - v_L)/(v_H - r_1)$, which is equal to $\lambda > \lambda'$. □

Proof of Lemma EC.6. First, we prove (a) by induction. For $N_L = 1$, there are two possible policies: $v_1 = v_L$, $v_2 = v_H$ and $v_1 = v_H$, $v_2 = v_L$. The total utility for the first and second policy are $v_L + v_H - \lambda\alpha(r_1 - v_L) + \alpha(v_H - (\theta r_1 + (1-\theta)v_L))$ and $v_L + v_H + \alpha(v_H - r_1) - \lambda\alpha(\theta r_1 + (1-\theta)v_H - v_L)$, respectively. Note $\alpha(v_H - (\theta r_1 + (1-\theta)v_L)) \geq \alpha(v_H - r_1)$ and $-\alpha(r_1 - v_L) \geq -\alpha(\theta r_1 + (1-\theta)v_H - v_L)$. Hence, the first policy is better than the second. Suppose it holds for $N_L = k$. Now, we prove it is true for $N_L = k+1$. If $v_1 = v_L$, the remaining performances should be arranged by: $v_i = v_L$ for $i = 2, 3, \dots, k+1$, and $v_i = v_H$ for $i = k+2$. If $v_1 = v_H$, the remaining performances should be arranged by $v_i = v_L$ for $i = 2, 3, \dots, k+2$. To complete the proof, we need to show the total utility of the former one is more than the latter one. The total utility of the former is

$$u_1 = (k+1)v_L + v_H - \sum_{i=1}^{k+1} \lambda\alpha\theta^{i-1}(r_1 - v_L) + \alpha(v_H - \theta^{k+1}r_1 - (1-\theta^{k+1})v_L),$$

and the total utility of the latter is

$$u_2 = (k+1)v_L + v_H + \alpha(v_H - r_1) - \sum_{i=1}^{k+1} \lambda\alpha\theta^{i-1}(r_2 - v_L),$$

where $r_2 = \theta r_1 + (1-\theta)v_H > r_1$. Note $v_H - \theta^{k+1}r_1 - (1-\theta^{k+1})v_L > v_H - r_1$ and $\theta^{i-1}(r_1 - v_L) \leq \theta^{i-1}(r_2 - v_L)$, thus we have $u_1 > u_2$.

Second, we prove (b). Since $N_L = 1$, then any policy is either U-shaped or crescendo pattern, i.e., any policy can be expressed as the first x performances are high-type, then 1 performance is low-type, the last y performances are high-type, where $x + y = N_H$ and $0 \leq x \leq N_H - 1$. Based on Lemma EC.5, the optimal policy is a U-shaped pattern if $\lambda \leq \lambda'$ and crescendo pattern otherwise. By the proof of Lemma EC.5, any possible policy has a utility function like (EC.7) with $N_L = 1$. From the analysis in the proof of Lemma EC.5, we know (EC.7) is concave, and its first order derivative is decreasing in λ for $\lambda < 1$ (the optimal design is a crescendo pattern when $\lambda \geq 1$). Thus x^* is decreasing in $\lambda < 1$. Note the order of low-type performance is $x^* + 1$, then it is always optimal to arrange the unique low-type performance earlier as λ becomes larger.

Last, we prove the last performance in the optimal design is a high-type performance by contradiction. Suppose the last performance in the optimal design is low-type performance, and $j = \max\{i | v_i = v_H\}$. Consider a new policy as same as the optimal policy except in periods j and N, i.e., $v_j = v_L$ and $v_N = v_H$. It is easy to see the utilities from the two policies are the same before period j; however, the utility from the new policy from period j to N is greater due to (a). □

Proof of Lemma EC.7. We prove it by induction. For $i=2$, on the one hand, r_2 is given by

$$r_2 = \begin{cases} \theta r_1 + (1-\theta)v_H, \text{ with probability } 1/2, \\ \theta r_1 + (1-\theta)v_L, \text{ with probability } 1/2. \end{cases} \tag{EC.9}$$

On the other hand, $\Theta_2 = \{1\}$, and

$$\delta_2 = \begin{cases} 0, \text{ with probability } 1/2, \\ 1, \text{ with probability } 1/2. \end{cases}$$

Hence, $r_2 = \theta r_1 + (1-\theta)v_H$ or $r_2 = \theta r_1 + (1-\theta)v_L$ with probability 1/2, which is equivalent to (EC.9). We suppose it is true for $i=n$ and verify Lemma EC.7 for $i=n+1$. Note $r_{n+1} = \theta r_n + (1-\theta)v_L$ or $r_{n+1} = \theta r_n + (1-\theta)v_H$ with probability $\frac{1}{2}$. Recall $r_n = \theta^{n-1}r_1 + \delta_n(1-\theta)v_L + (1-\delta_n(1-\theta)-\theta^{n-1})v_H$, thus $r_{n+1} = \theta^n r_1 + (1+\delta_n\theta)(1-\theta)v_L + \theta(1-\delta_n(1-\theta)-\theta^{n-1})v_H$ or $r_{n+1} = \theta^n r_1 + \delta_n\theta(1-\theta)v_L + (1-\delta_n\theta(1-\theta)-\theta^n)v_H$ with probability 1/2. It is equivalent to show $\theta_{n+1} = 1+\delta_n\theta$ or $\theta_{n+1} = \delta_n\theta$. For any $A \subset \Theta_n$, $\theta A \subset \Theta_{n+1}$ and $1+\theta A \subset \Theta_{n+1}$, we thus get the desired result.

Now consider $\mathbb{E}[\delta_i]$. Recall the definition of δ_i, we have $\mathbb{E}[\delta_i] = \frac{\sum_{j=0}^{i-1} jC_{i-1}^j}{2^{i-1}(i-1)}(1+\theta+\cdots+\theta^{i-2}) = \frac{1}{2}\cdot\frac{1-\theta^{i-1}}{1-\theta}$, which is due to $\sum_{j=0}^{i-1} jC_{i-1}^j = \frac{i-1}{2^{i-2}}$. Thus, $\mathbb{E}[r_i] = \theta^{i-1}r_1 + \frac{1}{2}(1-\theta^{i-1})v_L + \frac{1}{2}(1-\theta^{i-1})v_H$. $\square$

Proof of Lemma EC.8. By (EC.6), when $r_1 = \frac{1}{2}(v_H+v_L)$,

$$u_{se}(r_1, r_2, \ldots, r_N) = \frac{N}{2}(v_H+v_L) + \frac{N}{4}\alpha(1-\lambda)(v_H-v_L). \tag{EC.10}$$

If N is an odd number, then $\frac{N-1}{2}v_H + \frac{N+1}{2}v_L < Nr_1 = \frac{N}{2}(v_H+v_L) < \frac{N+1}{2}v_H + \frac{N-1}{2}v_L$, which implies $m = \frac{N+1}{2}$. By (EC.5), when $r_1 = \frac{1}{2}(v_H+v_L)$,

$$\begin{aligned} u_{si}(r_1) =& \frac{N}{2}(v_H+v_L) + \alpha\frac{\sum_{j=\frac{N+1}{2}}^{N} C_N^j[jv_H+(N-j)v_L-Nr_1] + \lambda\sum_{j=0}^{\frac{N-1}{2}} C_N^j[jv_H+(N-j)v_L-Nr_1]}{2^N} \\ =& \frac{N}{2}(v_H+v_L) + \alpha\frac{\left(N2^{N-2}(v_H+v_L) - \sum_{j=\frac{N+1}{2}}^{N} C_N^j Nr_1\right)(1+\lambda)}{2^N} = \frac{N}{2}(v_H+v_L), \end{aligned}$$

where the second equality is due to $\sum_{j=\frac{N+1}{2}}^{N} C_N^j j = \sum_{j=0}^{\frac{N-1}{2}} C_N^j j = N2^{N-2}$ and $\sum_{j=\frac{N+1}{2}}^{N} C_N^j(N-j) = \sum_{j=0}^{\frac{N-1}{2}} C_N^j(N-j) = N2^{N-2}$, and the last equality is due to $r_1 = \frac{1}{2}(v_H+v_L)$ and $\sum_{j=\frac{N+1}{2}}^{N} C_N^j = 2^{N-1}$.

If N is an even number, then $\frac{N}{2}v_H + \frac{N}{2}v_L = Nr_1 = \frac{N}{2}(v_H+v_L) < \frac{N+2}{2}v_H + \frac{N-2}{2}v_L$, which implies $m = \frac{N}{2}+1$. By (EC.5), when $r_1 = \frac{1}{2}(v_H+v_L)$,

$$
\begin{aligned}
u_{si}(r_1) =& \frac{N}{2}(v_H+v_L) + \alpha \frac{\sum_{j=\frac{N}{2}+1}^{N} C_N^j [jv_H + (N-j)v_L - Nr_1] + \lambda \sum_{j=0}^{\frac{N}{2}} C_N^j [jv_H + (N-j)v_L - Nr_1]}{2^N} \\
=& \frac{N}{2}(v_H+v_L) + \alpha \frac{\left(\frac{N}{4}(2^N - C_N^{N/2})(v_H+v_L) - \sum_{j=\frac{N}{2}+1}^{N} C_N^j N r_1\right)(1+\lambda)}{2^N} = \frac{N}{2}(v_H+v_L),
\end{aligned}
$$

where the second equality is due to $\sum_{j=\frac{N}{2}+1}^{N} C_N^j j = \sum_{j=0}^{\frac{N}{2}-1} C_N^j j = \frac{N}{4}(2^N - C_N^{N/2})$ and $\sum_{j=\frac{N+1}{2}}^{N} C_N^j (N-j) = \sum_{j=0}^{\frac{N-1}{2}} C_N^j (N-j) = \frac{N}{4}(2^N - C_N^{N/2})$, and the last equality is due to $r_1 = \frac{1}{2}(v_H+v_L)$ and $\sum_{j=\frac{N}{2}+1}^{N} C_N^j = \frac{1}{2}(2^N - C_N^{N/2})$. In summary, $u_{si}(r_1) = \frac{N}{2}(v_H+v_L)$ if $r_1 = \frac{1}{2}(v_H+v_L)$. Together with (EC.10), we get the desired results. □

Proof of Lemma EC.9. (a) By (EC.5) and (EC.6), we have

$$
\begin{aligned}
u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1) =& \alpha\Big[\frac{N}{4}(1-\lambda)(v_H - v_L) + \frac{1}{4}(1+\lambda)\frac{1-\theta^N}{1-\theta}(v_H + v_L - 2r_1)\Big] \\
& - \alpha \frac{\sum_{j=m}^{N} C_N^j [jv_H + (N-j)v_L - Nr_1] + \lambda \sum_{j=0}^{m-1} C_N^j [jv_H + (N-j)v_L - Nr_1]}{2^N}.
\end{aligned}
$$

Let

$$
\begin{aligned}
u(\lambda, r_1) =& \frac{N}{4}(1-\lambda)(v_H - v_L) + \frac{1}{4}(1+\lambda)\frac{1-\theta^N}{1-\theta}(v_H + v_L - 2r_1) \\
& - \frac{\sum_{j=m}^{N} C_N^j [jv_H + (N-j)v_L - Nr_1] + \lambda \sum_{j=0}^{m-1} C_N^j [jv_H + (N-j)v_L - Nr_1]}{2^N}.
\end{aligned} \tag{EC.11}
$$

Then we have

$$
\begin{aligned}
& \frac{\partial u(\lambda, r_1)}{\partial \lambda} \\
=& -\frac{N}{4}(v_H - v_L) + \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H + v_L - 2r_1) - \frac{\sum_{j=0}^{m-1} C_N^j [jv_H + (N-j)v_L - Nr_1]}{2^N} \\
=& -\frac{N}{4}(v_H - v_L) + \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H + v_L) - \frac{\sum_{j=0}^{m-1} C_N^j [jv_H + (N-j)v_L]}{2^N} - \Big(\frac{1}{2}\cdot\frac{1-\theta^N}{1-\theta} - \frac{\sum_{j=0}^{m-1} C_N^j N}{2^N}\Big) r_1,
\end{aligned}
$$

which is less than 0 if $\frac{1}{2}\cdot\frac{1-\theta^N}{1-\theta} \geq \frac{\sum_{j=0}^{m-1} C_N^j N}{2^N}$ due to $\frac{1-\theta^N}{1-\theta} = 1 + \theta + \cdots + \theta^{N_1} < N$. Note m is increasing in r_1, then $\frac{\partial u(\lambda,\theta,r_1)}{\partial \lambda}$ is increasing in r_1 if $\frac{1}{2}\cdot\frac{1-\theta^N}{1-\theta} < \frac{\sum_{j=0}^{m-1} C_N^j N}{2^N}$. Consider $r_1 = v_H$, then

$$\begin{aligned}\frac{\partial u(\lambda, r_1 = v_H)}{\partial \lambda} &= -\frac{N}{4}(v_H - v_L) - \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) + \frac{\sum_{j=0}^{N-1} C_N^j (N-j)(v_H - v_L)}{2^N}\\ &= -\frac{N}{4}(v_H - v_L) - \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) + \frac{N}{2}(v_H - v_L)\\ &= \frac{N}{4}(v_H - v_L) - \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) > 0,\end{aligned}$$

where the first equality is due to $m = N$ when $r_1 = v_H$, the second inequality is due to $\sum_{j=0}^{N-1} C_N^j (N - j) = \sum_{j=0}^{N} C_N^j (N-j) = 2^{N-1}N$, and the last inequality is due to $\frac{1-\theta^N}{1-\theta} = 1 + \theta + \cdots + \theta^{N_1} < N$. Recall Lemma EC.8, $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1)$ is decreasing in λ, i.e., $\frac{\partial u(\lambda, r_1)}{\partial \lambda} < 0$ when $r_1 = \frac{1}{2}(v_H + v_L)$. Denote $\bar{r}$ as the solution of $\frac{\partial u(\lambda, r_1)}{\partial \lambda} = 0$. Thus, $\frac{1}{2}(v_H + v_L) < \bar{r} < v_H$, if $r_1 \le \bar{r}$, $\frac{\partial u(\lambda, r_1)}{\partial \lambda} \le 0$, otherwise, $\frac{\partial u(\lambda, r_1)}{\partial \lambda} > 0$. Hence, we get the announced results.

(b) By (EC.11), we have

$$u(\lambda = 0, r_1) = \frac{N}{4}(v_H - v_L) + \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H + v_L - 2r_1) - \frac{\sum_{j=m}^{N} C_N^j [j v_H + (N-j) v_L - N r_1]}{2^N}.$$

Then

$$\frac{\partial u(\lambda = 0, r_1)}{\partial r_1} = -\frac{1}{2}\cdot\frac{1-\theta^N}{1-\theta} + \frac{\sum_{j=m}^{N} C_N^j N}{2^N},$$

which is decreasing in m, i.e., decreasing in r_1. Hence, $u(\lambda = 0, r_1)$ is concave in r_1. Consider $r_1 = v_L$, then

$$\begin{aligned}u(\lambda = 0, r_1 = v_L) &= \frac{N}{4}(v_H - v_L) + \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) - \frac{\sum_{j=1}^{N} C_N^j j (v_H - v_L)}{2^N}\\ &= \frac{N}{4}(v_H - v_L) + \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) - \frac{N}{2}(v_H - v_L)\\ &= \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) - \frac{N}{4}(v_H - v_L) < 0,\end{aligned}$$

where the second equality is due to $\sum_{j=1}^{N} C_N^j j = \sum_{j=0}^{N} C_N^j j = 2^{N-1}N$, the inequality is due to $\frac{1-\theta^N}{1-\theta} = 1 + \theta + \cdots + \theta^{N_1} < N$. Consider $r_1 = v_H$, then

$$u(\lambda = 0, r_1 = v_H) = \frac{N}{4}(v_H - v_L) - \frac{1}{4}\cdot\frac{1-\theta^N}{1-\theta}(v_H - v_L) > 0.$$

Recall Lemma EC.8, $u_{se}(r_1, r_2, \ldots, r_N) - u_{si}(r_1) > 0$ for $\lambda < 1$ when $r_1 = \frac{1}{2}(v_H + v_L)$. In other words, $u(\lambda = 0, r_1 = \frac{1}{2}(v_H + v_L)) > 0$. Denote $\underline{r}$ as the solution of $u(\lambda = 0, r_1) = 0$. Thus, $v_L < \underline{r} < \frac{1}{2}(v_H + v_L)$, if $r_1 < \underline{r}$, $u(\lambda = 0, r_1) < 0$, otherwise, $u(\lambda = 0, r_1) \ge 0$. Hence, we get the announced results. $\square$

(c) By (EC.11), we have

$$\frac{\partial u(\lambda,r_1)}{\partial r_1} = -\frac{1}{2}(1+\lambda)\frac{1-\theta^N}{1-\theta} + \frac{\sum_{j=m}^{N} C_N^j + \lambda\sum_{j=0}^{m-1} C_N^j}{2^N}N > -\frac{1}{2}(1+\lambda)N + \frac{2^N - \sum_{j=0}^{m-1} C_N^j + \lambda\sum_{j=0}^{m-1} C_N^j}{2^N}N$$

$$> -\frac{1}{2}(1+\lambda)N + \frac{2^N - \sum_{j=0}^{m-1} C_N^j + \lambda\sum_{j=0}^{m-1} C_N^j}{2^N}N = N(1-\lambda)\left(\frac{1}{2} - \frac{\sum_{j=0}^{m-1} C_N^j}{2^N}\right),$$

where the first inequality is due to $\frac{1-\theta^N}{1-\theta} = 1+\theta+\cdots+\theta^{N_1} < N$. The last term is greater than $\lambda\in[0,1]$ and $\sum_{j=0}^{m-1} C_N^j \le 2^{N-1}$, or $\lambda\in[1,+\infty)$ and $\sum_{j=0}^{m-1} C_N^j \ge 2^{N-1}$. Note $\sum_{j=0}^{m-1} C_N^j \le 2^{N-1}$ when $r_1 \le \frac{1}{2}(v_H+v_L)$ and $\sum_{j=0}^{m-1} C_N^j \ge 2^{N-1}$ when $r_1 \ge \frac{1}{2}(v_H+v_L)$. Hence, we get the announced results. □

E. Sensitivity Analysis: Heterogeneous Audience Members

We analyze in this section settings where audience members differ in their experienced utility. First, we consider settings where members do not share the same initial reference point r_1, or sensitivity to gains and losses λ. Then, we consider the case where all audience members share the same underlying parameters, but differ in the utility that each experience provides them, using idiosyncratic noise (i.e., audience member j's utility for the i-th experience is $v_i+\epsilon_{ij}$). Across all three models, we consider the setting where $N_L = N_H = 5$, with $\theta = 0.5$, $v_L = 1$, $v_H = 5$ and $\alpha = 5$.

In the parameter-heterogeneity case, we assume an experimental setup where an audience is made of 100 members, each with their own initial expectation r_1 and gain-loss sensitivity λ. To see how the optimal decisions changes based on the audience's heterogeneity, we compute the optimal strategy for four scenarios, henceforth denoted as Scenarios 1 through 4:

1. Homogeneous audience: all members have $r_1 = 2$ and $\lambda = 0.9$.
2. Heterogeneous initial reference: all members have $\lambda = 0.9$, but 75 have $r_1 = 1$ and the remaining 25 have $r_1 = 5$. This way, the average initial reference is $r_1 = 2$.
3. Heterogeneous gain-loss sensitivity: all members have $r_1 = 2$, but 60 have $\lambda = 0.5$ and 40 have $\lambda = 1.5$. This way, the average gain-loss sensitivity is $\lambda = 0.9$.
4. Heterogeneous audience: 60 members have $r_1 = 1$ and $\lambda = 0.5$, 15 members have $r_1 = 5$ and $\lambda = 0.5$, and the remaining 25 have $r_1 = 5$ and $\lambda = 1.5$. This way, the average initial reference and gain-loss sensitivity are identical to that of the homogeneous Scenario 1.

For the utility-heterogeneity case, we consider Scenario 1 and let every audience member have an idiosyncratic noise ϵ_j that they extract from every experience v_i in the sequence. Specifically, we assume that ϵ_{ij} can be with equal probability -0.5, 0, or 0.5. Each member then extracts $s(v_i+\epsilon_{ij}-r_i)$ from the experience and their expectations are correspondingly updated using $r_{i+1} =$

$\theta r_i + (1-\theta)(v_i + \epsilon_{ij})$. Therefore, just as in the heterogeneous r_1 case, audience members will have different expectations for every experience in the sequence.

E.1. News Release

E.1.1. Heterogeneous r_1 and λ. Consider the news release problem of Section 3. We find that in Scenarios 1-3, when the news is good it is optimal to reveal bad news in the first two periods (i.e., $t = 2$ in Proposition 1), and when the news is bad it is optimal to preempt them with positive news until the very last period. As a result, if at least one of the two characteristics r_1 and λ is shared across the entire audience, our experiments indicates that treating the entire audience as homogeneous does not imperil any experiential utility, and an optimal experience is still provided.

However, in Scenario 4, while the optimal way to release bad news is the same as in the homogeneous case, this is not true for releasing good news. Specifically, we find that when both r_1 and λ vary across the audience, it becomes optimal to reveal the good news immediate without preempting it with bad news. The utility of this preemptive release is 8.5% lower than that of revealing the good news immediately. As a result, most of the utility of the optimal release strategy for a heterogeneous audience is recovered by the optimal decision for a homogeneous audience. We summarize the results in Table EC.1.

We note that when deception is not allowed, the optimal news release strategy does not depend on the audience's characteristics. Therefore, the heterogeneity in the four scenarios would not affect the firm's release strategy if it makes decisions based on the audience's average characteristics.

E.1.2. Heterogeneous experiential utilities. We next assume that the audience is described by Scenario 1, but each member draws an idiosyncratic ϵ_{ij} noise in the experiential utility of every element in the sequence. Specifically, we assume that ϵ_{ij} is equally likely to be $-0.5, 0$ or 0.5. For the case of good news, we draw several random realizations of ϵ_{ij} and note that in every one, the optimal strategy is to deceive the audience with bad news in the first two periods, which was also the optimal strategy in the homogeneous audience case. Moreover, when the ultimate news is bad, we again see that in many realizations of the ϵ_{ij} it is still optimal to misguide the audience members with positive sentiment until the very last period. Therefore, when deception is allowed, we see that the idiosyncratic noise in experienced utility generally does not affect the optimal decision-making.

We also note that when deceiving is not allowed, the optimal news release strategy only depends on the sentiment of the ultimate news and not on the audience characteristics. Therefore, the heterogeneity in experienced utility would not change the firm's release strategy.

Table EC.1 Optimal news release for heterogeneous audiences (optimality gap with best strategy for homogeneous audience).

Good News	$r_1 = 2$	Heterogen. r_1
$\lambda = 0.9$	Deceive (0%)	Deceive (0%)
Heterogen. λ	Deceive (0%)	Immediate reveal (8.5%)

Bad News	$r_1 = 2$	Heterogen. r_1
$\lambda = 0.9$	Deceive (0%)	Deceive (0%)
Heterogen. λ	Deceive (0%)	Deceive (0%)

Note: The table at the top considers the case where the firm has good news to reveal. First, we consider 100 audience members that have the same r_1 and λ, represented by the top-left quadrant, and find that the optimal release includes deceiving the audience with bad news for the first two periods. Then, we let the members differ in these parameters. In the top-right quadrant, 75 members have $r_1 = 1$ and 25 members have $r_1 = 5$. In the bottom-left quadrant, 60 members have $\lambda = 0.5$, and 40 members have $\lambda = 1.5$. We find that in both of these scenarios, deceiving is still the optimal release strategy. However, when neither r_1 nor λ are homogeneous, the optimal strategy becomes to reveal the good news immediately. Nonetheless, the resulting total experienced utility for the audience when presenting them with the now suboptimal deceiving release strategy is only 8.5% lower than that achieved by the immediate release. The bottom table shows the result for the case of bad news and considers the same four heterogeneity scenarios. We see here that the audience characteristics do not change the optimal decision.

E.2. Event Organization

E.2.1. Heterogeneous r_1 and λ. We consider the scenario investigated in Section 4 using the model parameters described above. As indicated by Table 1, when $r_1 = 2$ and $\lambda = 0.9$, we have that the optimal sequence of performers, denoted henceforth by S_o^*, begins with two weak acts, followed by one strong act, then three weak ones and finally four strong acts. S_o^* symbolizes the original sequence when all audience members are characterized by a homogeneous r_1 and λ.

Next, we consider the numerical experiment where the audience is comprised of 100 members, each with possibly heterogeneous r_1 and λ. Unsurprisingly, when these parameters are equal for all members as in Scenario 1, S_o^* is the optimal sequence. However, we also check the optimal sequence for Scenario 2. We find that S_o^* still gives the optimal total experienced utility. We similarly check Scenario 3 and once again, we find that S_o^* remains optimal.

Lastly, we assume that the audience is heterogeneous in both initial reference and gain-loss sensitivities and consider Scenario 4. Here, the new optimal sequence, henceforth denoted by S_n^*, begins the event with a strong act, followed by five weak acts, and finishes with the four remaining strong acts. S_n^* symbolizes the new optimal sequence for the doubly-heterogeneous audience. Nonetheless,

Table EC.2 The optimal sequence of performances for heterogeneous audiences (optimality gap with best strategy for homogeneous audience).

	$r_1 = 2$	Heterogen. r_1
$\lambda = 0.9$	S_o^* (0%)	S_o^* (0%)
Heterogen. λ	S_o^* (0%)	S_n^* (7.1%)

Notes: First, we consider 100 audience members that have the same r_1 and λ, represented by the top-left quadrant, and find that the optimal sequence of strong (H) and weak (L) acts is $S_o^* =${L L H L L L H H H H}. Then, we let the members differ in these parameters. In the top-right quadrant, 75 members have $r_1 = 1$ and 25 members have $r_1 = 5$. In the bottom-left quadrant, 60 members have $\lambda = 0.5$ and 40 members have $\lambda = 1.5$. We find that in both of these scenarios, S_o^* is still the optimal sequence. However, when neither r_1 and λ are homogeneous, the optimal sequence becomes $S_n^* =${H L L L L L H H H H}. Nonetheless, the resulting total experienced utility for the audience when presenting them with the now suboptimal S_o^* is only 7.1% lower than that achieved by S_n^*.

the optimality gap of presenting the new heterogeneous audience with S_o^* is only 7.1%, showing that knowing the aggregate characteristics of the audience allows the event organizer to provide the audience a very large portion of their maximal experienced utility. We summarize these results in EC.2.

E.2.2. Heterogeneous experiential utilities. We next assume that the audience is described by Scenario 1, but each member draws an idiosyncratic ϵ_{ij} noise in the experiential utility of every element in the sequence. When ϵ_{ij} are equally probably to be $-0.5, 0$ and 0.5, we see that the optimal sequence is almost the same as it is in the homogeneous case, with the only exception that the interior peak occurs during the fourth element in the sequence, instead of the third. Moreover, the optimality gap of using the sequence S_o^* is only 0.03%. That is, 99.97% of the total utility provided by the optimal sequence for an audience that differs in the utility they extract from every performance in the event is captured by the optimal sequence for a homogeneous audience.

E.3. Simultaneous vs. Sequential Release

E.3.1. Heterogeneous r_1 and λ. We similarly repeat the above numerical experiment for the simultaneous and sequential release problem. For a homogeneous audience with $r_1 = 2$ and $\lambda = 0.9$, the *simultaneous* release of all items is optimal.

Next, we again consider the four scenarios described above. We show in Table EC.3 the resulting optimal release decision in each scenario. Surprisingly, we once again show that the optimal decision in the homogeneous case is still optimal when one of the audience's characteristic is heterogeneous across the population. However, when both the initial reference r_1 and gain-loss sensitivity λ are heterogeneous across the population as in Scenario 4, the sequential release becomes optimal.

Table EC.3 The optimal release decision for heterogeneous audiences (optimality gap with best strategy for homogeneous audience).

	$r_1 = 2$	Heterogen. r_1
$\lambda = 0.9$	Simultaneous (0%)	Simultaneous (0%)
Heterogen. λ	Simultaneous (0%)	Sequential (8.9%)

Note: First, we consider 100 audience members that have the same r_1 and λ, represented by the top-left quadrant, and find that a simultaneous release is optimal. Then, we let the members differ in these parameters. In the top-right quadrant, 75 members have $r_1 = 1$ and 25 members have $r_1 = 5$. In the bottom-left quadrant, 60 members have $\lambda = 0.5$ and 40 members have $\lambda = 1.5$. We find that in both of these scenarios, simultaneous release is still optimal. However, when neither r_1 and λ are homogeneous, a sequential release becomes optimal. Nonetheless, the resulting total experienced utility for the audience when presenting them with the now suboptimal simultaneous release is only 8.9% lower than that achieved by a sequential release.

Nonetheless, the optimality gap of choosing the simultaneous release is 8.9%., showing that once again a heavy majority of the maximal experienced utility that can be provided to the audience is captured by a decision that was guided under the homogeneous assumption.

E.3.2. Heterogeneous experiential utilities. We next assume that the audience is described by Scenario 1, but each member draws an idiosyncratic ϵ_{ij} noise in the experiential utility of every element in the sequence. Assuming that ϵ_{ij} are equally probably to be $-0.5, 0$ and 0.5, we draw several realizations of idiosyncratic noise for every audience member and note that in every single one, the simultaneous release is still optimal. Therefore, the optimal decision is still made if the musician believes that the audience is homogeneous, even when in reality every audience member may react differently to individual songs.

F. Concave Experienced Psychological Payoff Component for News Release

In this section, we consider the experienced psychological payoff component for news release model. Specifically, we replace (2) by

$$s(x_i) = \begin{cases} f(x_i) & \text{if } x_i > 0, \\ -\lambda f(-x_i) & \text{if } x_i \leq 0, \end{cases} \tag{EC.12}$$

where $f(\cdot)$ is concave. We also restrict ourselves to the scenario where deceiving is not allowed, and compare the results with Corollaries 2 and 3.

F.1. How to Release Good News

Denote $x_i = v_i - r_i \geq 0$ for $i \in \{1, 2, \ldots, N-1\}$, following the proof of Lemma EC.1, we have problem P_g is equivalent to the following problem:

$$P_g': \quad \max_{x_i} \quad v_H + \sum_{i=1}^{N-1} f(x_i) + f(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i)$$
$$s.t. \quad 0 \le \theta x_i \le x_{i+1},\ i \in \{1,2,\dots,N-2\},\ (1-\theta)(x_1 + x_2 + \cdots + x_{N-2}) + x_{N-1} \le v_H - r_1.$$

Corollary EC.1 (How to Release Good News without Deceiving). *If deceiving is not allowed (i.e., $\delta_1 \ge r_1$), then*

(a) If $f'(\frac{v_H - r_1}{\theta + (1-\theta)(N-1)}) < (1-\theta) f'(\frac{\theta(v_H - r_1)}{\theta + (1-\theta)(N-1)})$, it is optimal to release all information about the good news at once in the first period (i.e., $\delta_1 = v_H$, and $\delta_i = 0$ for any $i \in \{2,\dots,N\}$);

(b) Otherwise, there exists $\bar{j}^* \in \{2,\dots,N\}$[1], where it is optimal to release partial information about the good news at first $\bar{j}^* - 1$ periods then release all information about the good news at once in the $\bar{j}^*$-th period (the release policy is as follows: $\delta_1 = x_1^* + r_1$, and $\delta_i = (1-\theta)x_1^*$ for any $i \in \{2,\dots,\bar{j}^* - 1\}$, $\delta_{\bar{j}^*} = v_H - r_1 - [1 + (\bar{j}^* - 2)(1-\theta)x_1^*]$ and $\delta_i = 0$ for any $i \in \{\bar{j}^* + 1,\dots,N\}$, where $x_1^* > 0$ is given by (EC.13)).

Note the condition in Corollary EC.1(a) always holds if $f(x)$ is linear since $f'(x)$ is constant, which is indeed Corollary 2.

F.2. How to Release Bad News

Denote $x_i = r_i - v_i \ge 0$ for $i \in \{1,2,\dots,N-1\}$, following the proof of Lemma EC.3, we have problem P_b is equivalent to the following problem:

$$P_b': \quad \max_{x_i} \quad v_L - \lambda \sum_{i=1}^{N-1} f(x_i) - \lambda f(r_1 - v_L - (1-\theta)\sum_{i=1}^{N-1} x_i)$$
$$s.t. \quad 0 \le \theta x_i \le x_{i+1},\ i \in \{1,2,\dots,N-2\},\ (1-\theta)(x_1 + x_2 + \cdots + x_{N-2}) + x_{N-1} \le r_1 - v_L.$$

We find the result is consistent with Corollary 3, though we replace (2) by (EC.12).

Corollary EC.2 (How to Release Bad News without Deceiving). *If deceiving is not allowed (i.e., $\delta_1 \le r_1$), it is optimal to release no information about the bad news until the last period (i.e., $\delta_1 = r_1$, $\delta_i = 0$ for any $i \in \{2,\dots,N-1\}$, and $\delta_N = v_L - r_1$).*

F.3. Proofs in Sections F.1 and F.2

Proof of Corollary EC.1. By the proof of Corollary 2, we can show $v_i \ge r_i$ for $i \in \{1,2,\dots,N\}$ if deceiving is not allowed. Thus we should consider problem P_g'. Denoted the optimal solution to problem P_g' by x_i^*, where $i \in \{1,2,\dots,N-1\}$. We first prove the following lemma, which gives the structures of optimal solution.

[1] $\bar{j}^*$ is given by Lemma EC.11.

Lemma EC.10. *There exists a threshold $\bar{j}$, where $\bar{j} \in \{1, 2, \ldots, N\}$. If $i \le \bar{j} - 1$, $x_i^* = x_1^*$; if $i \ge \bar{j}$, $x_i^* = \theta^{i-\bar{j}} x_{\bar{j}}^*$. Moreover, $x_{\bar{j}}^* = v_H - r_1 - (\bar{j} - 1)(1-\theta)x_1^*$ for $\bar{j} \le N - 1$, and $x_1^* = v_H - r_1$ for $\bar{j} = 1$, is given by*

$$f'(x_1^*) - (1-\theta)\sum_{i=\bar{j}}^{N} \theta^{i-\bar{j}} f'(\theta^{i-\bar{j}}(v_H - r_1 - (\bar{j}-1)(1-\theta)x_1^*)) = 0. \tag{EC.13}$$

for $\bar{j} \ge 2$.

Proof of Lemma EC.10. We construct the Lagrangian function for problem P_g' as follows:

$$\begin{aligned} & L(x_1, x_2, \ldots, x_{N-1}) \\ = & \sum_{i=1}^{N-1} f(x_i) + f(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i) \\ & + \eta_1 x_1 + \sum_{i=2}^{N-1} \eta_i (x_i - \theta x_{i-1}) + \eta_N (v_H - r_1 - (1-\theta)\sum_{i=1}^{N-2} x_i - x_{N-1}), \end{aligned}$$

where $\eta_i \ge 0$ is the Lagrangian parameters. Then the optimal solution $(x_1^*, x_2^*, \ldots, x_{N-1}^*)$ to problem P_g' shall satisfy the first-order necessary conditions and the complementary slackness conditions, given as follows:

$$\left.\frac{\partial L}{\partial x_i}\right|_{x_i = x_i^*} = f'(x_i^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) + \eta_i - \theta\eta_{i+1} - (1-\theta)\eta_N = 0, \tag{EC.14}$$

$$\eta_1 x_1^* = 0, \tag{EC.15}$$

$$\eta_i (x_i^* - \theta x_{i-1}^*) = 0, \tag{EC.16}$$

$$\eta_N (v_H - r_1 - (1-\theta)\sum_{i=1}^{N-2} x_i^* - x_{N-1}^*) = 0. \tag{EC.17}$$

In the above conditions, (EC.14) holds for $i \in \{1, 2, \ldots, N-1\}$ are the first-order necessary conditions, (EC.16), (EC.17) holds for $i \in \{2, \ldots, N-2\}$, and (EC.17) are the complementary slackness conditions.

First, we show $\eta_1 = 0$ by induction. When $N = 2$, (EC.14)-(EC.17) reduces to

$$\left.\frac{\partial L}{\partial x_1}\right|_{x_1 = x_1^*} = f'(x_1^*) - (1-\theta) f'(v_H - r_1 - x_1^*) + \eta_1 - (1-\theta)\eta_2 = 0, \tag{EC.18}$$

$$\eta_1 x_1^* = 0, \tag{EC.19}$$

$$\eta_2 (v_H - r_1 - x_1^*) = 0. \tag{EC.20}$$

We show $\eta_1 = 0$ by contradiction. Suppose $\eta_1 > 0$, which implies $x_1^* = 0$ by (EC.19). By (EC.20), $\eta_2 = 0$. Hence, we have $f'(0) = (1-\theta) f'(v_H - r_1)$ by plugging $\eta_1 = \eta_2 = 0$ and $x_1^* = 0$ into (EC.18). However, $f'(0) > f'(v_H - r_1) > (1-\theta) f'(v_H - r_1)$, where the first due to $f(x)$ is concave, i.e., $f'(x)$

is decreasing in x. This is a contradiction. We suppose $\eta_1 = 0$ for $N = k$. Now we show it holds for $N = k+1$. We also show this by contradiction. Suppose $\eta_1 > 0$, which implies $x_1^* = 0$ by (EC.15). Now the problem P_g' with $N = k+1$ is equivalent to problem P_g' with $N = k$, then $\eta_2 = 0$ (this holds since we can regard x_2 as the first decision variable in the problem P_g' with $N = k$). Plugging $x_1^* = 0$ and $\eta_2 = 0$ into (EC.14) for $i = 1, 2$, we get $f'(0) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=2}^{N-1} x_i^*) = -\eta_1 + (1-\theta)\eta_N$ and $f'(x_2^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=2}^{N-1} x_i^*) = \theta\eta_3 + (1-\theta)\eta_N$. Note $f'(0) \geq f'(x_2^*)$ due to $f(x)$ being concave, i.e., $f'(x)$ is decreasing in x. We thus get $-\eta_1 + (1-\theta)\eta_N \geq \theta\eta_3 + (1-\theta)\eta_N$, which implies $-\eta_1 \geq \theta\eta_3 \geq 0$. This contradicts to $\eta_1 > 0$.

Second, let $\bar{j} = \max\{i | \eta_i = 0\}$, i.e., $\eta_i = 0$ for $i \leq \bar{j}$ and $\eta_{\bar{j}+1} > 0$, we show $\eta_i > 0$ for $i > \bar{j}+1$. We show this by induction. Consider $i = \bar{j}+2$ and prove $\eta_{\bar{j}+2} > 0$. Consider (EC.14) for $i = \bar{j}, \bar{j}+1$, we get $f'(x_{\bar{j}}^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) = -\eta_{\bar{j}} + \theta\eta_{\bar{j}+1} + (1-\theta)\eta_N$ and $f'(x_{\bar{j}+1}^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) = -\eta_{\bar{j}+1} + \theta\eta_{\bar{j}+2} + (1-\theta)\eta_N$. Note $\eta_{\bar{j}+1} > 0$ implies $x_{\bar{j}+1}^* = \theta x_{\bar{j}}^*$. Hence, $f'(x_{\bar{j}}^*) < f'(x_{\bar{j}+1}^*)$ due to $f(x)$ being concave, i.e., $f'(x)$ is decreasing in x and $x_{\bar{j}+1}^* = \theta x_{\bar{j}}^* < x_{\bar{j}}^*$. We thus get $-\eta_{\bar{j}} + \theta\eta_{\bar{j}+1} + (1-\theta)\eta_N < -\eta_{\bar{j}+1} + \theta\eta_{\bar{j}+2} + (1-\theta)\eta_N$, which implies $\eta_{\bar{j}} + \theta\eta_{\bar{j}+2} > (1+\theta)\eta_{\bar{j}+1} > 0$. Then $\eta_{\bar{j}+2} > 0$ since $\eta_{\bar{j}} = 0$. Suppose $\eta_{\bar{j}+n} > 0$ for $n \leq k$, we prove $\eta_{\bar{j}+k+1} > 0$. Consider (EC.14) for $i = \bar{j}, \bar{j}+k$, we get $f'(x_{\bar{j}}^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) = -\eta_{\bar{j}} + \theta\eta_{\bar{j}+1} + (1-\theta)\eta_N$ and $f'(x_{\bar{j}+k}^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) = -\eta_{\bar{j}+k} + \theta\eta_{\bar{j}+k+1} + (1-\theta)\eta_N$. Note $\eta_{\bar{j}+n} > 0$ for $n \leq k$ implies $x_{\bar{j}+k}^* = \theta x_{\bar{j}+k-1}^* = \cdots = \theta^k x_{\bar{j}}^*$ by (EC.16). Hence, $f'(x_{\bar{j}}^*) < f'(x_{\bar{j}+k}^*)$ due to $f(x)$ being concave, i.e., $f'(x)$ is decreasing in x and $x_{\bar{j}+k}^* = \theta^k x_{\bar{j}}^* < x_{\bar{j}}^*$. We thus get $-\eta_{\bar{j}} + \theta\eta_{\bar{j}+1} + (1-\theta)\eta_N < -\eta_{\bar{j}+k} + \theta\eta_{\bar{j}+k+1} + (1-\theta)\eta_N$, which implies $\eta_{\bar{j}} + \theta\eta_{\bar{j}+k+1} > (1+\theta)\eta_{\bar{j}+1} + \eta_{\bar{j}+k} > 0$. Then $\eta_{\bar{j}+k+1} > 0$ since $\eta_{\bar{j}} = 0$.

Last, we complete the proof by considering the following three cases: (1) If $\bar{j} = 1$, i.e., $\eta_i = 0$ for $i > 1$. Plugging $\eta_i > 0$ into (EC.16) and (EC.17), we have $x_i^* = \theta^{i-1} x_1^*$ and $x_1^* = v_H - r_1$. (2) If $2 \leq \bar{j} \leq N-1$, i.e., $\eta_N > 0$. On the one hand, plugging $\eta_i = 0$ into (EC.14) for $i \leq \bar{j}-1$, we have $f'(x_i^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*) = (1-\theta)\eta_N$, which implies $x_i^* = x_1^*$ for $i \leq \bar{j}-1$ and $\eta_N = [f'(x_i^*) - (1-\theta) f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x_i^*)]/(1-\theta)$. On the other hand, plugging $\eta_i > 0$ into (EC.16) for $i \geq \bar{j}+1$ and $\eta_N > 0$ into (EC.17), we have $x_i^* = \theta^{i-\bar{j}} x_{\bar{j}}^*$ for $\bar{j} \leq i \leq N-1$ and $v_H - r_1 - (1-\theta)\sum_{i=1}^{N-2} x_i^* - x_{N-1}^* = 0$. Hence, we obtain $x_{\bar{j}}^* = v_H - r_1 - (\bar{j}-1)(1-\theta) x_1^*$. Consequently, $x_i^* = \theta^{i-\bar{j}}[v_H - r_1 - (\bar{j}-1)(1-\theta) x_1^*]$ for $\bar{j} \leq i \leq N-1$. Plugging $\eta_i = 0$ for $i \leq \bar{j}-1$, $x_i^* = x_1^*$ for $i \leq \bar{j}-1$ and $x_i^* = \theta^{i-\bar{j}}[v_H - r_1 - (\bar{j}-1)(1-\theta) x_1^*]$ for $\bar{j} \leq i \leq N-1$ into (EC.14), we get (EC.13). (3) If $\bar{j} = N$, i.e., $\eta_i = 0$ for any i. Following the analysis above, we get $x_i^* = x_1^*$ for $i \leq N-1$ and (EC.13) still holds. In sum, we get the announced results in Lemma EC.10. $\square$

We denote the policy described in Lemma EC.10 as policy $G_{\bar{j}}$ and Lemma EC.10 shows that the optimal policy can be restricted to policy $G_{\bar{j}}$. The following lemma provides the conditions for choosing optimal $\bar{j}^*$.

Lemma EC.11. *(a) If* $f'(v_H - r_1) > (1-\theta)\sum_{i=2}^{N}\theta^{i-2} f'(\theta^{i-1}(v_H - r_1))$, $\bar{j}^* = 1$;

(b) If $f'(\frac{v_H - r_1}{\bar{j}-(\bar{j}-1)\theta}) > (1-\theta)\sum_{i=\bar{j}+1}^{N}\theta^{i-\bar{j}-1} f'(\frac{\theta^{i-\bar{j}}(v_H-r_1)}{\bar{j}-(\bar{j}-1)\theta})$ *and* $f'(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) \le (1-\theta)\sum_{i=\bar{j}}^{N}\theta^{i-\bar{j}} f'(\frac{\theta^{i-\bar{j}+1}(v_H-r_1)}{\bar{j}-1-(\bar{j}-2)\theta})$ *holds for* $\bar{j}\in\{2,\dots,N-1\}$, $\bar{j}^* = \bar{j}$;

(c) If $f'(\frac{v_H-r_1}{N-1-(N-2)\theta}) \le (1-\theta) f'(\frac{\theta(v_H-r_1)}{N-1-(N-2)\theta})$, $\bar{j}^* = N$.

Proof of Lemma EC.11. First, we show the condition in Lemma EC.11(b) is the condition of policy $G_{\bar{j}}$ being feasible for $\bar{j}\in\{2,\dots,N\}$, and the condition in Lemma EC.11(c) is the condition of policy G_N being feasible for $\bar{j}\in\{2,\dots,N-1\}$. Note $x^*_{\bar{j}} = v_H - r_1 - (\bar{j}-1)(1-\theta)x^*_1$ from the previous paragraph, with $x^*_{\bar{j}} \ge \theta x^*_{\bar{j}-1} = x^*_1$, we thus have $x^*_1 \le \frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}$. Consider (EC.14) for $i=\bar{j}-1$ and $i=\bar{j}$, we have $f'(x^*_{\bar{j}-1}) - (1-\theta)f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x^*_i) + \eta_{\bar{j}-1} - \theta\eta_{\bar{j}} - (1-\theta)\eta_N = 0$ and $f'(x^*_{\bar{j}}) - (1-\theta)f'(v_H - r_1 - (1-\theta)\sum_{i=1}^{N-1} x^*_i) + \eta_{\bar{j}} - \theta\eta_{\bar{j}+1} - (1-\theta)\eta_N = 0$. Hence, $f'(x^*_{\bar{j}}) - f'(x^*_{\bar{j}-1}) = \theta\eta_{\bar{j}+1} > 0$ due to $\eta_{\bar{j}-1} = \eta_{\bar{j}} = 0$. Consequently, $x^*_{\bar{j}} < x^*_{\bar{j}-1} = x^*_1$ due to $f(x)$ being concave, i.e., $f'(x)$ is decreasing in x. Combining $x^*_{\bar{j}} = v_H - r_1 - (\bar{j}-1)(1-\theta)x^*_1$ and $x^*_{\bar{j}} < x^*_1$, we have $x^*_1 > \frac{v_H-r_1}{\bar{j}-(\bar{j}-1)\theta}$. Note that the left hand of (EC.13) is decreasing in x^*_1, then the condition of $\frac{v_H-r_1}{\bar{j}-(\bar{j}-1)\theta} < x^*_1 \le \frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}$ is indeed the condition in Lemma EC.11(b). If $\bar{j}=N$, we have $x^*_i = x^*_1$ for $i \le N-1$ by Lemma EC.10. Recall $(1-\theta)(x_1 + x_2 + \cdots + x_{N-2}) + x_{N-1} \le v_H - r_1$, then $x^*_1 \le \frac{v_H-r_1}{N-1-(N-2)\theta}$. Consequently, $f'(\frac{v_H-r_1}{N-1-(N-2)\theta}) \le (1-\theta)f'(\frac{\theta(v_H-r_1)}{N-1-(N-2)\theta})$ by (EC.13).

Note the conditions in Lemma EC.11(a), (b), (c) are mutually exclusive, from the former paragraph, only policy G_1 is feasible when $f'(v_H - r_1) > (1-\theta)\sum_{i=2}^{N}\theta^{i-2}f'(\theta^{i-1}(v_H-r_1))$, we thus get Lemma EC.11(a). If $f'(v_H - r_1) \le (1-\theta)\sum_{i=2}^{N}\theta^{i-2}f'(\theta^{i-1}(v_H-r_1))$, i.e., the condition in Lemma EC.11(b) or (c) is satisfied, then only policies G_1 and $G_{\bar{j}}$ are feasible. We now show policy $G_{\bar{j}}$ dominates G_1. We denote

$$u_j(x) = v_H + (j-1)f(x) + \sum_{i=j}^{N} f(\theta^{i-j}(v_H - r_1 - (j-1)(1-\theta)x)). \tag{EC.21}$$

Let $j=\bar{j}$, $x^*_i = x^*_1$ for $i \le \bar{j}-1$, $x^*_i = \theta^{i-\bar{j}}[v_H - r_1 - (\bar{j}-1)(1-\theta)x^*_1]$ for $i \ge \bar{j}$ in the objective function of problem P'_g, we get the objective value under policy $G_{\bar{j}}$ is $u_{\bar{j}}(x^*_1)$. Note $u'_{\bar{j}}(x) = f'(x) - (1-\theta)\sum_{i=\bar{j}}^{N}\theta^{i-\bar{j}}f'(\theta^{i-\bar{j}}(v_H - r_1 - (\bar{j}-1)(1-\theta)x))$, which is the left hand of (EC.13) and is decreasing in x. Therefore, $u_{\bar{j}}(x)$ is maximized at $x = x_1^*$, where x_1^* is given by (EC.13). Moreover, it is easy to see that

$$u_j(\frac{v_H - r_1}{j-1-(j-2)\theta}) = u_{j-1}(\frac{v_H - r_1}{j-1-(j-2)\theta}), \tag{EC.22}$$

$$u'_j(\frac{v_H - r_1}{j-1-(i-2)\theta}) = u'_{j-1}(\frac{v_H - r_1}{j-1-(j-2)\theta}). \tag{EC.23}$$

Note $f'(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) \leq (1-\theta)\sum_{i=\bar{j}}^{N}\theta^{i-\bar{j}}f'(\frac{\theta^{i-\bar{j}+1}(v_H-r_1)}{\bar{j}-1-(\bar{j}-2)\theta})$ is equivalent to $u'_{\bar{j}}(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) = u'_{\bar{j}-1}(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) \leq 0$. Hence, $u'_{\bar{j}-1}(x) \leq 0$ for $x \geq \frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}$ due to $u_j(x)$ being decreasing in x. We thus have $u_{\bar{j}-1}(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) > u_{\bar{j}-1}(\frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta}) = u_{\bar{j}-2}(\frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta})$, where the equality is due to (EC.22) holding for $j = \bar{j}-1$. Furthermore, $0 \geq u'_{\bar{j}-1}(\frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta}) = u'_{\bar{j}-2}(\frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta})$, where the inequality is due to $u_{\bar{j}-1}(x)' \leq 0$ for $x \geq \frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta}$, and the equality is due to (EC.23) holding for $j = \bar{j}-1$. Hence, $u'_{\bar{j}-2}(x) \leq 0$ for $x \geq \frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta}$ due to $u_j(x)$ being decreasing in x. We thus have $u_{\bar{j}-2}(\frac{v_H-r_1}{\bar{j}-2-(\bar{j}-3)\theta}) > u_{\bar{j}-2}(\frac{v_H-r_1}{\bar{j}-3-(\bar{j}-4)\theta}) = u_{\bar{j}-3}(\frac{v_H-r_1}{\bar{j}-3-(\bar{j}-4)\theta})$, where the equality is due to (EC.22) holding for $j = \bar{j}-2$. Repeating this process, we get $u_{\bar{j}}(\frac{v_H-r_1}{\bar{j}-(\bar{j}-1)\theta}) > u_{\bar{j}-1}(\frac{v_H-r_1}{\bar{j}-1-(\bar{j}-2)\theta}) > \cdots > u_2(v_H - r_1)$. On the one hand, $u_{\bar{j}}(x_1^*) \geq u_{\bar{j}}(\frac{v_H-r_1}{\bar{j}-(\bar{j}-1)\theta})$ due to $u_{\bar{j}}(x_1^*)$ being maximized at $x = x_1^*$, where x_1^* is given by (EC.13). On the other hand, $u_2(v_H - r_1) = v_H + \sum_{i=1}^{N} f(\theta^{i-1}(v_H - r_1 - (j-1)(1-\theta)x))$, which is the objective value of policy G_1. We thus obtain policy $G_{\bar{j}}$ dominates G_1. □

Now we will complete the proof of Corollary EC.1. By Lemma EC.10 and Lemma EC.11(a), if $f'(v_H - r_1) > (1-\theta)\sum_{i=2}^{N}\theta^{i-2}f'(\theta^{i-1}(v_H - r_1))$, it is optimal to let $x_1^* = v_H - r_1$ and $x_i^* = \theta^{i-1}x_1^*$ for $i \in \{2, \ldots, N-1\}$. Recall $x_i = v_i - r_i$ for $i \in \{2, \ldots, N-1\}$, we have $v_i = v_H$ for $i \in \{2, \ldots, N\}$. Consequently, $\delta_1 = v_H$, and $\delta_i = 0$ for any $i \in \{2, \ldots, N\}$. If $f'(v_H - r_1) \leq (1-\theta)\sum_{i=2}^{N}\theta^{i-2}f'(\theta^{i-1}(v_H - r_1))$, it is optimal to let $x_i^* = x_1^*$ for $i \leq \bar{j}^* - 1$, $x_i^* = \theta^{i-\bar{j}^*}x_{\bar{j}^*}^*$ for $i \geq \bar{j}^*$, and $x_{\bar{j}^*}^* = v_H - r_1 - (\bar{j}^* - 1)(1-\theta)x_1^*$ for $\bar{j}^* \leq N-1$, where x_1^* is given by (EC.13). Recall $x_i = v_i - r_i$ for $i \in \{2, \ldots, N-1\}$, we have $v_1 = x_1^* + r_1$, $v_i = [1 + (i-1)(1-\theta)]x_1^* + r_1$ for $2 \leq i \leq \bar{j}^* - 1$, and $v_i = v_H$ for $i \geq \bar{j}^*$. Consequently, $\delta_1 = x_1^* + r_1$, and $\delta_i = (1-\theta)x_1^*$ for any $i \in \{2, \ldots, \bar{j}^* - 1\}$, $\delta_{\bar{j}^*} = v_H - r_1 - [1 + (\bar{j}^* - 2)(1-\theta)x_1^*]$ and $\delta_i = 0$ for any $i \in \{\bar{j}^* + 1, \ldots, N\}$. Note the left hand of (EC.13) being decreasing in x_1^* and $f'(0) - (1-\theta)\sum_{i=\bar{j}^*}^{N}\theta^{i-\bar{j}^*}f'(\theta^{i-\bar{j}^*}(v_H - r_1)) > f'(0) - (1-\theta)\sum_{i=\bar{j}^*}^{N}\theta^{i-\bar{j}^*}f'(0) = (1 - \theta^{N-\bar{j}^*+1})f'(0) > 0$, where the first inequality is due to $f'(x)$ is decreasing in x. Then $x_1^* > 0$ and we thus get the desired results. □

Proof of Corollary EC.2. By the proof of Corollary 3, we can show $v_i \leq r_i$ for $i \in \{1, 2, \ldots, N\}$ if deceiving is not allowed. Thus we should consider problem P'_b.

We first show $f(x_1) + f(x_2) + \cdots + f(x_n) \geq f(x_1 + x_2 + \cdots + x_n)$ by induction. For $n = 2$, $\frac{f(x_1)-f(0)}{x_1-0} \geq \frac{f(x_1+x_2)-f(x_2)}{(x_1+x_2)-x_2}$ due to $f(x)$ being concave. Hence, $f(x_1) + f(x_2) \geq f(x_1 + x_2)$ since $f(0) =$

0. Suppose $f(x_1)+f(x_2)+\cdots+f(x_n)\geq f(x_1+x_2+\cdots+x_n)$ for $n\leq k$. Then $f(x_1)+f(x_2)+\cdots+f(x_k)+f(x_{k+1})\geq f(x_1+x_2+\cdots+x_k)+f(x_{k+1})\geq f(x_1+x_2+\cdots+x_k+x_{k+1})$.

We have $v_L-\lambda\sum_{i=1}^{N-1}f(x_i)-\lambda f(r_1-v_L-(1-\theta)\sum_{i=1}^{N-1}x_i)\leq v_L-\lambda f(x_1+x_2+\cdots+x_{N-1})-\lambda f(r_1-v_L-(1-\theta)\sum_{i=1}^{N-1}x_i)\leq v_L-\lambda f(r_1-v_L+\theta(x_1+x_2+\cdots+x_{N-1}))\leq v_L-\lambda f(r_1-v_L)$, where the first and second equalities are due to $f(x_1)+f(x_2)+\cdots+f(x_n)\geq f(x_1+x_2+\cdots+x_n)$, and the last inequality is due to $x_i\geq 0$ and $f(x)$ is increasing in x. Hence, the optimal objective value of problem P_b' is $v_L-\lambda f(r_1-v_L)$, which can be archived by $x_i=0$ for $i=1,2,\ldots,N-1$ and $x_N=r_1-v_L$. It is easy to verify $x_i=0$ for $i=1,2,\ldots,N-1$ and $x_N=r_1-v_L$ is equivalent to $\delta_1=r_1$, $\delta_i=0$ for any $i\in\{2,\ldots,N-1\}$ and $\delta_N=v_L-r_1$. We thus get the desired result. □

G. Implications of the Peak-End Rule

We assume in our analysis that the update of reference points over a sequence of experiences follows the weighted-average smoothing specified in (1), and note that our results may differ under alternative reference structures. One popular such choice is the peak-end rule (see, e.g., Nasiry and Popescu 2011) given by

$$r_i(r_1,r_2,\ldots,r_{i-1},v_1,v_2,\ldots,v_{i-1};\theta)=\theta\max_{j<i}v_j+(1-\theta)v_{i-1}, \tag{EC.24}$$

which assumes that only the most recent and most pleasant experiences impact the audience's changing expectations and references. We present below several instances where the peak-end rule can alter the optimal decision in the three applications of our general framework. For example, in the case of the performance sequencing problem in Section 4, assuming that the audience's reference point follows (EC.24) could significantly alter the results in some instances. Specifically, U-shaped patterns with consecutive low-type performances in the middle portion of the sequence may no longer be optimal for sufficiently gain-seeking audiences. Recall that this pattern is optimal because every low-type performance in the middle portion of the sequence reduces the audience's expectation when the references are exponentially smoothed (i.e., every past experience matters in forming expectations), and once expectations are low enough, the final high-type performances in the tail-end of the pattern provide the audience a very pleasant experience. If references were instead only dependent on the strongest and most recent experiences, then after any low-type performances in the middle portion of the pattern, the audience would have the same reference point. This is because their most recent experience would have been a low type, and their strongest one would have been a high type. As a result, the consecutive low-type performances do not reduce expectations to the same degree as they would under exponentially-smoothed reference points.

Therefore, the tail-end of the U-shape would not necessarily be as pleasant for the audience, and an alternative sequence, such as a crescendo, could provide them with a more pleasant experience.

Nonetheless, in some instances, we can expect that the peak-end rule will have no impact on the audience's total utility. Such a scenario could occur in the song release problem of Section 5, where changing the reference-updating rule should not change the audience's total utility for a simultaneous release, as the audience's reference is not updated with every song. However, the total utility of a sequential release could change under an alternative reference structure. Therefore, if the peak-end rule described by (EC.24) is used, the comparison between u_{se} and u_{si} could change, and accordingly, the resulting optimal release strategy. We can think of one instance where an early song in the sequence is realized to be of a high valuation, which would result in the audience maintaining a higher reference for the rest of the sequence than they would have under an exponentially smoothed reference point. In such a case, the experienced utility from every consequent song will be lower, which decreases the total utility extracted from the sequential release. As a result, a simultaneous release could result in a more pleasant experience for the audience.

In the case of news release in Section 3, if we assume that the audience's reference point follows (EC.24), by the discretized revelation of news we have that $r_i = v_{i-1}$ for all i in the good news case. Here, the peak-end rule reduces our general framework to a special case where $\theta = 0$. However, when releasing bad news, we have that $r_i = \theta v_1 + (1-\theta)v_{i-1}$ since the initial update must have the most pleasant sentiment. As a result, the structure of the problem changes since past references no longer affect future utilities, and the optimal sequence could also take a different structure.