E-Companion

"Regulation of Privatized Public Service Systems"

A. Two Practical Examples

We present two practical examples, then calibrate our model with empirical data and finally use the calibrated model to conduct counterfactual analysis.

A.1. Toll Tunnels

In this subsection, we present the example of tunnel operations in Hong Kong and calibrate our model to this example to demonstrate the potential efficiency of different regulation schemes. The data is downloaded from the official webpage of the Hong Kong Transportation Department.[1] The dataset contains the financial and operating statistics of five tunnels: Aberdeen Tunnel, Lion Rock Tunnel, Shing Mun Tunnel, Tseung Kwan O Tunnel, and Route 8 (Kowloon section). The Hong Kong government owns the tunnels and outsources the operations and management to third-party companies, such as Chun Wo Tunnel Management Limited and Transport Infrastructure Management Limited. The service fee for the five tunnels ranged from HK$3 to HK$8 in 2017.

We use the average value of the data collected from the five tunnels to calibrate the value of parameters in our model. From the dataset, we find that the average number of vehicles using the tunnels in 2017 was 26.1 million, the average revenue was HK$150.52 million, and the average expense was HK$114.74 million, comprising the staff cost of HK$2.5 million, and the administration expense (such as equipment operation, maintenance, depreciation cost, and all other costs except human resources) of HK$112.24 million. For ease of exposition, we converted the data from yearly figures to hourly by dividing them by 8760. (There are $24 \times 365 = 8760$ hours in a year.) The following table presents the data we used in this example, in which the average service price is calculated by dividing "Revenue" by "Number of Vehicles Using Tunnels."

Table EC.1 Summary of the data (hourly)

Number of Vehicles Using Tunnels	Revenue (HK$)	Staff Cost (HK$)	Administration Expense (HK$)	Service Price (HK$)
2,979	17,183	285.388	12,812.785	5.767

Next, we calibrate the model parameters. In our model, there are two types of cost parameters,

[1] https://www.td.gov.hk/filemanager/en/content_20/govt%20toll%20tunnels%20(oa%202016-17)%20eng.pdf. For translation, use https://translate.google.com/?sl=auto&tl=en&op=docs.

c and γ. In particular, γ is the service capacity cost rate, which is mainly driven by staffing;[2] c then represents the administration expense per vehicle. Using the data provided in Table EC.1, we have the administration cost rate $c = 12,812.785/2,979 \approx 4.3$. We then turn to the service cost rate γ. We find from the report[3] that the median Hong Kong hourly wage in 2017 was HK\$68. Because the staff cost was HK\$285.388 (per hour), there were about 4 staff working in a tunnel station on average. Roughly speaking, it takes only 2 to 6 seconds for a vehicle to pass the tunnel station. Thus, the total service rate (of 4 staff) per hour ranges from 2400 to 7200, which gives the service cost rate as $\gamma \in [0.04, 0.119]$. In this example, we try different values for γ and select one of them based on the so-called consistency criterion. This criterion requires that after we feed the values into our model, the resulting service rate and the associated service cost shall be consistent with the data provided in Table EC.1, i.e., $\gamma \cdot \mu =$ HK\$285.388. We approximate the delay cost per unit of time by using the median hourly wage in Hong Kong; that is, $h =$ HK\$68. There are two parameters regarding the market status left to be calibrated, a and Λ. We try a variety of combinations of them until the resulting number of vehicles using tunnels and the service price match those in Table EC.1. Because the government owns the tunnels and outsources the management to third-party companies, and because the government has commitment power (the service price had not changed since 2010), we consider Scenario SP for this example. Using these figures, we calibrate the model parameters as follows. In particular, $a = 27.8$, as the upper bound of the uniform distribution, means that the average value for customers to use the tunnel is HK\$13.9.

Table EC.2 Summary of parameter calibration

c	γ	h	a	Λ
4.3	0.0847	68	27.8	3789

Using the data in Table EC.2, we evaluate the performance of different regulation schemes and a joint venture. The performances are summarized in Table EC.3, where the column beginning with "β (%)" reports how much of the share the government must take under the joint venture to achieve the same social welfare as under a regulation scenario. We observe that among the four regulation scenarios (SP, SW, NP, and NW), Scenario SP, in which the government acts as the Stackelberg leader to regulate the service price, leads to the highest social welfare. Moreover, comparing it with Scenario C, we find that Scenario SP is very effective, achieving 98.79% of the maximum possible social welfare. We also observe that the value of the government's commitment

[2] In this example, the number of lanes in each tunnel could not be expanded after the tunnel had been built, and the fixed cost of building the lanes is sunk. Tunnel operators can decide the number of lanes opened, which determines the corresponding staffing level for each day (each lane requires one staff member).

[3] https://www.statistics.gov.hk/pub/B10500142017AN17B0100.pdf (page 9). For translation, use https://translate.google.com/?sl=auto&tl=en&op=docs.

power (i.e., not pursuing short-sighted objectives) is significant when the government adopts price regulation. The social welfare under NP is about one-quarter of that under SPand about one-third of that without any regulation (as in Scenario D). However, when the government chooses to regulate wait time, the government's commitment power has little value. Next, we conduct a counterfactual analysis by assuming that the government and the firm jointly own the tunnels. In our example, the government needs to take 87.685% share of the project under the joint venture to achieve the same social welfare obtained in Scenario SP. Hence, switching from price regulation to joint venture may not be a good choice for a government with a limited budget. Nevertheless, the joint venture with even a small government investment ensures that social welfare will be greater than it would be without regulation (i.e., under Scenario D), and it eliminates the possibility of regulation backfire, which happens when the government regulates the price and does not have commitment power (i.e., under Scenario NP).

Table EC.3 Performance of regulations and joint venture in the tunnel example

Scenario	p	μ	λ	w (in second)	SW	π	CS	β (%)
C	4.427	4,778	3,180	2.25	37,092	0	37,092	100
D	16.062	2,722	1,592	3.18	27,783	18,490	9,293	0
SP	5.767	3,368	2,979	9.26	36,645	4,085	32,560	87.685
SW	16.068	2,977	1,592	2.6	27,788	18,486	9,302	0.05
NP	4.385	1,602	1,596	619.81	9,341	0	9,341	NA
NW	16.071	3,192	1,593	2.25	27,786	18,479	9,307	0.035

A.2. General Outpatient Clinics

Now we consider another example, that of general outpatient clinics in Hong Kong. We focus on private clinics because public clinics are usually subsidized by the government, and our model does not consider subsidy. According to the public information available online, the service price of clinics ranges from HK$150 to HK$270. This price covers the service of basic diagnosis only. If more tests and drugs are required, patients will be charged additional amounts. Basic diagnosis has a different cost structure from that of the tunnel service; staff cost takes a more significant share. On the other hand, the administration cost is low. If more tests and drugs are needed, the administration cost increases, but the patients will be charged for these additional services. We thus ignore the revenue and cost from subsequent tests and drugs; see also, e.g., Dai et al. (2017). In fact, when patients who have flu symptoms decide whether to visit a clinic, they barely consider the expense of tests and drugs because most of the expense is covered by insurance. We assume that $c = 10$; that is, the administration cost for providing a diagnosis to a patient is HK$10. On the other hand, the staff cost can be high because serving a patient involves several staff members, including nurses, physicians, and other medical workers. Following Adida and Bravo (2019), we approximate a nurse's hourly wage using the median hourly wage of those who worked in human health activities in 2017,[4] which was HK$84. Physicians' and pharmacists' wages are higher than nurses'. For example, Adida and Bravo (2019) assume that a pharmacist's hourly wage is twice that of a nurse's. Also, staff in private clinics usually receive a higher hourly wage than those who work for public hospitals. We assume that the total hourly wage of the staff is $84 \times 7 = 588$. Because about 4 patients can be served in an hour, this gives us the service cost rate as $\gamma = 588/4 = 147$. In a way similar to the one we used in the tunnel example, we set $h = 68$. Because the service price ranges from HK$150 to HK$270, we assume that $a = 300$, which indicates that the average service valuation of the patients is HK$150. For the potential market size, we set $\Lambda = 30$ such that the expected wait time in the clinic is about 40 minutes under Scenario D.

Table EC.4 Performance of regulations and joint venture in the clinic example

Scenario	p	μ	λ	w (in minute)	SW	π	CS	β (%)
C	197	8.11	6.39	35	204	0	204	100
D	206	6.39	4.89	40	137	18	119	0
SP	194	6.71	5.39	46	154	8	146	9.973
SW	211	7.33	5.39	31	152	6	146	8.942
NP	190	6.73	5.49	48	151	0	151	8.417
NW	212	7.55	5.49	29	151	0	151	8.417

Table EC.4 summarizes the performance measures of various scenarios. We observe that the performance of wait time regulation is very close to that of price regulation in this example. Interestingly, we observe in this clinic example that the government can obtain greater social welfare under joint venture by taking only a small share (about 10%) of the project than it can under any regulation.

The clinic example shows that wait time regulation may work well, whereas the tunnel example shows that it may not. This is because the service cost rate is low and/or the market size is large in the tunnel example, so that the firm has a strong incentive to invest in service capacity even without regulation. However, that is not the case in the clinic example, where privatization results in a very congested system. Wait time regulation of the clinic requires the firm to invest more in service capacity to ease the congestion (which also induces the firm to increase the service price). Given that the system is very congested under privatization, wait time regulation will significantly increase social welfare. Similarly, under joint venture, increasing the government's share in the project will boost the investment in service capacity, which will significantly improve social welfare given the very congested system under privatization. Finally, while service capacity in the two

[4] `https://www.statistics.gov.hk/pub/B10500142017AN17B0100.pdf` (page 12). For translation, use `https://translate.google.com/?sl=auto&tl=en&op=docs`.

examples depends only on the human resources, we would like to point out that other factors can contribute to service capacity as well. For instance, if tunnels can increase the number of lanes, the service capacity cost also includes the expense of building the lanes and deploying the associated facilities. Similarly, the amount of equipment plays an important role in the service capacity of health care facilities to provide imaging and laboratory tests, and the service capacity cost includes the expense of purchasing the equipment. Our model can be used to study these scenarios as well. Because it is difficult for us at present to collect data on service capacity cost in those scenarios, we leave that for future research.

B. The Rest of Proofs

For the ease of exposition, we prove Corollary 1 before Proposition 1.

Proof of Corollary 1. We begin with Scenario C. For any given w, plugging

$$\lambda = \Lambda\left(1 - \frac{hw}{a} - \frac{p}{a}\right) \tag{EC.1}$$

into Π yields

$$\Pi(w,p) = \Lambda\left(1 - \frac{hw+p}{a}\right)(p-c-\gamma) - \frac{\gamma}{w}, \tag{EC.2}$$

of which $\lambda \geq 0$ requires that $p \leq a - hw$. We observe that for any given w, $\Pi(w,p)$ is quadratically concave in p. Thus, it is easy to obtain the maximum profit with any given w, which is $\frac{\Lambda}{4a}\Delta$ where $\Delta = (a-hw-c-\gamma)^2 - \frac{4a\gamma}{w\Lambda}$. Apparently, when $\Delta < 0$, $\Pi \geq 0$ is never satisfied. Conditional on $\Delta \geq 0$, there are two roots to $\Pi(w,p) = 0$ with respect to (w.r.t.) p, and they are $\underline{p}(w) = \frac{a-hw+c+\gamma-\sqrt{\Delta}}{2}$ and $\overline{p}(w) = \frac{a-hw+c+\gamma+\sqrt{\Delta}}{2}$ with $\underline{p}(w) \leq \overline{p}(w)$. Therefore, the firm's participation constraint $\Pi \geq 0$ requires that $\Delta \geq 0$ and

$$\underline{p}(w) \leq p \leq \min\{a - hw, \overline{p}(w)\}. \tag{EC.3}$$

Note that if $0 \leq w \leq \frac{a-c-\gamma}{h}$, $\min\{a-hw, \overline{p}(w)\} = \overline{p}(w)$, and then (EC.3) is equivalent to

$$\underline{p}(w) \leq p \leq \overline{p}(w). \tag{EC.4}$$

Otherwise, if $w > \frac{a-c-\gamma}{h}$, $a - hw < \underline{p}(w)$, and then $\Pi \geq 0$ cannot hold.

Next, we derive the feasible domain of w for $\Delta \geq 0$. Given $0 \leq w \leq \frac{a-c-\gamma}{h}$, $\Delta \geq 0$ is equivalent to

$$a - hw - c - \gamma \geq \sqrt{\frac{4a\gamma}{w\Lambda}}. \tag{EC.5}$$

Let $L(w)=a-hw-c-\gamma-\sqrt{\frac{4a\gamma}{w\Lambda}}$. Then $\Delta\geq 0$ is equivalent to $L(w)\geq 0$. Now, $\frac{\mathrm{d}L(w)}{\mathrm{d}w}=-h+\sqrt{\frac{a\gamma}{\Lambda}}w^{-3/2}$, implying that $\frac{\mathrm{d}L(w)}{\mathrm{d}w}$ decreases in w. The maximizer of $L(w)$ is $w_l^*=\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}$, and, correspondingly, the maximum value of $L(w)$ satisfies $L(w_l^*)=a-c-\gamma-3\left(\frac{ah\gamma}{\Lambda}\right)^{1/3}$. In addition, $L(0)=-\infty$, and $L(\frac{a-c-\gamma}{h})=-\sqrt{\frac{4ah\gamma}{(a-c-\gamma)\Lambda}}<0$. Hence, if $L(w_l^*)\geq 0$, then there exist two roots to $L(w)=0$ w.r.t. w on the interval $(0,\frac{a-c-\gamma}{h})$, denoted as $\underline{w}_l$ and $\overline{w}_l$, where $\underline{w}_l\in(0,\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}]$ and $\overline{w}_l\in[\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3},\frac{a-c-\gamma}{h})$. It follows that $L(w)\geq 0$ requires that $\Lambda\geq\frac{27ah\gamma}{(a-c-\gamma)^3}$ (i.e., $L(w_l^*)\geq 0$), or equivalently $\Theta\geq 1$, and

$$\underline{w}_l\leq w\leq\overline{w}_l. \tag{EC.6}$$

To conclude, when $\Theta<1$, $\Pi\geq 0$ is never satisfied. Suppose that $\Theta\geq 1$. According to the previous analysis, the decision problem under Scenario C becomes $\max_{p,w}\ SW$ subject to (EC.4) and (EC.6). Plugging (EC.1) into SW and differentiating SW w.r.t. p yields $\frac{\partial SW}{\partial p}=\frac{\Lambda}{a}(c+\gamma-p)$. Based on (EC.6), it is clear that $\underline{p}(w)>c+\gamma$. It follows that SW is decreasing in $p\in[\underline{p}(w),\overline{p}(w)]$. Therefore, the optimal price is $p^C(w)=\underline{p}(w)$. Correspondingly, $\Pi^C=0$, the effective arrival rate is

$$\lambda(w)=\Lambda\left(1-\frac{hw+\underline{p}(w)}{a}\right), \tag{EC.7}$$

and the social welfare is

$$SW(w,\underline{p}(w))=\frac{a\lambda^2}{2\Lambda}. \tag{EC.8}$$

Differentiating $SW(w,\underline{p}(w))$ w.r.t. w yields

$$\frac{\mathrm{d}SW(w,\underline{p}(w))}{\mathrm{d}w}=\frac{\lambda}{2h}\left(-1+\frac{\frac{2a\gamma}{h\Lambda w^2}-(a-hw-c-\gamma)}{\sqrt{(a-hw-c-\gamma)^2-\frac{4a\gamma}{w\Lambda}}}\right).$$

Then $\mathrm{d}SW(w,\underline{p}(w))/\mathrm{d}w=0$ can be simplified as

$$h\Lambda\left(1-\frac{2hw+c+\gamma}{a}\right)=\frac{\gamma}{w^2}, \tag{EC.9}$$

and $\mathrm{d}SW(w,\underline{p}(w))/\mathrm{d}w<0$ can be simplified as

$$h\Lambda\left(1-\frac{2hw+c+\gamma}{a}\right)>\frac{\gamma}{w^2}. \tag{EC.10}$$

Solving (EC.10) yields $\underline{w}^C<w<\overline{w}^C$, where $\underline{w}^C\in(0,\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}]$ and $\overline{w}^C\in[\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3},\frac{a-c-\gamma}{2h})$ are two roots to (EC.9) w.r.t. w on the interval $(0,\frac{a-c-\gamma}{2h})$. When (EC.9) holds, we have $(a-hw-$

$c-\gamma)^2 = \left(hw + \frac{a\gamma}{h\Lambda w^2}\right)^2 > \frac{4a\gamma}{w\Lambda}$, implying that $\Delta > 0$. It then follows that $\underline{w}_l < \underline{w}^C$ and $\overline{w}^C < \overline{w}_l$. Therefore, $\frac{\mathrm{d}SW(w,\underline{p}(w))}{\mathrm{d}w} \geq 0$, $\frac{\mathrm{d}SW(w,\underline{p}(w))}{\mathrm{d}w} < 0$, and $\frac{\mathrm{d}SW(w,\underline{p}(w))}{\mathrm{d}w} \geq 0$ hold on intervals $[\underline{w}_l, \underline{w}^C]$, $(\underline{w}^C, \overline{w}^C)$ and $[\overline{w}^C, \overline{w}_l]$ regarding w, respectively. In addition, when $w = \underline{w}_l$, $\lambda = \frac{\Lambda}{2a}(a+c+\gamma-h\underline{w}_l)$, and when $w = \overline{w}_l$, $\lambda = \frac{\Lambda}{2a}(a+c+\gamma-h\overline{w}_l) < \frac{\Lambda}{2a}(a+c+\gamma-h\underline{w}_l)$, which, according to (EC.8), implies that $SW(\underline{w}_l, \underline{p}(\underline{w}_l)) \geq SW(\overline{w}_l, \underline{p}(\overline{w}_l))$. Hence, the optimal wait time w^C equals $\underline{w}^C$, or equivalently $w^C = \Phi^C \frac{a-c-\gamma}{h}$, where, according to (EC.9), Φ^C is the unique root to the following equation in the range $(0, \frac{1}{3}\Theta^{-\frac{1}{3}}]$,

$$\Theta(1-2\Phi^C) = \frac{1}{27(\Phi^C)^2}. \tag{EC.11}$$

The optimal price p^C equals $\underline{p}(w^C)$. From (EC.9), we have

$$\underline{p}(w^C) = \frac{a - hw^C + c + \gamma - \sqrt{\left(\frac{a\gamma}{h\Lambda(w^C)^2} - hw^C\right)^2}}{2} = hw^C + c + \gamma, \tag{EC.12}$$

or equivalently

$$p^C = \Phi^C(a-c-\gamma) + c + \gamma. \tag{EC.13}$$

From (EC.1), (EC.9), and (EC.12), the resulting effective arrival rate satisfies

$$\lambda^C = \Lambda\left(1 - \frac{hw^C + p^C}{a}\right) = \frac{\gamma}{h(w^C)^2}, \tag{EC.14}$$

or equivalently,

$$\lambda^C = \frac{1}{(\Phi^C)^2}\frac{K^2}{h\gamma}, \tag{EC.15}$$

where $K = \frac{h\gamma}{a-c-\gamma}$. From (EC.8) and (EC.14), the maximum social welfare satisfies

$$SW^C = \frac{a\gamma^2}{2h^2\Lambda(w^C)^4}. \tag{EC.16}$$

Plugging $w^C = \Phi^C\frac{a-c-\gamma}{h}$ and $\Lambda = \Theta\Lambda_0$ into (EC.16) yields $SW^C = \frac{K}{54\Theta(\Phi^C)^4}$. From (EC.11), we have $\Theta = \frac{1}{27(1-2\Phi^C)(\Phi^C)^2}$. It follows that SW^C can be rearranged as $SW^C = \left(\frac{1}{2(\Phi^C)^2} - \frac{1}{\Phi^C}\right)K$.

Next, we turn to Scenario D. Differentiating $\Pi(w,p)$ given by (EC.2) w.r.t. p yields $\partial\Pi(w,p)/\partial p = \Lambda(a+c+\gamma-hw-2p)/a$, which implies that the maximizer is

$$p_f(w) = (a+c+\gamma-hw)/2, \tag{EC.17}$$

and correspondingly, the effective arrival rate is

$$\lambda_f(w) = \frac{\Lambda}{2}\left(1 - \frac{hw + c + \gamma}{a}\right), \tag{EC.18}$$

requiring that w is on the interval $[0, \frac{a-c-\gamma}{h}]$. Plugging (EC.17) into $\Pi(w,p)$ yields $\Pi(w, p_f(w)) = \frac{a\Lambda}{4}\left(1 - \frac{hw+c+\gamma}{a}\right)^2 - \frac{\gamma}{w}$. Taking the first and second derivatives of $\Pi(w, p_f(w))$ w.r.t. to w yields

$$\frac{\mathrm{d}\Pi(w, p_f(w))}{\mathrm{d}w} = -\frac{h\Lambda}{2}\left(1 - \frac{hw + c + \gamma}{a}\right) + \frac{\gamma}{w^2}, \quad \frac{\mathrm{d}^2\Pi(w, p_f(w))}{\mathrm{d}w^2} = \frac{h^2\Lambda}{2a} - \frac{2\gamma}{w^3}.$$

Clearly, $\frac{\mathrm{d}\Pi(w,p_f(w))}{\mathrm{d}w}$ is first decreasing and then increasing in w. Because $\frac{\mathrm{d}\Pi(w,p_f(w))}{\mathrm{d}w}|_{w=0} = +\infty$, and $\frac{\mathrm{d}\Pi(w,p_f(w))}{\mathrm{d}w}|_{w=\frac{a-c-\gamma}{h}} = \frac{h^2\gamma}{(a-c-\gamma)^2} > 0$, it follows that the optimal wait time w^D is the smaller root to

$$\frac{h\Lambda}{2}\left(1 - \frac{hw + c + \gamma}{a}\right) = \frac{\gamma}{w^2}, \tag{EC.19}$$

or equivalently $w^D = \Phi^D \frac{a-c-\gamma}{h}$ where Φ^D is the smaller root to

$$\Theta(1 - \Phi^D) = \frac{2}{27(\Phi^D)^2}, \tag{EC.20}$$

provided that the associated profit is nonnegative. From (EC.18) and (EC.19), we have $\lambda^D = \frac{\gamma}{h(w^D)^2}$, and accordingly, $\Pi^D = \frac{a\gamma^2}{h^2\Lambda(w^D)^4} - \frac{\gamma}{w^D}$, and $SW^D = \frac{3a\gamma^2}{2h^2\Lambda(w^D)^4} - \frac{\gamma}{w^D}$. Solving $\Pi^D \geq 0$ yields $w^D \leq (\frac{a\gamma}{h^2\Lambda})^{\frac{1}{3}}$, or equivalently $\Phi^D \leq \frac{1}{3}\Theta^{-\frac{1}{3}}$, which gives us $\Theta \geq 1$. Plugging $w^D = \Phi^D \frac{a-c-\gamma}{h}$ and $\Lambda = \Theta\Lambda_0$ into λ^D yields $\lambda^D = \frac{1}{(\Phi^D)^2}\frac{K^2}{h\gamma}$. Plugging $w^D = \Phi^D \frac{a-c-\gamma}{h}$, $\Lambda = \Theta\Lambda_0$ and $\Theta = \frac{2}{27(1-\Phi^D)(\Phi^D)^2}$ respectively into Π^D and SW^D yields

$$\Pi^D = \left(\frac{1}{2(\Phi^D)^2} - \frac{3}{2\Phi^D}\right)K, \tag{EC.21}$$

$$SW^D = \left(\frac{3}{4(\Phi^D)^2} - \frac{7}{4\Phi^D}\right)K. \tag{EC.22}$$

Proof of Proposition 1. Let $F(w,p) = \beta SW + (1-\beta)\Pi$. We first relax the firm's participation constraint and get the optimal solution (w^B, p^B) that maximizes $F(w,p)$. Differentiating $F(w,p)$ w.r.t. p yields $\frac{\partial F(w,p)}{\partial p} = (\beta - 2)p + (1-\beta)(a - hw) + c + \gamma$. It follows from the first-order condition that the optimal price is given by

$$p^*(w) = ((1-\beta)(a - hw) + c + \gamma)/(2 - \beta).$$

Correspondingly, we have

$$\lambda(w,p^*(w)) = \frac{\Lambda}{2-\beta}\left(1-\frac{hw+c+\gamma}{a}\right), \tag{EC.23}$$
$$\Pi(w,p^*(w)) = \frac{(1-\beta)\Lambda}{a(2-\beta)^2}(a-hw-c-\gamma)^2 - \frac{\gamma}{w}, \tag{EC.24}$$
$$SW(w,p^*(w)) = \frac{(3/2-\beta)\Lambda}{a(2-\beta)^2}(a-hw-c-\gamma)^2 - \frac{\gamma}{w}. \tag{EC.25}$$

Plugging $p^*(w)$ into $F(w,p)$ yields $F(w,p^*(w))$. Differentiating $F(w,p^*(w))$ w.r.t. w yields

$$\begin{aligned}\frac{\mathrm{d}F(w,p^*(w))}{\mathrm{d}w} &= \frac{\mathrm{d}\Pi(w,p^*(w))}{\mathrm{d}w} + \frac{\beta\Lambda\lambda(w,p^*(w))}{a}\frac{\mathrm{d}\lambda(w,p^*(w))}{\mathrm{d}w}\\ &= -\frac{2(1-\beta)h\Lambda}{(2-\beta)^2}\left(1-\frac{hw+c+\gamma}{a}\right)+\frac{\gamma}{w^2}-\frac{\beta h\Lambda}{2-\beta}\left(1-\frac{hw+c+\gamma}{a}\right)\\ &= -\frac{h\Lambda}{2-\beta}\left(1-\frac{hw+c+\gamma}{a}\right)+\frac{\gamma}{w^2}.\end{aligned}$$

The optimal wait time w^B is the smaller root w.r.t. $w \in (0, \frac{a-c-\gamma}{h})$ to

$$\frac{h\Lambda}{2-\beta}\left(1-\frac{hw+c+\gamma}{a}\right) = \frac{\gamma}{w^2}. \tag{EC.26}$$

It is clear that $\frac{\mathrm{d}w^B}{\mathrm{d}\beta} < 0$. From (EC.23), we have

$$\frac{\mathrm{d}\lambda^B}{\mathrm{d}\beta} = \frac{1}{(2-\beta)^2}\left(1-\frac{hw^B+c+\gamma}{a}\right) - \frac{h\Lambda}{a(2-\beta)}\frac{\mathrm{d}w^B}{\mathrm{d}\beta} > 0.$$

Let $\Pi^B = \Pi(w^B, p^*(w^B))$. From (EC.24), we have

$$\begin{aligned}\frac{\mathrm{d}\Pi^B}{\mathrm{d}\beta} &= \frac{\partial\Pi(w^B,p^*(w^B))}{\partial\beta} + \frac{\mathrm{d}\Pi(w^B,p^*(w^B))}{\mathrm{d}w^B}\frac{\mathrm{d}w^B}{\mathrm{d}\beta}\\ &= \frac{-\beta\Lambda}{(2-\beta)^3}\left(1-\frac{hw^B+c+\gamma}{a}\right)^2 + \frac{\beta h\Lambda}{2-\beta}\left(1-\frac{hw^B+c+\gamma}{a}\right)\frac{\mathrm{d}w^B}{\mathrm{d}\beta} < 0.\end{aligned}$$

Let $SW^B = SW(w^B, p^*(w^B))$. From (EC.25), we have

$$\begin{aligned}\frac{\mathrm{d}SW^B}{\mathrm{d}\beta} &= \frac{\partial SW(w^B,p^*(w^B))}{\partial\beta} + \frac{\mathrm{d}SW(w^B,p^*(w^B))}{\mathrm{d}w^B}\frac{\mathrm{d}w^B}{\mathrm{d}\beta}\\ &= \frac{(1-\beta)\Lambda}{a(2-\beta)^3}(a-hw^B-c-\gamma)^2 + \frac{(\beta-1)h\Lambda}{(2-\beta)^2}\left(1-\frac{hw^B+c+\gamma}{a}\right)\frac{\mathrm{d}w^B}{\mathrm{d}\beta} > 0.\end{aligned}$$

Next, we take the firm's participation constraint into account. From (EC.24) and (EC.26), we have $\Pi^B = \frac{(1-\beta)a\gamma^2}{h^2\Lambda(w^B)^4} - \frac{\gamma}{w^B}$. Then $\Pi^B \geq 0$ requires that

$$w^B \leq \left(\frac{(1-\beta)a\gamma}{h^2\Lambda}\right)^{\frac{1}{3}}, \tag{EC.27}$$

or equivalently, $\Phi^B \leq \frac{1}{3}(1-\beta)^{\frac{1}{3}}\Theta^{-\frac{1}{3}}$. From (EC.26), (EC.27) is equivalent to $\Lambda \geq \left((1-\beta)^{\frac{1}{3}} + (2-\beta)(1-\beta)^{-\frac{2}{3}}\right)^3 \frac{ah\gamma}{(a-c-\gamma)^3}$, or equivalently, $\Theta \geq \Theta_B = \left(\frac{1}{3}(1-\beta)^{\frac{1}{3}} + \frac{2-\beta}{3}(1-\beta)^{-\frac{2}{3}}\right)^3$. Therefore, if $\Theta \geq \Theta_B$, the firm's participation constraint is redundant under the solution (w^B, p^B), and hence, (w^B, p^B) is optimal.

Consider that $1 \leq \Theta < \Theta_B$. In this case, the firm's participation constraint is binding, i.e., $\Pi(w,p) = 0$. According to the preceding analysis, the optimal price satisfies $p_B = \underline{p}(w)$, which is the smaller root to $\Pi(w,p) = 0$. Then, the optimal wait time w^B solves the following optimization problem:

$$\max_w \; SW = \frac{a\lambda^2}{2\Lambda} \quad \text{s.t. } p = \underline{p}(w),$$

which is the same as the decision model under Scenario C. Hence, we can conclude that when $1 \leq \Theta < \Theta_B$, the optimal solution under Scenario B is identical to that under Scenario C such that the outcomes are invariant to β.

Note that $\Theta_B = \left(\frac{1}{3}(1-\beta)^{\frac{1}{3}} + \frac{2-\beta}{3}(1-\beta)^{-\frac{2}{3}}\right)^3$ increases in β and $\Theta_B \geq 1$. We can, respectively, rewrite $\Theta < \Theta_B$ and $\Theta \geq \Theta_B$ as $\beta > \beta_0$ and $\beta \leq \beta_0$, where $\beta_0 \in [0,1)$ solves $\Theta = \Theta_B$.

Proof of Proposition 2. We begin with Scenario SP. Given p, the firm's optimal wait time

$$w_f(p) = \sqrt{\frac{a\gamma}{(p-c-\gamma)h\Lambda}} \tag{EC.28}$$

is determined by the first-order condition of Π w.r.t. w: $\frac{\partial \Pi}{\partial w} = -\frac{h\Lambda}{a}(p-c-\gamma) + \frac{\gamma}{w^2} = 0$. Next, we derive the government's optimal decision about p. Because under the firm's best response, there exists a one-to-one mapping between p and w via (EC.28). We can change the government's decision variable from p to w by substituting the following for p:

$$p_g(w) = \frac{a\gamma}{h\Lambda w^2} + c + \gamma. \tag{EC.29}$$

Thus, the effective arrival rate is

$$\lambda = \Lambda\left(1 - \frac{hw + c + \gamma}{a} - \frac{\gamma}{h\Lambda w^2}\right). \tag{EC.30}$$

Plugging (EC.29) and (EC.30) into Π yields $\Pi(w,p_g(w)) = \left(1-\frac{hw+c+\gamma}{a}-\frac{\gamma}{h\Lambda w^2}\right)\frac{a\gamma}{hw^2}-\frac{\gamma}{w}$, and then the constraint $\Pi(w,p_g(w))\geq 0$ can be simplified as

$$h\Lambda\left(1-\frac{2hw+c+\gamma}{a}\right)\geq\frac{\gamma}{w^2},$$

which is equivalent to $\underline{w}^C\leq w\leq\overline{w}^C$. Recall that $\underline{w}^C\in(0,\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}]$ and $\overline{w}^C\in[\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3},\frac{a-c-\gamma}{2h})$ are the two roots to (EC.9) w.r.t. $w\in(0,\frac{a-c-\gamma}{2h})$.

Plugging (EC.29) and (EC.30) into SW, the government's decision problem becomes maximizing

$$SW(w,p_g(w)) = \Lambda\left(1-\frac{hw+c+\gamma}{a}-\frac{\gamma}{h\Lambda w^2}\right)\left(\frac{a-hw-c-\gamma}{2}+\frac{a\gamma}{2h\Lambda w^2}\right)-\frac{\gamma}{w}$$

w.r.t. w, subject to $w\in[\underline{w}^C,\overline{w}^C]$. Differentiating $SW(w,p_g(w))$ w.r.t. w yields

$$\frac{\mathrm{d}SW(w,p_g(w))}{\mathrm{d}w} = -h\Lambda\left(1-\frac{hw+c+\gamma}{a}\right)+\frac{2a\gamma^2}{h^2\Lambda w^5}+\frac{\gamma}{w^2}.$$

Given $\Theta\geq 1$, there exist two solutions to the first-order condition,

$$h\Lambda\left(1-\frac{hw+c+\gamma}{a}\right)=\frac{2a\gamma^2}{h^2\Lambda w^5}+\frac{\gamma}{w^2}, \tag{EC.31}$$

w.r.t. $w\in(0,\frac{a-c-\gamma}{h})$, and the smaller root, denoted as $w^S_{P,1}$, is the maximizer. It follows that w^S_P is either $\underline{w}^C$, $\overline{w}^C$, or $w^S_{P,1}$. Since $\underline{w}^C\leq w^D\leq\overline{w}^C$, we prove $w^S_P=\min\{w^S_{P,1},\overline{w}^C\}$ by illustrating $w^S_{P,1}>w^D$. From (EC.31) and (EC.19), $w^S_{P,1}>w^D$ is equivalent to $\frac{2\gamma}{(w^D)^2}<\frac{2a\gamma^2}{h^2\Lambda(w^D)^5}+\frac{\gamma}{(w^D)^2}$. Solving this inequality yields $w^D<(\frac{2a\gamma}{h^2\Lambda})^{\frac{1}{3}}$, which is true because $w^D\leq(\frac{a\gamma}{h^2\Lambda})^{\frac{1}{3}}$. Hence, $w^S_{P,1}>w^D$.

Plugging $w=w^S_{P,1}$ into $\Pi(w,p_g(w))$ yields $\Pi^S_{P,1}=\frac{2a^2\gamma^3}{h^4\Lambda^2(w^S_{P,1})^7}-\frac{\gamma}{w^S_{P,1}}$. Solving $\Pi^S_{P,1}\geq 0$ yields $w^S_{P,1}\leq(\frac{\sqrt{2}a\gamma}{h^2\Lambda})^{\frac{1}{3}}$, which gives $\Lambda\geq(2^{-1/3}+2^{7/6})^3\frac{ah\gamma}{(a-c-\gamma)^3}$, or $\Theta\geq\frac{(2^{-1/3}+2^{7/6})^3}{27}=\Theta'$. It follows that

$$\min\{w^S_{P,1},\overline{w}^C\}=\begin{cases}\overline{w}^C, & if\ 1\leq\Theta\leq\Theta'\\ w^S_{P,1}, & if\ \Theta>\Theta'\end{cases}.$$

Equivalently, letting $w^S_P=\Phi^S_P\frac{a-c-\gamma}{h}$, if $1\leq\Theta\leq\Theta'$, Φ^S_P is the unique root to (EC.11) in the range $[\frac{1}{3}\Theta^{-\frac{1}{3}},\frac{1}{2})$; if $\Theta>\Theta'$, Φ^S_P is the unique root to

$$\Theta(1-\Phi^S_P)=\frac{1}{27(\Phi^S_P)^2}\left(1+\frac{2}{27\Theta(\Phi^S_P)^3}\right) \tag{EC.32}$$

in the range $(0, \frac{2^{1/6}}{3}\Theta^{-\frac{1}{3}})$. Plugging $w_P^S = \Phi_P^S \frac{a-c\gamma}{h}$ into (EC.29) yields

$$p_P^S = \frac{a-c-\gamma}{27\Theta(\Phi_P^S)^2} + c + \gamma. \tag{EC.33}$$

Accordingly, if $1 \le \Theta \le \Theta'$, $\Pi_P^S = 0$. From (EC.9) and (EC.30), it follows that $\lambda_P^S = \frac{h\Lambda w_P^S}{a}$. Plugging $w_P^S = \Phi_P^S \frac{a-c-\gamma}{h}$, $\Lambda = \Theta\Lambda_0$, and $\Theta = \frac{1}{27(\Phi_P^S)^2(1-2\Phi_P^S)}$ into λ_P^S yields $\lambda_P^S = \frac{1}{\Phi_P^S(1-2\Phi_P^S)}\frac{K^2}{h\gamma}$, and the equilibrium social welfare satisfies

$$SW_P^S = \frac{K}{2(1-2\Phi_P^S)}. \tag{EC.34}$$

If $\Theta > \Theta'$, from (EC.30) and (EC.31), it follows that $\lambda_P^S = \frac{2a\gamma^2}{h^3\Lambda(w_P^S)^5}$, or equivalently,

$$\lambda_P^S = \frac{2}{27\Theta(\Phi_P^S)^5}\frac{K^2}{h\gamma}. \tag{EC.35}$$

The equilibrium profit satisfies $\Pi_P^S = \Pi_{P,1}^S = \left(\frac{2}{(27\Theta)^2(\Phi_P^S)^7} - \frac{1}{\Phi_P^S}\right)K$. The equilibrium social welfare satisfies $SW_P^S = \left(\frac{2}{(27\Theta)^2(\Phi_P^S)^7}\left(\frac{1}{27\Theta(\Phi_P^S)^3}+1\right) - \frac{1}{\Phi_P^S}\right)K$.

Next we determine the equilibrium under Scenario SW. From the proof of Corollary 1, given w, the price maximizing the profit and the resulting effective arrival rate are given by (EC.17) and (EC.18), and constraint $\Pi(w, p_f(w)) \ge 0$ can be simplified as (EC.5). It follows that $\Pi(w, p_f(w)) \ge 0$ requires that $\Theta \ge 1$ and $w \in [\underline{w}_l, \overline{w}_l]$. Recall that $\underline{w}_l \in (0, \left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}]$ and $\overline{w}_l \in [\left(\frac{a\gamma}{h^2\Lambda}\right)^{1/3}, \frac{a-c-\gamma}{h})$ are two roots to (EC.5) with equality. Differentiating $SW(w, p_f(w))$ w.r.t. w yields

$$\frac{\mathrm{d}SW(w,p_f(w))}{\mathrm{d}w} = -\frac{3h\Lambda}{4}\left(1 - \frac{hw+c+\gamma}{a}\right) + \frac{\gamma}{w^2}.$$

Given $\Theta \ge 1$, there exist two roots to $\frac{\mathrm{d}SW(w,p_f(w))}{\mathrm{d}w} = 0$ w.r.t. $w \in (0, \frac{a-c-\gamma}{h})$, and the smaller root, say $w_{W,1}^S$, is the maximizer. It is easy to check that $w_{W,1}^S < w^D < \overline{w}_l$. It follows that $w_W^S = w_{W,1}^S$, if

$$\Pi_{W,1}^S = \frac{\Lambda(a - hw_{W,1}^S - c - \gamma)^2}{4a} - \frac{\gamma}{w_{W,1}^S} = \frac{4a\gamma^2}{9h^2\Lambda}\frac{1}{(w_{W,1}^S)^4} - \frac{\gamma}{w_{W,1}^S} \ge 0,$$

which requires $w_{W,1}^S \le (\frac{4a\gamma}{9h^2\Lambda})^{\frac{1}{3}}$, or equivalently, $\Theta \ge \frac{256}{243} = \Theta''$; otherwise, $w_W^S = \underline{w}_l$.

Let $w_W^S = \Phi_W^S \frac{a-c-\gamma}{h}$. When $1 \le \Theta \le \Theta''$, from $w_W^S = \underline{w}_l$, Φ_W^S is the unique solution to

$$\Theta(1-\Phi_W^S)^2 = \frac{4}{27\Phi_W^S} \tag{EC.36}$$

in the range $\Phi \in (0, \Phi^D]$. When $\Theta > \Theta''$, from $w_W^S = w_{W,1}^S$, Φ_W^S is the unique solution to

$$\Theta(1-\Phi_W^S)=\frac{4}{81(\Phi_W^S)^2} \tag{EC.37}$$

in the range $\Phi\in(0,(\frac{243}{4}\Theta)^{-\frac{1}{3}})$.

Plugging $w_W^S=\Phi_W^S\frac{a-c-\gamma}{h}$ into $p_f(w)$ yields $p_W^S=(a+c+\gamma-\Phi_W^S(a-c-\gamma))/2$. Accordingly, if $1\le\Theta\le\Theta'$, $\Pi_W^S=0$. Plugging $\underline{w}_l$ into $\lambda_f(w)$ yields $\lambda_W^S=\sqrt{\frac{\gamma\Lambda}{aw}}$, or equivalently,

$$\lambda_W^S=\frac{2}{(1-\Phi_W^S)\Phi_W^S}\frac{K^2}{h\gamma}. \tag{EC.38}$$

The equilibrium social welfare satisfies $SW_W^S=\frac{\gamma}{2w_W^S}=\frac{1}{2\Phi_W^S}K$. If $\Theta>\Theta'$, plugging w_W^S into $\lambda_f(w)$ yields $\lambda_W^S=\frac{2\gamma}{3h(w_W^S)^2}$, or equivalently $\lambda_W^S=\frac{2}{3(\Phi_W^S)^2}\frac{K^2}{h\gamma}$. The equilibrium profit satisfies $\Pi_W^S=\Pi_{W,1}^S=\left(\frac{1}{3(\Phi_W^S)^2}-\frac{4}{3\Phi_W^S}\right)K$. The equilibrium social welfare satisfies $SW_W^S=\left(\frac{1}{2(\Phi_W^S)^2}-\frac{3}{2\Phi_W^S}\right)K$.

Proof of Theorem 1. The proofs of Corollary 1 and Proposition 2 have shown that $w^C\le w^D$ and $w_W^S\le w^D\le w_P^S$, we thus need to compare w^C and w_W^S. Note from the proof of Corollary 1 that $w^C=\underline{w}^C$. According to the proof of Proposition 2, when $\Theta\le\Theta''$, $w_W^S=\underline{w}_l\le\underline{w}^C=w^C$. Consider that $\Theta>\Theta''$. Note that $w^C\ge w_W^S$ is equivalent to $\Phi^C\ge\Phi_W^S$, which, according to (EC.11) can be rewritten as

$$\Theta(1-2\Phi_W^S)\le\frac{1}{27(\Phi_W^S)^2}. \tag{EC.39}$$

Since Φ_W^S satisfies (EC.37), then (EC.39) can be simplified as $\Phi_P^S\ge\frac{1}{5}$, or equivalently $\Theta\le\frac{125}{81}$. In summary, $\max\{w^C,w_W^S\}\le w^D\le w_P^S$, and

$$\max\{w^C,w_W^S\}=\begin{cases} w^C,\ if\ 1\le\Theta\le\frac{125}{81} \\ w_W^S,\ if\ \Theta>\frac{125}{81}\end{cases}.$$

Next we compare the optimal prices. Given $p^C=hw^C+c+\gamma$ and $p^D=\frac{a+c+\gamma-hw^D}{2}$, it follows that $p^C\le p^D$ is equivalent to $w^C\le\frac{a-c-\gamma-hw^D}{2h}$, taking which into (EC.9) yields

$$(a-hw^D-c-\gamma)^2w^D\le(4a\gamma)/\Lambda. \tag{EC.40}$$

Given that w^D satisfies (EC.19), (EC.40) can be simplified as $w^D\le\left(\frac{a\gamma}{h^2\Lambda}\right)^{\frac{1}{3}}$, which is true.

Note that the optimal price and expected wait time under Scenarios D and SW satisfy (EC.17), and $w^D\ge w_W^S$; then it follows that $p^D\le p_W^S$. From (EC.17) and (EC.19), we can rewrite the optimal price under Scenario D as $p^D=\frac{a\gamma}{h\Lambda(w^D)^2}+c+\gamma$. Given that p_P^S and w_P^S satisfy (EC.29), i.e., $p_P^S=\frac{a\gamma}{h\Lambda(w_P^S)^2}+c+\gamma$, and $w^D\le w_P^S$, it follows that $p^D\ge p_P^S$. Finally, we compare p^C and p_P^S. Given that p^C satisfies (EC.13) and p_P^S satisfies (EC.33), $p^C \geq p_P^S$ can be simplified as $\Phi^C \geq \frac{1}{27\Theta(\Phi_P^S)^2}$, which, according to (EC.11), is equivalent to

$$1 - \frac{2}{27\Theta(\Phi_P^S)^2} \leq 27\Theta(\Phi_P^S)^4 \tag{EC.41}$$

When $1 \leq \Theta \leq \Theta'$, note that Φ_P^S is the unique solution to (EC.11) in the range $\Phi \in [\frac{1}{3}\Theta^{-\frac{1}{3}}, \frac{1}{2})$. Rewriting (EC.11) as $\Theta = \frac{1}{27(1-2\Phi_P^S)(\Phi_P^S)^2}$ and plugging it into (EC.41), we obtain

$$1 - 2(1 - 2\Phi_P^S) \leq \frac{(\Phi_P^S)^2}{1 - 2\Phi_P^S}, \tag{EC.42}$$

which is true and the equality holds only when $\Phi_P^S = 1/3$, i.e., $\Theta = 1$. Hence, we have $p^C \geq p_P^S$ when $1 \leq \Theta \leq \Theta'$.

Consider that $\Theta > \Theta'$. Note that Φ_P^S is the unique solution to (EC.32) in the range $\Phi \in (0, \frac{2^{1/6}}{3}\Theta^{-\frac{1}{3}})$. To compare p_P^S and p^C, we rewrite (EC.41) as $27^2(\Phi_P^S)^6\Theta^2 - 27(\Phi_P^S)^2\Theta + 2 \geq 0$, which is equivalent to $\Phi_P^S \geq \frac{1}{2\sqrt{2}}$ or $\Phi_P^S < \frac{1}{2\sqrt{2}}$ and

$$\Theta \leq \frac{1 - \sqrt{1 - 8(\Phi_P^S)^2}}{54(\Phi_P^S)^4}. \tag{EC.43}$$

Plugging (EC.43) into (EC.32) yields $\frac{1-\sqrt{1-8(\Phi_P^S)^2}}{2(\Phi_P^S)^2}(1 - \Phi_P^S) \geq 1 + \frac{4\Phi_P^S}{(1-\sqrt{1-8(\Phi_P^S)^2})}$, solving which yields $\Phi_P^S \geq \Phi_0 \approx 0.3532$, or equivalently $\Theta \leq \Theta_0 \approx 1.1367$.

In summary, the optimal prices satisfy $\max\{p^C, p_P^S\} \leq p^D \leq p_W^S$, and

$$\max\{p^C, p_P^S\} = \begin{cases} p^C, \ if \ \Theta \leq \Theta_0 \\ p_P^S, \ if \ \Theta > \Theta_0 \end{cases}.$$

It is straightforward to have $SW^D \leq \min\{SW_P^S, SW_W^S\} \leq SW^C$ and $\Pi^C = 0 \leq \max\{\Pi_P^S, \Pi_W^S\} \leq \Pi^D$. Consequently, $CS^D \leq \min\{CS_P^S, CS_W^S\} \leq CS^C$ and $\lambda^D \leq \min\{\lambda_P^S, \lambda_W^S\} \leq \lambda^C$. In the following, we determine the comparison results of the effective arrival rate, profit, and social welfare under Scenarios SP and SW.

(i) Given $1 \leq \Theta \leq \Theta'$, $\Pi_P^S = \Pi_W^S = 0$ (by Proposition 2). From (EC.32) and (EC.36), we have $(1 - 2\Phi_P^S)(2\Phi_P^S)^2 = \Phi_W^S(1 - \Phi_W^S)^2$ which gives us $1 - 2\Phi_P^S = \Phi_W^S$. It follows that $\lambda_P^S = \lambda_W^S$ and $SW_P^S = SW_W^S$.

(ii) Given $\Theta' < \Theta \leq \Theta''$, $\Pi_P^S > \Pi_W^S = 0$ (by Proposition 2). From the proof of Proposition 2, the equilibrium social welfare under Scenario SP, in this case, satisfies $SW_P^S = SW(w_{P,1}^S, p_g(w_{P,1}^S)) >$

$SW(\overline{w}^C, p_g(\overline{w}^C))$. As shown in (i), the equilibrium social welfare under Scenario SW satisfies $SW_W^S = SW(\overline{w}^C, p_g(\overline{w}^C))$. It follows that $SW_P^S > SW_W^S$.

Given that λ_P^S satisfies (EC.35) and λ_W^S satisfies (EC.38), $\lambda_P^S < \lambda_W^S$ is equivalent to

$$27\Theta(\Phi_P^S)^5 > (1-\Phi_W^S)\Phi_W^S. \tag{EC.44}$$

From (EC.36), we have $\Theta = \frac{4}{27(1-\Phi_W^S)^2\Phi_W^S}$, plugging which into (EC.44) yields

$$\Phi_P^S > 4^{-\frac{1}{5}}(1-\Phi_W^S)^{\frac{3}{5}}(\Phi_W^S)^{\frac{2}{5}}. \tag{EC.45}$$

From (EC.32), (EC.45) is equivalent to

$$2^{\frac{4}{5}}(1-\Phi_W^S)^{\frac{4}{5}}(\Phi_W^S)^{\frac{1}{5}} + 2^{\frac{8}{5}}(1-\Phi_W^S)^{\frac{3}{5}}(\Phi_W^S)^{\frac{2}{5}} - 2\Phi_W^S - 2 > 0. \tag{EC.46}$$

When $\Theta = \Theta'$, $\Phi_W^S = \frac{1}{1+2\sqrt{2}}$ (which can be obtained from $\Pi_P^S = 0$ and $1 - 2\Phi_P^S = \Phi_W^S$). When $\Theta = \Theta''$, $\Phi_W^S = \frac{1}{4}$ (which can be obtained from $\Pi_W^S = 0$). It is easy to check that (EC.46) holds for $\Phi_W^S \in [\frac{1}{4}, \frac{1}{1+2\sqrt{2}})$. Hence, $\lambda_P^S < \lambda_W^S$ for $\Theta' < \Theta \leq \Theta''$.

(iii) Given $\Theta > \Theta''$, in this case, $\Phi_W^S < \frac{1}{4}$. We rewrite (EC.37) as

$$\Theta = \frac{4}{81(1-\Phi_W^S)(\Phi_W^S)^2}. \tag{EC.47}$$

Plugging (EC.47) into (EC.32), we have

$$\frac{4(1-\Phi_P^S)}{3(1-\Phi_W^S)(\Phi_W^S)^2} = \frac{1}{(\Phi_P^S)^2} + \frac{3(1-\Phi_W^S)(\Phi_W^S)^2}{2(\Phi_P^S)^5}. \tag{EC.48}$$

To compare the effective arrival rates, plugging (EC.47) into λ_P^S yields $\lambda_P^S = \frac{3(\Phi_W^S)^2(1-\Phi_W^S)}{2(\Phi_P^S)^5}$. So, $\lambda_P^S > \lambda_W^S$ becomes

$$\Phi_P^S < (\frac{9}{4})^{\frac{1}{5}}(\Phi_W^S)^{\frac{4}{5}}(1-\Phi_W^S)^{\frac{1}{5}}. \tag{EC.49}$$

From (EC.48), condition (EC.49) is equivalent to

$$3(\frac{2}{3})^{\frac{4}{5}}(1-\Phi_W^S)^{\frac{3}{5}}(\Phi_W^S)^{\frac{2}{5}} + 4(\frac{3}{2})^{\frac{2}{5}}(\Phi_W^S)^{\frac{4}{5}}(1-\Phi_W^S)^{\frac{1}{5}} - 2\Phi_W^S - 2 < 0. \tag{EC.50}$$

Solving (EC.50) yields $\Phi_W^S < \Phi_1$ (≈ 0.2458), or equivalently, $\Theta > \Theta_1$ (≈ 1.0837).

Next we compare the equilibrium profits. Plugging (EC.47) into Π_P^S, $\Pi_P^S < \Pi_W^S$ is equivalent to

$$\frac{9(1-\Phi_W^S)^2(\Phi_W^S)^4}{8(\Phi_P^S)^7} - \frac{1}{\Phi_P^S} < \frac{1}{3(\Phi_W^S)^2} - \frac{4}{3\Phi_W^S}. \tag{EC.51}$$

Let $\Phi_P^S = k\Phi_W^S$, where $k > 1$. Then (EC.48) is reduced to

$$\frac{3}{2k^5}(1-\Phi_W^S)^2 + \frac{1}{k^2}\Phi_W^S(1-\Phi_W^S) - \frac{4}{3}\Phi_W^S(1-k\Phi_W^S) = 0, \tag{EC.52}$$

and (EC.51) is reduced to

$$\frac{9}{8k^7}(1-\Phi_W^S)^2 - \frac{1}{k}(\Phi_W^S)^2 < \frac{1}{3}\Phi_W^S - \frac{4}{3}(\Phi_W^S)^2. \tag{EC.53}$$

Let $A = \frac{3}{2} + \frac{4}{3}k^6 - k^3$, $B = 3 + \frac{4}{3}k^5 - k^3$. Solving (EC.52) yields $\Phi_W^S(k) = \frac{B-\sqrt{B^2-6A}}{2A}$ with $k > k'$, where $k = k'$ is determined by $\Phi_W^S(k) = \frac{1}{4}$. Plugging $\Phi_W^S(k)$ into (EC.53) and solving (EC.53) yields $k > k_1$ (≈ 1.4789), or equivalently, $\Theta > \Theta_2$ (≈ 1.1374) .

Plugging (EC.47) into SW_P^S and SW_W^S, it follows that $SW_P^S < SW_W^S$ is equivalent to

$$\frac{27(1-\Phi_W^S)^3(\Phi_W^S)^6}{32(\Phi_P^S)^{10}} + \frac{9(1-\Phi_W^S)^2(\Phi_W^S)^4}{8(\Phi_P^S)^7} - \frac{1}{\Phi_P^S} < \frac{1}{2(\Phi_W^S)^2} - \frac{3}{2\Phi_W^S}. \tag{EC.54}$$

There exists no solution w.r.t. $\Phi_W^S < \frac{1}{4}$ that satisfies (EC.48) and (EC.54) simultaneously. Hence, we can conclude that $SW_P^S > SW_W^S$.

Proof of Proposition 3. We first determine the equilibrium under Scenario NP. According to the proof of Corollary 1, given the expected wait time set by the firm, it is optimal for the government to set $p = \underline{p}(w)$, which equals the minimum of the roots to $\Pi(w,p) = 0$. Given the service price set by the government, the firm's optimal wait time strategy $w_f(p)$ is given by (EC.28). Since $w = w_f(p)$ can be changed into $p = p_g(w)$ (given by (EC.29)), then the equilibrium expected wait time under Scenario NP satisfies $\underline{p}(w) = p_g(w)$, which is equivalent to

$$a - hw - c - \gamma - \frac{2a\gamma}{h\Lambda w^2} = \sqrt{(a-hw-c-\gamma)^2 - \frac{4a\gamma}{w\Lambda}}, \tag{EC.55}$$

with

$$a - hw - c - \gamma - \frac{2a\gamma}{h\Lambda w^2} \geq 0. \tag{EC.56}$$

(EC.55) can be further simplified as (EC.9). Given that the equilibrium expected wait time under

Scenario NP satisfies (EC.56) and (EC.9), it is clear that $w_P^N = \overline{w}^C$. Thus we find that the equilibrium under Scenario NP for $\Theta \geq 1$ is identical to the equilibrium under Scenario SP with $1 \leq \Theta \leq \Theta'$.

Next we determine the equilibrium under Scenario NW. Given the expected time set by the government, the price decision of the firm $p_f(w)$ is given by (EC.17). Taking the derivative of $SW(w,p)$ w.r.t. w yields

$$\frac{\partial SW(w,p)}{\partial w} = -h\Lambda\left(1 - \frac{hw+c+\gamma}{a}\right) + \frac{\gamma}{w^2}.$$

Clearly, the government's best response is independent of p. Given $\Lambda \geq \frac{27ah\gamma}{2(a-c-\gamma)^2}$, or equivalently $\Theta \geq \frac{1}{2}$, there exist two roots to first-order condition,

$$h\Lambda\left(1 - \frac{hw+c+\gamma}{a}\right) = \frac{\gamma}{w^2}, \tag{EC.57}$$

w.r.t. $w \in (0, \frac{a-c-\gamma}{h})$, and the smaller root, denoted as $w_{W,1}^N$, is the maximizer. Then we can conclude that the government's optimal decision about w is either equal to $w_{W,1}^N$ or equal to the smaller root of $\Pi(w, p_f(w)) = 0$, depending on whether $\Pi(w_{W,1}^N, p_f(w_{W,1}^N))$ is larger than zero. Note that $\Pi(w_{W,1}^N, p_f(w_{W,1}^N)) \geq 0$ is equivalent to

$$\frac{\Lambda(a - hw_{W,1}^N - c - \gamma)^2}{4a} - \frac{\gamma}{w_{W,1}^N} \geq 0. \tag{EC.58}$$

From (EC.57), (EC.58) is equivalent to $w_{W,1}^N \leq (\frac{a\gamma}{4h^2\Lambda})^{\frac{1}{3}}$, which gives us $\Lambda \geq \frac{125ah\gamma}{4(a-c-\gamma)^3}$, or equivalently $\Theta \geq \frac{125}{108}$. Hence, when $\Theta \geq \frac{125}{108}$, $w_W^N = w_{W,1}^N$. Let $w_W^N = \Phi_W^N \frac{a-c-\gamma}{h}$. Then when $\Theta \geq \frac{125}{108}$, Φ_W^N is the unique solution to

$$\Theta(1 - \Phi_W^N) = \frac{1}{27(\Phi_W^N)^2} \tag{EC.59}$$

in the range $\Phi \in (0, \frac{1}{3}(4\Theta)^{-\frac{1}{3}}]$. The equilibrium price satisfies

$$p_W^N = (a + c + \gamma - \Phi_W^N(a - c - \gamma))/2. \tag{EC.60}$$

The equilibrium arrival rate, profit, and social welfare are

$$\lambda_W^N = \frac{1}{(\Phi_W^N)^2} \frac{K^2}{2h\gamma}, \tag{EC.61}$$

$$\Pi_W^N = \left(\frac{1}{4(\Phi_W^N)^2} - \frac{5}{4\Phi_W^N}\right) K, \tag{EC.62}$$

$$SW_W^N = \left(\frac{3}{8(\Phi_W^N)^2} - \frac{11}{8\Phi_W^N}\right)K. \tag{EC.63}$$

When $\Theta \in [1, \frac{125}{108})$, it is easy to check that the equilibrium under Scenario NW is identical to that under Scenario SW with $\Theta \le \Theta''$.

Proof of Theorem 2. Following the proof of Theorem 1, it is easy to check that $w_W^N \le w^C \le w^D \le w_P^N$ and $p_W^N \ge p^D \ge p^C \ge p_P^N$; $\Pi^C = \Pi_P^N = 0 \le \Pi_W^N \le \Pi^D$ and $SW^C \ge \max\{SW_P^N, SW_W^N, SW^D\}$. It follows that $\lambda^C \ge \max\{\lambda_P^N, \lambda_W^N, \lambda^D\}$. In the following, we determine the comparison results of the effective arrival rate and of social welfare under Scenarios NP, NW, and D.

(i) Given that $1 \le \Theta \le \frac{125}{108}$, the proof of Theorem 1 shows that $\lambda_P^N = \lambda_W^N \ge \lambda^D$, and $SW_P^N = SW_W^N \ge SW^D$.

(ii) Given that $\Theta > \frac{125}{108}$, in this case, $\Phi_W^N < \frac{1}{5}$ (which is obtained from $\Pi_W^N = 0$). We first compare the effective arrival rate. Since the effective arrival rate and the expected wait time under Scenarios D and NW satisfy (EC.18), then $w_W^N \le w^D$ implies that $\lambda_W^N \ge \lambda^D$. Given that Φ_P^N satisfies (EC.11) and Φ_W^N satisfies (EC.59), we have

$$(1 - 2\Phi_P^N)(\Phi_P^N)^2 = (1 - \Phi_W^N)(\Phi_W^N)^2. \tag{EC.64}$$

$\lambda_P^N > \lambda_W^N$ is equivalent to

$$(1 - 2\Phi_P^N)(\Phi_P^N) < 2\Phi_W^N. \tag{EC.65}$$

Solving (EC.64) and (EC.65) yields $\Phi^C < \frac{1}{5}$. Hence, $\lambda_P^N > \lambda_W^N$ holds for $\Theta > \frac{125}{108}$. In summary, $\lambda_P^N \ge \lambda_W^N \ge \lambda^D$ for $\Theta \ge 1$.

Next we compare social welfare. We first compare it under Scenarios NW and NP. Given that SW_W^N satisfies (EC.63) and SW_P^N satisfies (EC.34), $SW_W^N > SW_P^N$ is equivalent to

$$\Phi_P^N < \frac{1}{2} - \frac{2(\Phi_W^N)^2}{3 - 11\Phi_W^N}. \tag{EC.66}$$

Solving (EC.64) and (EC.66) yields $\Phi^C < \frac{1}{5}$. Hence, $SW_W^N > SW_P^N$ holds for $\Theta > \frac{125}{108}$. Next we compare social welfare under Scenarios NW and D. Because Φ^D satisfies (EC.20) and Φ_W^N satisfies (EC.59), we have

$$2(1 - \Phi_W^N)(\Phi_W^N)^2 = (1 - \Phi^D)(\Phi^D)^2, \tag{EC.67}$$

and so, $SW_W^N > SW^D$ is equivalent to

$$\frac{3}{2(\Phi_W^N)^2} - \frac{11}{2\Phi_W^N} > \frac{3}{(\Phi^D)^2} - \frac{7}{\Phi^D}. \tag{EC.68}$$

Let $\Phi^D = l\Phi_W^N$. Then (EC.67) implies that $\Phi^C = \frac{l^2-2}{l^3-2}$ with $l > \sqrt{2}$, and (EC.68) is equivalent to

$$3l^5 - 11l^4 + 8l^3 + 16l^2 - 28l + 12 > 0,$$

which holds for all $l > \sqrt{2}$. Hence, $SW_W^N > SW^D$ for $\Theta \geq \frac{125}{108}$. At last, we compare social welfare under Scenarios NP and D. Because Φ^D satisfies (EC.20) and Φ_P^N satisfies (EC.11), we have

$$2(1-2\Phi_P^N)(\Phi_P^N)^2 = (1-\Phi^D)(\Phi^D)^2, \tag{EC.69}$$

and so, $SW_P^N < SW_D$ is equivalent to

$$\Phi_P^N < \frac{1}{2} - \frac{(\Phi^D)^2}{3-7\Phi^D}. \tag{EC.70}$$

Solving (EC.69) and (EC.70) yields $\Phi^D < \Phi_3 (\approx 0.2930)$, or equivalently, $\Theta > \Theta_3$ $(\approx 1.2204 > \frac{125}{108})$. In summary, $SW_W^N \geq \max\{SW_P^N, SW^D\}$, where

$$\max\{SW_P^N, SW^D\} = \begin{cases} SW_P^N, \; if \; 1 \leq \Theta \leq \Theta_3 \\ SW^D, \; if \; \Theta > \Theta_3 \end{cases}.$$

Proof of Theorem 3. (i) The result follows immediately from Theorems 1 and 2.

(ii) From part (i), we have $SW_p^S \geq SW_w^S$ and $SW_w^N \geq SW_p^N$. Summing them up yields $SW_p^S + SW_w^N \geq SW_w^S + SW_p^N$, or equivalently, $SW_p^S - SW_p^N \geq SW_w^S - SW_w^N$.

Proof of Corollary 4. (i) When $\beta = 0$, $SW^B = SW^D < SW_j^S$ for $j = P, W$. When $\beta = 1$, the decision model under Scenario B is identical to that under Scenario C, from which we have $SW_j^S \leq SW^C = SW^B$. Combining the property that SW^B is increasing in β, we can conclude there exists a unique root denoted by β_j^S w.r.t. $\beta \in [0,1]$ to $SW^B = SW_j^S$, such that $SW^B < SW_j^S$ for $\beta \in [0, \beta_j^S)$ and $SW^B > SW_j^S$ for $\beta \in (\beta_j^S, 1]$.

(ii) Following property 5 of Theorem 2 and the proof of part (i) in Corollary 4, we get the comparison result between SW^B and SW_j^N where $j = P, W$.

C. Alternative Models and Their Equivalences

In this section, we show that the current modeling settings for regulation are equivalent to alternative settings imposing price or wait time cap.

The equivalence between the price-cap regulation without myopic adjustment and regulation SP. Given the price-cap regulation with $p \le \hat{p}$, the firm's optimal wait time decision $w_f(p)$ is given by (EC.28). Plugging (EC.28) into Π yields that $\Pi(w_f(p),p) = \Lambda\left(1-\frac{p}{a}\right)(p-c-\gamma) - 2\sqrt{\frac{h\gamma\Lambda(p-c-\gamma)}{a}}$.

Differentiating $\Pi(w_f(p),p)$ about p yields that

$$\frac{\mathrm{d}\Pi(w_f(p),p)}{\mathrm{d}p} = \frac{\Lambda}{a}(a+c+\gamma-2p) - \sqrt{\frac{h\gamma\Lambda}{a(p-c-\gamma)}}$$

There exist two roots to $\frac{\mathrm{d}\Pi(w_f(p),p)}{\mathrm{d}p}=0$ w.r.t. $p \in (c+\gamma, a)$, which are denoted by p_1^D and p^D with $p_1^D \le p^D$. We can show that $\frac{\mathrm{d}\Pi(w_f(p),p)}{\mathrm{d}p} < 0$, $\frac{\mathrm{d}\Pi(w_f(p),p)}{\mathrm{d}p} > 0$ and $\frac{\mathrm{d}\Pi(w_f(p),p)}{\mathrm{d}p} < 0$ respectively hold for $p \in (c+\gamma, p_1^D)$, $p \in (p_1^D, p^D)$ and $p \in (p^D, a)$. Note that for $\Theta \ge 1$, $\Pi(w_f(p),p)|_{p=c+\gamma} = 0$, $\Pi(w_f(p),p)|_{p=p^D} = \Pi^D \ge 0$, and $\Pi(w_f(p),p)|_{p=a} < 0$. Then it follows that the maximizer of $\Pi(w_f(p),p)$ is p^D and there exist two roots of $\Pi(w_f(p),p)=0$, denoted by $\underline{p}^D$ and $\overline{p}^D$, where $\underline{p}^D \in (p_1^D, p^D]$ and $\overline{p}^D \in [p^D, a)$.

If $\hat{p} < \underline{p}^D$, $\Pi(w_f(p),p) < 0$ for $p \le \hat{p}$, then the firm will not participate in the project. If $\underline{p}^D \le \hat{p} < p^D$, $\Pi(w_f(p),p) \ge 0$ increases in $p \in [\underline{p}^D, \hat{p}]$, then the firm's optimal price decision is $p_f^{PC} = \hat{p}$. If $\hat{p} \ge p^D$, the the firm's optimal price decision is $p_f^{PC} = p^D$.

To guarantee the firm's participation, the price-cap should be set lager than $\underline{p}^D$, i.e. $\hat{p} \ge \underline{p}^D$. Given the firm's best response, the social welfare is

$$SW(w_f(p_f^{PC}), p_f^{PC}) = \begin{cases} SW(w_f(\hat{p}),\hat{p}), & if\ \underline{p}^D \le \hat{p} < p^D \\ SW^D, & if\ \hat{p} \ge p^D \end{cases}.$$

Recall that $\arg\max_{\underline{p}^D \le \hat{p} \le p^D} SW(w_f(\hat{p}),\hat{p}) = p_P^S$. Hence, the government will set $\hat{p} = p_P^S$. Correspondingly, the firm will set $p = \hat{p} = p_P^S$ and $w = w_P^S$. That is, the price-cap regulation will lead to the same equilibrium as that in regulation SP.

The equivalence between the time-cap regulation without myopic adjustment and regulation SW. Given the time-cap regulation with $w \le \hat{w}$, the firm's optimal price decision $p_f(w)$ is given by (EC.17). According to the proof of Corollary 1, the maximizer of $\Pi(w, p_f(w))$ is w^D, and given $\Theta \ge 1$, $\Pi(w, p_f(w)) \ge 0$ requires that $w \in [\underline{w}^D, \overline{w}^D]$, where $\underline{w}^D$ and $\overline{w}^D$ are two roots to $\Pi(w, p_f(w)) = 0$ w.r.t $w \in (0, \frac{a-c-\gamma}{h})$.

If $\hat{w} < \underline{w}^D$, $\Pi(w, p_f(w)) < 0$ for $w \in (0, \hat{w}]$, then the firm will not participate in the project. If $\underline{w}^D \le \hat{w} < w^D$, $\Pi(w, p_f(w)) \ge 0$ increases in $w \in [\underline{w}^D, \hat{w}]$, then the firm's optimal wait time decision is $w_f^{WC} = \hat{w}$. If $\hat{w} \ge w^D$, then it is straightforward that $w_f^{WC} = w^D$.

To guarantee the firm's participation, the wait time cap should be set lager than $\underline{w}^D$, i.e. $\hat{w} \geq \underline{w}^D$. Given the firm's best response, the social welfare is

$$SW(w_f^{WC}, p_f(w_f^{WC})) = \begin{cases} SW(\hat{w}, p_f(\hat{w})), \; if \;\; \underline{w}^D \leq \hat{w} < w^D \\ SW^D, \qquad\qquad\quad if \;\; \hat{w} \geq w^D \end{cases}.$$

Recall that $\arg\max_{\underline{w}^D \leq \hat{w} \leq w^D} SW(\hat{w}, p_f(\hat{w})) = w_W^S$. Hence, the government will set $\hat{w} = w_W^S$. Correspondingly, the firm will set $w = \hat{w} = w_W^S$ and $p = p_W^S$. That is, the time-cap regulation will lead to the same equilibrium as that in regulation SW.

The equivalence between the price-cap regulation with myopic adjustment and regulation NP. In the presence of myopic adjustment, the government and the private firm take turns to adjust their own decision to maximize their interest, i.e., following the best response, taking the other party's decision as given. Under the price-cap regulation with myopic adjustment, the starting point is $(w, \hat{p}) = (w_P^S, p_P^S)$. In the previous analysis, we have derived the firm's best response to the government's price cap as follows:

$$(w, p) = \begin{cases} (+\infty, +\infty), & \hat{p} < \underline{p}^D \\ (w_f(\hat{p}), \hat{p}), & \underline{p}^D \leq \hat{p} < p^D \\ (w_f(p^D), p^D), & \hat{p} \geq p^D \end{cases},$$

where $(w, p) = (+\infty, +\infty)$ indicates that the firm does not participate. Moreover, $p_P^S \leq p^D$ holds such that it is optimal for the firm to set the price at the price cap when $\hat{p} = p_P^S$. Observing the firm's decision, the government's best response is to solve $\max_{\hat{p}} SW(w, \hat{p})$ subject to $\Pi(w, \hat{p}) \geq 0$, of which the solution is $\hat{p} = \underline{p}(w)$ (by which the firm makes zero profit). Note that $\underline{p}(w_P^S)$ is the smaller root to $\Pi(w_P^S, p) = 0$ w.r.t p and $\Pi(w_P^S, p_P^S) \geq 0$, then it follows that $\underline{p}(w_P^S) \leq p_P^S$. Hence, observing $w = w_P^S$, it is optimal for the government to reduce the price cap from p_P^S to $\underline{p}(w_P^S)$. Now, consider the firm's problem. Because $\underline{p}(w_P^S) \leq p_P^S \leq p^D$ and $w_f(\hat{p})$ decreases in $\hat{p}$, the firm will set the price at the price cap and increase the wait time. In the next round of the government's problem, because the firm's wait time is increased (such that $w \geq w^D$ continues to hold), it is optimal for it to reduce the price cap (such that the firm makes zero profit). The process repeats until the Nash equilibrium is attained. Furthermore, it is clear that during this process, the price cap is always below p^D such that the firm always sets the price at the price cap. Therefore, the price-cap regulation with myopic adjustment is equivalent to regulation NP.

The equivalence between the time-cap regulation with myopic adjustment and regulation NW. In the presence of myopic adjustment, the government and the private firm take turns to adjust their own decision to maximize their interest. Under the time-cap regulation with myopic adjustment, the starting point is $(\hat{w}, p) = (w_W^S, p_W^S)$. In the previous analysis, we have derived the firm's best response to the government's wait time cap as follows:

$$(w,p) = \begin{cases} (+\infty, +\infty), & \hat{w} < \underline{w}^D \\ (\hat{w}, p_f(\hat{w})), & \underline{w}^D \leq \hat{w} < w^D \\ (w^D, p_f(w^D)), & \hat{w} \geq w^D \end{cases},$$

where $(w,p) = (+\infty, +\infty)$ indicates that the firm does not participate. Moreover, $w_W^S \leq w^D$ holds such that it is optimal for the firm to set the wait time at the cap when $\hat{w} = w_W^S$. Observing the firm's decision, the government's best response is to solve $\max_{\hat{w}} SW(\hat{w}, p)$ subject to $\Pi(\hat{w}, p) \geq 0$, of which the solution is $\max\{w_{W,1}^N, \underline{w}(p)\}$ where $w_{W,1}^N$ is independent of p. Note that $\underline{w}(p_W^S)$ is the smaller root to $\Pi(w, p_W^S) = 0$ w.r.t w and $\Pi(w_W^S, p_W^S) \geq 0$, then it follows that $\underline{w}(p_W^S) \leq w_W^S$. Note also that $w_{W,1}^N \leq w_W^S$. Then we can conclude that $\max\{w_{W,1}^N, \underline{w}(p_W^S)\} \leq w_W^S$. Hence, observing $p = p_W^S$, it is optimal for the government to reduce the wait time cap from w_W^S to $\max\{w_{W,1}^N, \underline{w}(p_W^S)\}$. Now, consider the firm's problem. Because $\max\{w_{W,1}^N, \underline{w}(p_W^S)\} \leq w_W^S \leq w^D$ and $p_f(\hat{w})$ decreases in $\hat{w}$, the firm will set the wait time at the cap and increase the price. In the next round of the government's problem, because the firm's price is increased (such that $p \geq p^D$ continues to hold), it is optimal for it to reduce the wait time cap. The process repeats until the Nash equilibrium is attained. Furthermore, it is clear that during this process, the wait time cap is always below w^D such that the firm always sets the wait time at the cap. Therefore, the time-cap regulation with myopic adjustment is equivalent to regulation NW.

D. The Impact of Myopic Adjustment

D.1. Price Regulation

Observing the firm's decision $w = w_P^S$, the government's myopic adjustment is to solve $\max_p SW(w_P^S, p)$ subject to $\Pi(w_P^S, p) \geq 0$, of which the solution is $p = \underline{p}(w_P^S)$ (by which the firm makes zero profit). Observing the government's decision, it is optimal for the firm to adjust wait time from w_P^S to $w_f(\underline{p}(w_P^S))$. Thus, we can conclude that $(w_1, p_1) = (w_f(\underline{p}(w_P^S)), \underline{p}(w_P^S))$.

Next, we compare the performance of (w_1, p_1) to that of (w_P^S, p_P^S). Recall that $p_P^S = \arg\max_{p \in [\underline{p}^D, \overline{p}^D]} SW(w_f(p), p)$, where $p \in [\underline{p}^D, \overline{p}^D]$ is the solution of $\Pi(w_f(p), p) \geq 0$. Since

$\Pi(w_f(p_1),p_1)\geq 0$, it is straightforward that $p_1\geq \underline{p}^D$. Since p_1 is the smaller root of $\Pi(w_P^S,p)=0$ w.r.t. p and $\Pi(w_P^S,p_P^S)\geq 0$, we can conclude that $p_1\leq p_P^S$. Therefore, given $\underline{p}^D\leq p_1\leq p_P^S$, we can conclude that $SW(w_f(p_P^S),p_P^S)\geq SW(w_f(p_1),p_1)$, i.e., $SW_P^S\geq SW(w_1,p_1)$. Recall that $\Pi(w_f(p),p)$ is increasing in $p\in[\underline{p}^D,p^D]$. Then given $\underline{p}^D\leq p_1\leq p_P^S\leq p^D$, we can conclude that $\Pi(w_f(p_P^S),p_P^S)\geq \Pi(w_f(p_1),p_1)$, i.e., $\Pi_P^S\geq\Pi(w_1,p_1)$.

D.2. Wait Time Regulation

Observing the firm's decision, the government's myopic adjustment is to solve $\max_w SW(w,p_W^S)$ subject to $\Pi(w,p_W^S)\geq 0$, of which the solution is $w_1'=\max\{w_{W,1}^N,\underline{w}(p_W^S)\}$, where $w_{W,1}^N$ is independent of p and $\underline{w}(p_W^S)$ is the smaller root of $\Pi(w,p_W^S)=0$ w.r.t. $w\in(0,\frac{a-p_W^S}{h})$. Observing the government's decision, it is optimal for the firm to adjust price from p_W^S to $p_f(w_1')$. Thus, we can conclude that $(w_1',p_1')=(\max\{w_{W,1}^N,\underline{w}(p_W^S)\},p_f(w_1'))$.

Next, we compare the performance of (w_1',p_1') to that of (w_W^S,p_W^S). Recall that $w_W^S=\arg\max_{w\in[\underline{w}_l,\overline{w}_l]} SW(w,p_f(w))$, where $w\in[\underline{w}_l,\overline{w}_l]$ is the solution of $\Pi(w,p_f(w))\geq 0$. Since $\Pi(w_l',p_f(w_l'))\geq 0$, we can conclude that $w_1'\geq \underline{w}_l$. Then given $\underline{w}_l\leq w_1'\leq w_W^S$, it follows that $SW(w_W^S,p_f(w_W^S))\geq SW(w_1',p_f(w_1'))$, i.e., $SW_W^S\geq SW(w_1',p_1')$. Note also that $\Pi(w,p_f(w))$ is increasing in $w\in[\underline{w}_l,w^D]$. Then given $\underline{w}_l\leq w_1'\leq w_W^S\leq w^D$, we can conclude that $\Pi(w_W^S,p_f(w_W^S))\geq \Pi(w_1',p_f(w_1'))$, i.e., $\Pi_W^S\geq\Pi(w_1',p_1')$.

The analysis shows that the government's myopic adjustment reduces the private firm's profit and social welfare eventually under both price regulation and wait time regulation.

E. Robustness Checks

E.1. Non-uniform Valuation Distributions

Let us assume that the valuations of the customers follows a continuous distribution with a support of $[0,a]$, a probability density function (PDF) f and a cumulative distribution function (CDF) F. The complementary cumulative distribution function (CCDF) $\bar{F}$ is defined as $\bar{F}=1-F$. Then, the effective arrival rate equals

$$\lambda=\begin{cases}\Lambda\bar{F}(p+hw), & p+hw\leq a\\ 0, & p+hw>a\end{cases}. \tag{EC.71}$$

Suppose that $p+hw\leq a$. The total customer surplus per unit of time equals

$$CS=\Lambda\int_{p+hw}^{a}(v-p-hw)f(v)\,\mathrm{d}v=\Lambda\int_{p+hw}^{a}\bar{F}(v)\,\mathrm{d}v,$$

and the profit and social welfare per unit of time equal

$$\Pi = \Lambda\bar{F}(p+hw)(p-c-\gamma) - \frac{\gamma}{w},$$
$$SW = \Lambda\bar{F}(p+hw)(p-c-\gamma) - \frac{\gamma}{w} + \Lambda\int_{p+hw}^{a}\bar{F}(v)\,\mathrm{d}v.$$

Note that

$$\frac{\partial SW}{\partial p} = -(p-c-\gamma)f(p+hw)\Lambda$$

is negative for all $p > c+\gamma$. Meanwhile, $\Pi \geq 0$ requires $p > c+\gamma$. Hence, we can conclude that under the optimal solution of Scenarios C and NP, $\Pi = 0$ holds.

Nevertheless, both Π and SW are not jointly concave in (p,w) in general. In fact, even when the service valuation is uniformly distributed, they are not jointly concave in (p,w) neither. Hence, we resort to numerical study to identify the optimal solution. For robustness checking, we consider different shapes of the valuation distributions. We assume that the valuation distribution follows the Beta distribution with a support of $[0,1]$. As such, we have

$$f(v) = \frac{1}{B(\alpha,\beta)}v^{\alpha-1}(1-v)^{\beta-1}, \quad 0 \leq v \leq 1, \quad \alpha,\beta > 0$$

where

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx, \alpha,\beta > 0$$

is the beta function;

$$F(v) = \int_0^v f(x)dx = \frac{B_v(\alpha,\beta)}{B(\alpha,\beta)}, \quad \bar{F}(v) = 1 - F(v) = 1 - \frac{B_v(\alpha,\beta)}{B(\alpha,\beta)},$$

where

$$B_v(\alpha,\beta) = \int_0^v x^{\alpha-1}(1-x)^{\beta-1}dx, \quad 0 \leq v \leq 1, \quad \alpha,\beta > 0.$$

Suppose that $p + hw \leq 1$. The total customer surplus per unit of time equals

$$CS = \Lambda\frac{B(\alpha+1,\beta) - B_{p+hw}(\alpha+1,\beta)}{B(\alpha,\beta)} - (p+hw)\Lambda\bar{F}(p+hw),$$

and the profit and social welfare per unit of time equal

$$\Pi = \Lambda(p-c-\gamma)\left(1-\frac{B_{p+hw}(\alpha,\beta)}{B(\alpha,\beta)}\right)-\frac{\gamma}{w},$$
$$SW = \Lambda\frac{B(\alpha+1,\beta)-B_{p+hw}(\alpha+1,\beta)}{B(\alpha,\beta)}-(c+\gamma+hw)\Lambda\left(1-\frac{B_{p+hw}(\alpha,\beta)}{B(\alpha,\beta)}\right)-\frac{\gamma}{w}.$$

As mentioned, both Π and SW are not jointly concave in (p,w). It is challenging to design fast algorithms to compute the optimal solution for each scenario under general distributions. Following the literature, we assume that the valuation distribution has an increasing failure rate (i.e., $f/\bar{F}$ is an increasing function); see, e.g., Buzacott and Zhang (2004) and Dong and Zhu (2007). The uniform distribution we consider in the basic model has an increasing failure rate. This assumption ensures that Π is (partially) quasi-concave in p (such that the profit-maximizing price is unique and easily to numerically compute), because

$$\frac{\partial \Pi}{\partial p} = \frac{\Lambda}{\bar{F}(p+hw)}\left[1-(p-c-\gamma)\frac{f(p+hw)}{\bar{F}(p+hw)}\right],$$

whose sign changes once when the valuation distribution has an increasing failure rate. The failure rate function of the Beta distribution is

$$r(v)=\frac{f(v)}{\bar{F}(v)}=\frac{v^{\alpha-1}(1-v)^{\beta-1}}{B(\alpha,\beta)-B_v(\alpha,\beta)}, \quad 0\le v\le 1, \quad \alpha,\beta>0.$$

According to Ghitany (2004) (Theorem 5), $r(v)$ is increasing in v if $\alpha \ge 1$. (Note that the Beta distribution with $\alpha=\beta=1$ is equal to the uniform distribution with support $[0,1]$.) Thus, throughout the numerical study, we focus on the scenarios with $\alpha \ge 1$. Furthermore, to investigate the impact of shapes of the valuation distributions, we consider the following values of α and β for the Beta distribution as shown in Figure EC.1.

In Figure EC.1, we set $\alpha+\beta=5$. Note the mean of the Beta distribution equals $\alpha/(\alpha+\beta)$. Thus, in Figure EC.1, as the value of α increases, the mean increases. Moreover, we can observe that as α increases, the proportion of high valuations increases.

We first investigate the impact of distribution parameters on the performance in Scenario B. Recall that we use β to represent the share the government holds in the project. To distinguish it from the parameter in the Beta distributions, we denote the government's share in joint venture as β_j (where the subscript "j" is short for joint venture). We conducted a large number of numerical instances and obtained the following observations. First, given the value of the sum of α and β,

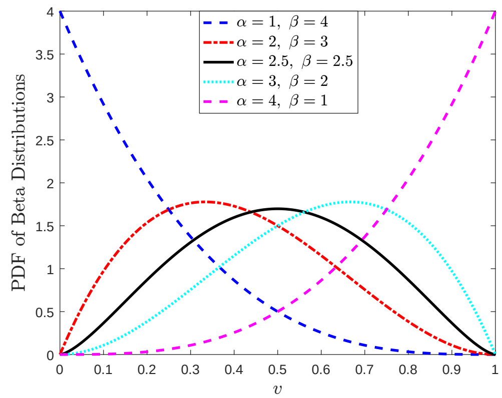


Figure EC.1 Probability density functions with different parameters

as α increases (such that the mean increases), the profit and social welfare increase for any given β_j. Second, as α increases, it requires larger share the government holds in Scenario to achieve the social welfare level in Scenario C. See Figure EC.2 for the illustration: the value of the share is approximately 0.74 in Figure EC.2(a), and it increases from Figure EC.2(a) to Figure EC.2(e). Third, we observe that $(SW^C - SW^D)/SW^D$ decreases in α because the blue curve is more flat and has a larger intercept as α increases; that is, the improvement of social welfare by increasing the government's share is less significant as α increases.

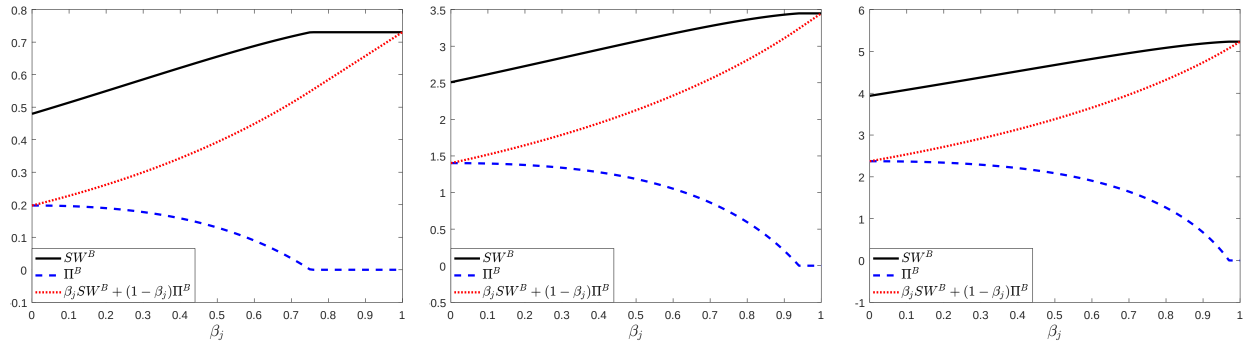
Next, we compare the performance of the four regulation schemes under the Beta distribution. We use the indicator RI defined in Section 5.1 to indicate the efficiency of different schemes:

$$RI = \frac{x - SW^D}{SW^C - SW^D}, \text{ where } x \in \{SW_P^S, SW_W^S, SW_P^N, SW_W^N\}.$$

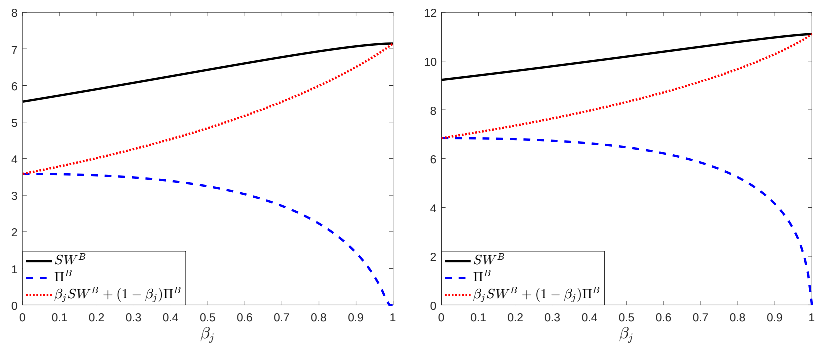
The numerical result is illustrated in Figure EC.3, from which we can have the following observations. First, consistent with Theorems 1 and 2, price regulation is superior (resp., inferior) to wait time regulation when the government has (resp., lacks) commitment power (of not pursuing short-sighted objectives); when the government lacks commitment power, price regulation backfires if the market size Λ is sufficiently large. Hence, our main results hold regardless of the shape of valuation distributions. In addition, we observe that Scenario NW can backfire as well; see Figure EC.3(e) for an illustration. Interestingly, we observe that the degree of backfire is bounded in Scenario NW but unbounded in Scenario NP. Finally, Scenario SP achieves the maximal RI (or equivalently, social welfare) among the four regulation schemes.

E.2. Impact of Non-zero Outside Options

Because the firm has sufficient capital, it may have outside options so that its participation constraint becomes $\Pi \geq \pi_0$, where $\pi_0 > 0$ represent the largest profit derived from the outside options;



(a) $\alpha = 1,\ \beta = 4$ (b) $\alpha = 2,\ \beta = 3$ (c) $\alpha = 2.5,\ \beta = 2.5$

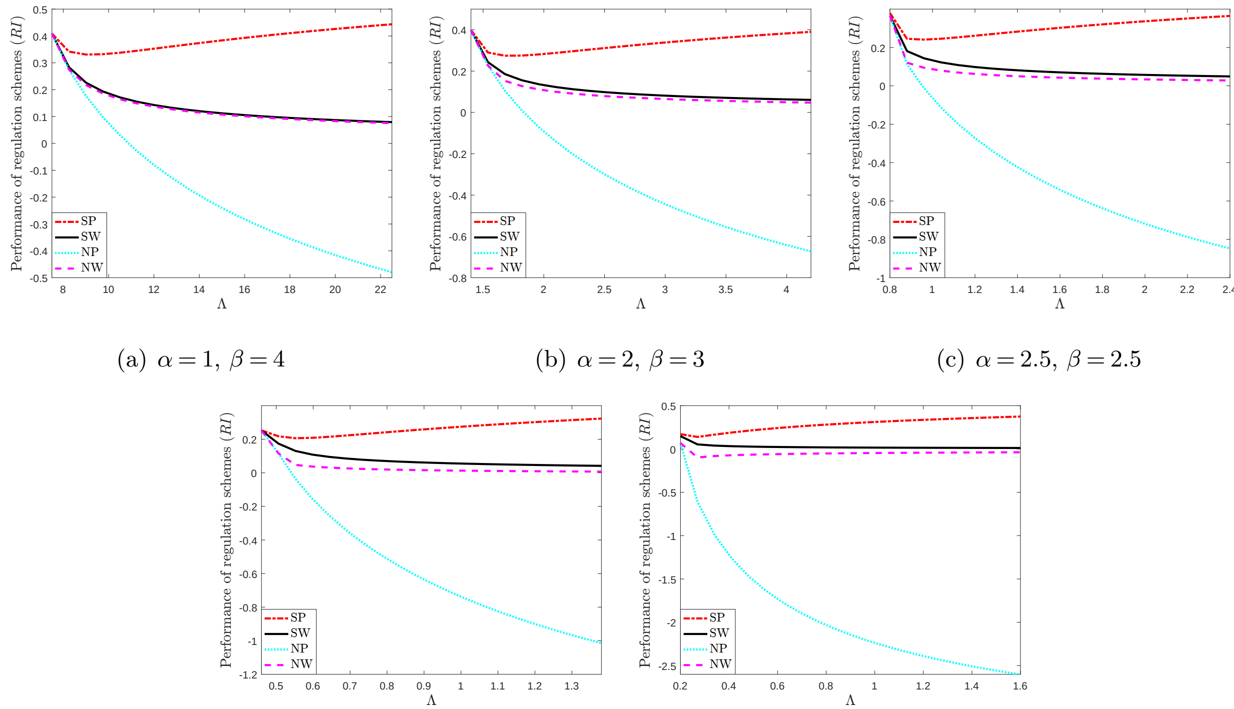


(d) $\alpha = 3,\ \beta = 2$ (e) $\alpha = 4,\ \beta = 1$

Figure EC.2 The equilibrium social welfare and profit in Scenario B under the Beta distributions: $h = 0.1$, $c = 0.1$, $\gamma = 0.1$, $\Lambda = 20$

that is, the firm has a positive reservation profit. Clearly, if $\pi_0 > \Pi^D$, the firm will never join the project of public service provision; if $\pi_0 = \Pi^D$, all schemes lead to the same result as that in Scenario D because there is only one feasible solution (i.e., $(w,p) = (w^D, p^D)$) for all schemes. Throughout this subsection, we focus on the case with $\pi_0 < \Pi^D$. Apparently, π_0 does not play a role in the decision of Scenario D. Recall that $SW(w,p)$ decreases in p for any $p > c + r$ and $\Pi \geq 0$ requires $p > c + r$. Thus, we can conclude that at the optimum of Scenario C or the equilibrium of Scenario NP, $\Pi = \pi_0$ holds. However, further theoretical analysis is intractable, and we have to resort to numerical study. Following Section E.1, we assume that the service valuation follows the Beta distribution.

We begin with investigating the impact of the non-zero outside option on the performance of joint venture. Recall that Π^B decreases in β_j. It is easy to have the following results. As π_0 increases, the threshold of β_j to bind the constraint $\Pi = \pi_0$ will be lower. When β_j is lower than this threshold, the optimal solution and the associated social welfare are not affected by π_0. When β_j is higher than this threshold, the optimal solution and the associated social welfare are invariant to β_j, and the associated social welfare decreases with π_0. In the following, we take the case with the Beta

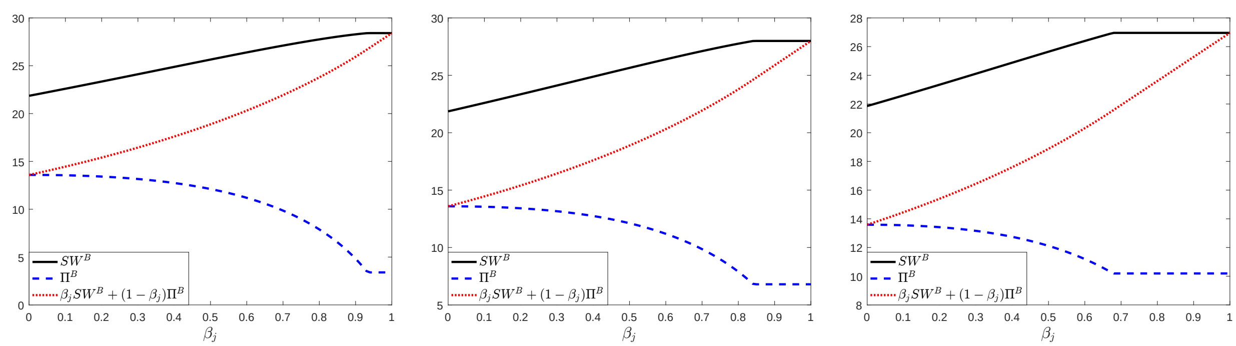


(a) $\alpha=1,\ \beta=4$ (b) $\alpha=2,\ \beta=3$ (c) $\alpha=2.5,\ \beta=2.5$

(d) $\alpha=3,\ \beta=2$ (e) $\alpha=4,\ \beta=1$

Figure EC.3 Social welfare improvement of the regulation schemes: $h=0.1$, $c=0.1$, $\gamma=0.1$

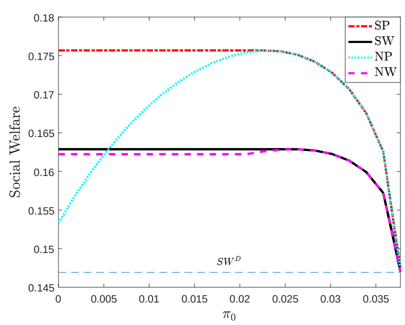
distribution with $\alpha=\beta=2.5$ as an example to illustrate this result; see Figure EC.4.



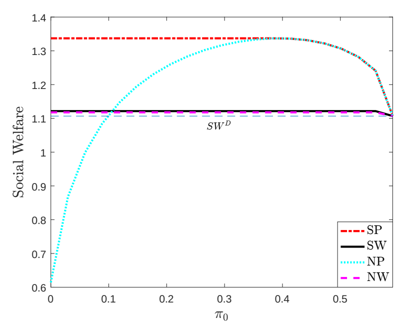
(a) $\pi_0=0.25\Pi^D$ (b) $\pi_0=0.5\Pi^D$ (c) $\pi_0=0.75\Pi^D$

Figure EC.4 The equilibrium social welfare and profit in Scenario B with a positive outside option: $h=0.1$, $c=0.1$, $\gamma=0.1$, $\Lambda=100$

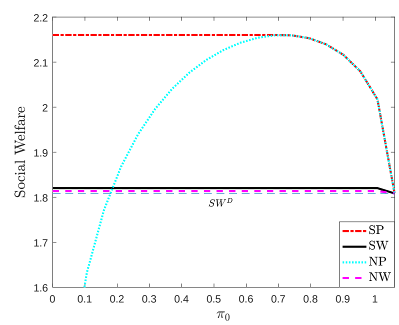
Next, we investigate the impact of the non-zero outside option on the social welfare of the regulation schemes. In Figure EC.5, we allow π_0 to take value from $[0,\Pi^D)$. Note that π_0 does not play a role in Scenario D so that SW^D is a flat line in the figures. Through the numerical study, we observe four key results. First, when the government has commitment power, as the firm requires a larger reservation profit π_0, the social welfare in price or wait time regulation decreases. This is because fewer options are available to the government for the regulation as π_0 increases. Second, when the government lacks commitment power, the social welfare in price or wait time regulation can increase with π_0, and Scenario NP is more sensitive to the change in π_0 than Scenario NW. Recall that when the government lacks commitment power, it will aggressively cut the price or wait time down, which discourages the firm to serve customers. As π_0 increases, the government cannot be too aggressive, which creates incentives for the firm to serve more customers or/and expand the capacity. Third, the commitment power is useless when π_0 is sufficiently large. As shown in the figures, the social welfare of Scenarios SP and NP or Scenario SW and NW are the same when π_0 is sufficiently large, and this phenomenon is more likely to occur under price regulation. Four, Scenario NP can outperform Scenarios SW and NW. This result is different from that in the basic model. Nevertheless, we still observe that Scenario SP performs the best.



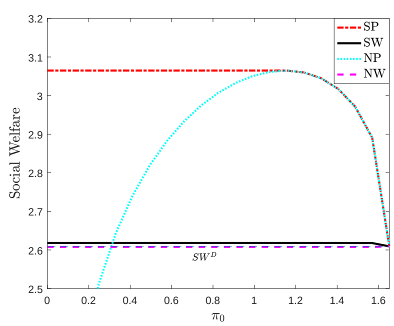
(a) $\alpha = 1,\ \beta = 4$



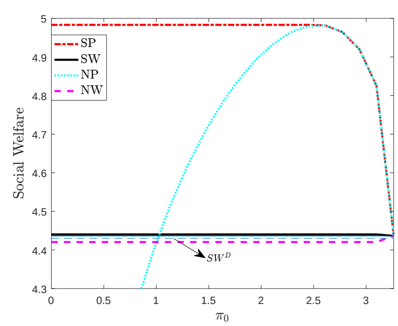
(b) $\alpha = 2,\ \beta = 3$



(c) $\alpha = 2.5,\ \beta = 2.5$



(d) $\alpha = 3,\ \beta = 2$



(e) $\alpha = 4,\ \beta = 1$

Figure EC.5 Equilibrium Social welfare of the regulation schemes: $h = 0.1$, $c = 0.1$, $\gamma = 0.1$, $\Lambda = 10$

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