Online Appendices for "Privacy Management in Service Systems" by Hu, Momot, Wang

Appendix A: Proofs of the Main Results

A.1. Proof of Lemma 1 (Page 12)

Denote $\mu_H = \mu$, $\mu_N = \bar{\mu} = \sigma\mu/(1+\alpha(\sigma-1))$, and $\mu_L = 1/\sigma\mu$ the service rates of type-i requests, for $i = H$, N, or L. Denote $B_i = 1/\mu_i$ the expected service time of type-i requests. Then, from (7.15) of Adan and Resing (2015), the residual service time of type-N requests is

$$R_N = \frac{E\left(B_N^2\right)}{2B_N} = \frac{\alpha\frac{1}{\mu^2} + (1-\alpha)\frac{1}{(\sigma\mu)^2}}{\alpha\frac{1}{\mu} + (1-\alpha)\frac{1}{\sigma\mu}} = \frac{1+\alpha\left(\sigma^2-1\right)}{\sigma\mu\left(1+\alpha(\sigma-1)\right)}.$$

We have three priority classes with the following characteristics:

Type	Arrival Rate	Exp. Service Time	Exp. Residual Service Time	Workload
H	$\alpha\gamma\lambda$	$B_H = \frac{1}{\mu}$	$R_H = \frac{1}{\mu}$	$\rho_H = \frac{\alpha\gamma\lambda}{\mu}$
N	$(1-\gamma)\lambda$	$B_N = \frac{1+\alpha(\sigma-1)}{\sigma\mu}$	$R_N = \frac{1+\alpha\left(\sigma^2-1\right)}{\sigma\mu(1+\alpha(\sigma-1))}$	$\rho_N = \frac{(1+\alpha(\sigma-1))(1-\gamma)\lambda}{\sigma\mu}$
L	$(1-\alpha)\gamma\lambda$	$B_L = \frac{1}{\sigma\mu}$	$R_L = \frac{1}{\sigma\mu}$	$\rho_L = \frac{(1-\alpha)\gamma\lambda}{\sigma\mu}$

From Ch. 9.2 of Adan and Resing (2015), we obtain the expected waiting time of each priority class:

Type	Exp. Waiting Time
H	$W_H = \frac{\rho_H R_H}{1-\rho_H} + B_H = \frac{1}{\mu-\alpha\lambda\gamma}$
N	$W_N = \frac{\rho_H R_H + \rho_N R_N}{(1-\rho_H-\rho_N)(1-\rho_H)} + \frac{B_N}{1-\rho_H} = \frac{\sigma\mu(1+\alpha(\sigma-1)) - \alpha\lambda(\alpha-1)(\sigma-1)(\sigma+\gamma-1)}{\sigma(\mu-\alpha\lambda\gamma)(\sigma\mu-\lambda(\alpha\sigma+(1-\gamma)(1-\alpha)))}$
L	$W_L = \frac{\rho_H R_H + \rho_N R_N + \rho_L R_L}{(1-\rho_H-\rho_N-\rho_L)(1-\rho_H-\rho_N)} + \frac{B_L}{1-\rho_H-\rho_N} = \frac{\sigma(\mu+\alpha\lambda(\sigma-1))}{(\sigma\mu-\lambda(1+\alpha(\sigma-1)))(\sigma\mu-\lambda(\alpha\sigma+(1-\gamma)(1-\alpha)))}$

A customer observes type-H request w.p. α, and type-L request w.p. $1-\alpha$. If an infinitesimal customer discloses information, this customer falls either into type-H or type-L queue depending on the request type. The expected utility of disclosing customer is thus $U_d = R - c(\alpha W_H + (1-\alpha)W_L)$. If a customer does not disclose information, this customer's request will be considered type-N. The expected utility of an infinitesimal customer withholding information is then $U_w = R - cW_N$. Substituting W_H, W_N, and W_L in U_d and U_w, we obtain (2)—functions of γ and λ—disclosure probability and effective arrival rate in the population.

A.2. Proof of Proposition 1 (Page 12)

When deciding whether to disclose or withhold information, infinitesimal customer compares $U_d(\gamma,\lambda)$ with $U_w(\gamma,\lambda)$. Hence, customer's behavior is defined by the sign of the difference:

$$U_d(\gamma,\lambda) - U_w(\gamma,\lambda) = \frac{c\lambda}{\sigma\left(\sigma\mu-(1+\alpha(\sigma-1))\lambda\right)\left(\sigma\mu-\lambda\left(\alpha\sigma+(1-\gamma)(1-\alpha)\right)\right)(\mu-\alpha\lambda\gamma)} \cdot \Xi(\gamma,\lambda), \tag{6}$$

where

$$\Xi(\gamma,\lambda) = \alpha(1-\alpha)\left(\sigma(\sigma-1)\mu + \left(1-\alpha(1-\sigma)\left(\sigma^2+1\right)\right)\lambda\right)\gamma +$$
$$\left(\sigma\left(\alpha^2\sigma^2+\alpha(\alpha-1)\sigma-(\alpha-1)(2\alpha-1)\right)\mu - \alpha\left(\alpha\sigma^3+(\alpha-1)^2\sigma^2-(\alpha-1)(2\alpha-1)\sigma+(\alpha-1)^2\right)\lambda\right).$$

the denominator of the first multiplier of Expression 6 is positive. We then have the following scenarios: (i) if $\Xi(0,\lambda) < 0$ then $\gamma^* = 0$ can be supported as an equilibrium; (ii) if $\Xi(1,\lambda) > 0$ then $\gamma^* = 1$ can be supported as an equilibrium; (iii) if $\exists\hat{\gamma}\in(0,1)$ s.t. $\Xi(\hat{\gamma},\lambda)=0$ then $\hat{\gamma}$ can be supported as an equilibrium (only if $\Xi(1,\lambda)<0$). Several of those scenarios can hold true simultaneously. Define:

$$G_0(\alpha)\equiv\frac{\mu\sigma\left(\alpha^2\sigma^2-(\alpha-1)^2-\alpha(1-\alpha)(\sigma-1)\right)}{\alpha\left(\alpha\sigma^3+(\alpha-1)^2\sigma^2-(\alpha-1)(2\alpha-1)\sigma+(\alpha-1)^2\right)},$$
$$G_1(\alpha)\equiv\frac{\mu\left(\alpha^2\sigma^2-(\alpha-1)^2\right)}{\alpha\left(\alpha^2\sigma^2-(\alpha-1)\sigma-(\alpha-1)^2\right)}.$$

– solutions wrt to λ to $\Xi(0,\lambda)=0$ and $\Xi(1,\lambda)=0$ respectively. We have the following two technical lemmas characterizing behavior of functions $\Xi(\gamma,\lambda)$ and $G_0(\alpha),G_1(\alpha)$ in the points of interest:

Lemma 2. *The linear function $\Xi(\gamma,\lambda)$ satisfies the following properties:*

1. $\Xi(0,\lambda)<0\Leftrightarrow\lambda>G_0(\alpha)$*;*
2. $\Xi(1,\lambda)\geq 0\Leftrightarrow\lambda\leq G_1(\alpha)$ *if either (i)* $\sigma>1$ *or (ii)* $\sigma<1$ *and* $\alpha>\frac{1}{\sigma+1}$*; in all other regions* $\Xi(1,\lambda)<0,\forall\lambda$*.*

Proof. Proofs of all technical results are provided in Online Supplement (https://www.dropbox.com/s/dh9mab85tdahn8k/Online_Suppliment_Privacy_Queues.pdf).

Notice that interior $\hat{\gamma}$ s.t. $\hat{\gamma}\in(0,1)$ and $\Xi(\hat{\gamma},\lambda)=0$ exists only if either (i) $\Xi(0,\lambda)<0$ and $\Xi(1,\lambda)>0$ or (ii) $\Xi(0,\lambda)>0$ and $\Xi(1,\lambda)<0$, but only the latter can be supported as an equilibrium. Next technical lemma specifies behavior of functions $G_0(\alpha),G_1(\alpha)$.

Lemma 3. *Functions $G_0(\alpha),G_1(\alpha)$ exhibit the following behavior:*

1. $G_0(\alpha)>0$ *iff* $\alpha>\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$*;*
2. $G_1(\alpha)>0$ *iff either (i)* $\sigma>1$ *and* $\alpha\geq\frac{1}{1+\sigma}$ *or (ii)* $\sigma<1$ *and either* $\alpha\leq\frac{2-\sigma-\sigma\sqrt{5-4\sigma}}{2(1-\sigma^2)}<\frac{1}{1+\sigma}$ *(in this case also* $G_1(\alpha)\geq\bar{\mu}>0$*) or* $\alpha>\frac{1}{\sigma+1}$*;*
3. $G_0(\alpha)>G_1(\alpha)$ *if* $\frac{2-\sigma-\sigma\sqrt{5-4\sigma}}{2(1-\sigma^2)}\leq\alpha<\frac{1}{\sigma^{3/2}+1}$ *and* $G_0(\alpha)<G_1(\alpha)$ *if* $\frac{1}{\sigma^{3/2}+1}\leq\alpha$*.*

Additionally, we have:

$$\begin{cases}\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}<\frac{1}{\sigma+1}<\frac{1}{\sigma^{3/2}+1} & \text{if } \sigma<1\\ \frac{1}{\sigma^{3/2}+1}<\frac{1}{\sigma+1}<\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)} & \text{if } \sigma>1\end{cases}\tag{7}$$

Using results of the Lemmas above and condition 7, we consider the following distinct cases.

Case A: $\sigma<1$

A1) if $0<\alpha\leq\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$ then we have $\Xi(1,\lambda)<0$ and $G_0(\alpha)\leq 0$, which leads to $\Xi(0,\lambda)<0$ because $\lambda>0\geq G_0(\alpha)$. Then, from the fact that $\Xi(\gamma,\lambda)$ is linear in γ, we have $\gamma^*=0$.

A2) if $\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}<\alpha\leq\frac{1}{\sigma+1}$, then $G_0(\alpha)>0$ and $G_1(\alpha)\notin[0,\bar{\mu}]$ (i.e., $G_1(\alpha)<0$ or $G_1(\alpha)>\bar{\mu}$). If $0<\lambda\leq G_0(\alpha)$ then $\Xi(0,\lambda)\geq 0$ and $\Xi(1,\lambda)<0$. The unique equilibrium is $\gamma^*=\hat{\gamma}\in(0,1)$, where:

$$\hat{\gamma}\equiv\frac{\sigma\left(\alpha^2\sigma^2+\alpha(\alpha-1)\sigma-(\alpha-1)(2\alpha-1)\right)\mu-\alpha\left(\alpha\sigma^3+(\alpha-1)^2\sigma^2-(\alpha-1)(2\alpha-1)\sigma+(\alpha-1)^2\right)\lambda}{\alpha(1-\alpha)\left(\sigma(1-\sigma)\mu+\left(\alpha(1-\sigma)(\sigma^2+1)-1\right)\lambda\right)},$$

If $\lambda>G_0(\alpha)$ then $\Xi(0,\lambda)<0$ and $\Xi(1,\lambda)<0$, hence $\gamma^*=0$.

A3) if $\frac{1}{\sigma+1} < \alpha \leq \frac{1}{\sigma^{3/2}+1}$, then $0 < G_1(\alpha) \leq G_0(\alpha)$. If $0 < \lambda \leq G_1(\alpha)$ then $\Xi(0,\lambda) \geq 0$ and $\Xi(1,\lambda) \geq 0$, hence $\gamma^* = 1$. If $G_1(\alpha) < \lambda \leq G_0(\alpha)$ then $\Xi(0,\lambda) \geq 0$ and $\Xi(1,\lambda) < 0$, hence there is a unique equilibrium $\gamma^* = \hat{\gamma}$. If $\lambda > G_0(\alpha)$ then $\Xi(0,\lambda) < 0$ and $\Xi(1,\lambda) < 0$, hence $\gamma^* = 0$.

A4) if $\alpha > \frac{1}{\sigma^{3/2}+1}$, then $0 < G_0(\alpha) < G_1(\alpha)$. If $0 < \lambda \leq G_0(\alpha)$ then $\Xi(0,\lambda) \geq 0$ and $\Xi(1,\lambda) > 0$, thus $\gamma^* = 1$. If $G_0(\alpha) < \lambda \leq G_1(\alpha)$ then $\Xi(0,\lambda) < 0$ and $\Xi(1,\lambda) \geq 0$, there are two equilibria $\gamma^* = 0$ and $\gamma^* = 1$. Since the system follows SPT rule (i.e., $\sigma < 1$), customers have higher utility at $\gamma^* = 1$. If $\lambda > G_1(\alpha)$ then $\Xi(0,\lambda) < 0$ and $\Xi(1,\lambda) < 0$, hence $\gamma^* = 0$.

Case B: $\sigma > 1$

B1) if $0 < \alpha \leq \frac{1}{\sigma+1}$, then $G_1(\alpha) \leq 0 < \lambda$ and hence $\Xi(1,\lambda) < 0$. Also $G_0(\alpha) < 0 < \lambda$ thus $\Xi(0,\lambda) < 0$. Since $\Xi(\gamma,\lambda)$ is linear in γ, hence we should have $\Xi(\gamma,\lambda) < 0, \forall \gamma \in [0,1]$, thus $\gamma^* = 0$.

B2) if $\frac{1}{\sigma+1} < \alpha \leq \frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$, then $G_0(\alpha) \leq 0 < G_1(\alpha)$. Then, it must be $\lambda > G_0(\alpha) \Leftrightarrow \Xi(0,\lambda) < 0$. If $0 < \lambda \leq G_1(\alpha) \Leftrightarrow \Xi(1,\lambda) \geq 0$, there are two equilibria: $\gamma^* = 0$ and $\gamma^* = 1$. Since the system follows LPT rule (i.e., $\sigma > 1$), customers have higher utility at $\gamma^* = 0$. If $\lambda > G_1(\alpha) \Leftrightarrow \Xi(1,\lambda) < 0$, and $\gamma^* = 0$.

B3) if $\alpha > \frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$, then $0 < G_0(\alpha) < G_1(\alpha)$. If $0 < \lambda \leq G_0(\alpha) \Leftrightarrow \Xi(0,\lambda) \geq 0$ and $\Xi(1,\lambda) > 0$, hence $\gamma^* = 1$. If $G_0(\alpha) < \lambda \leq G_1(\alpha) \Leftrightarrow \Xi(0,\lambda) < 0$ and $\Xi(1,\lambda) \geq 0$, there are two equilibria: $\gamma^* = 0$ and $\gamma^* = 1$. Since the system follows LPT rule (i.e., $\sigma > 1$), customers have higher utility at $\gamma^* = 0$. If $\lambda > G_1(\alpha) \Leftrightarrow \Xi(0,\lambda) < 0$ and $\Xi(1,\lambda) < 0$, hence $\gamma^* = 0$.

We define the inverse function of $G_0(\alpha)$ and $G_1(\alpha)$: $F_0(\lambda) \equiv G_0^{-1}(\alpha)$ and $F_1(\lambda) \equiv G_1^{-1}(\alpha)$. The intersection point of F_0 and F_1 can be derived as

$$\tilde{\lambda} = \frac{\mu\left(\sigma^{\frac{3}{2}} - \sqrt{\sigma} + 1 - \sigma^2\right)}{\sigma^{\frac{3}{2}} - \sigma + 1},$$

which is positive iff $\sigma < 1$. Then, we obtain (3) by summarizing all the cases and defining $\underline{\alpha}(\lambda) = \min(F_0(\lambda), F_1(\lambda))$, $\bar{\alpha}(\lambda) = \max(F_0(\lambda), F_1(\lambda))$.

A.3. Proof of Theorem 1 (Page 15)

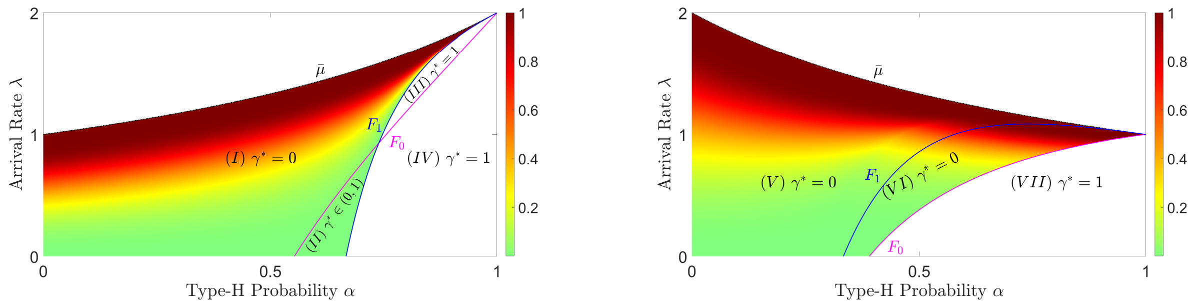
The expected customer surplus can be derived as the weighted average of the utility of customers who disclose and withhold their personal information:

$$\mathrm{CS}(\gamma) = \lambda\left(\gamma U_d(\gamma,\lambda) + (1-\gamma) U_w(\gamma,\lambda)\right). \tag{8}$$

Hence, $\mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*)$, $\mathrm{CS}_{\mathrm{discl}} = \mathrm{CS}(1)$, and $\mathrm{CS}_{\mathrm{wthl}} = \mathrm{CS}(0)$. Function $\mathrm{CS}(\gamma)$ has the following property.

Lemma 4. *Total customer surplus* $\mathrm{CS}(\gamma)$ *increases with* $\gamma \in [0,1]$ *if* $\sigma < 1$ *and decreases otherwise.*

We have the equilibrium information disclosure strategy $\gamma^* \leq 1$ from Proposition 1. Thus, when $\sigma < 1$, $\mathrm{CS}_{\mathrm{wthl}} = \mathrm{CS}(0) \leq \mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) \leq \mathrm{CS}_{\mathrm{discl}} = \mathrm{CS}(1)$ because $\mathrm{CS}(\gamma)$ increases with γ. When $\sigma > 1$, $\mathrm{CS}_{\mathrm{wthl}} = \mathrm{CS}(0) \geq \mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) \geq \mathrm{CS}_{\mathrm{discl}} = \mathrm{CS}(1)$ because $\mathrm{CS}(\gamma)$ decreases with γ. Inequalities are strict if $\gamma^* < 1$.



(a) SPT discipline, $\sigma < 1$. (b) LPT discipline, $\sigma > 1$.

Figure 5 Monetary incentive P_d needed to induce full information disclosure as a function of the arrival rate λ and the probability α of a type-H request.

A.4. Proof of Proposition 2 (Page 17)

From Lemma 4, one can easily prove the following result regarding the socially optimal information disclosure strategy.

Corollary 1. *Socially optimal information disclosure strategy is $\gamma^{\mathrm{SO}} = 1$ if $\sigma < 1$, and $\gamma^{\mathrm{SO}} = 0$ otherwise.*

Define $\alpha^{\mathrm{p.s.}}(\lambda;\sigma) = \begin{cases} F_1(\lambda) & \text{if } \sigma < 1 \\ F_0(\lambda) & \text{if } \sigma > 1 \end{cases}$. When $\sigma < 1$, we have $\gamma^* = \gamma^{\mathrm{SO}} = 1$ if $\alpha > \alpha^{\mathrm{p.s.}}(\lambda)$. In this case, $\mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) = \mathrm{CS}_{\mathrm{social}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right)$. Otherwise, if $\alpha \le \alpha^{\mathrm{p.s.}}(\lambda)$, we have $\gamma^* < \gamma^{\mathrm{SO}} = 1$ and $\mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) < \mathrm{CS}_{\mathrm{social}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right)$.

When $\sigma > 1$, we have $\gamma^* = \gamma^{\mathrm{SO}} = 0$ if $\alpha < \alpha^{\mathrm{p.s.}}(\lambda)$. Here $\mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) = \mathrm{CS}_{\mathrm{social}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right)$. Otherwise, if $\alpha \ge \alpha^{\mathrm{p.s.}}(\lambda)$, we have $\gamma^* < \gamma^{\mathrm{SO}} = 1$ and $\mathrm{CS}_{\mathrm{control}} = \mathrm{CS}(\gamma^*) < \mathrm{CS}_{\mathrm{social}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right)$.

A.5. Proof of Proposition 3 (Page 19)

When all customers choose to withhold information in equilibrium, i.e., $\gamma^* = 0$, we must have $U_d(0,\lambda) < U_w(0,\lambda)$. To induce customers who withhold information to disclose it, the monetary incentive paid to each customer not only needs to turn full disclosure into an equilibrium but also needs to make this equilibrium Pareto dominant compared to withholding information. Thus, we have

$$P_d = \max\left(\lambda\left(U_w(1,\lambda) - U_d(1,\lambda)\right), \mathrm{CS}_{\mathrm{withhold}} - \mathrm{CS}_{\mathrm{discl}}\right),$$

where $\mathrm{CS}_{\mathrm{withhold}} = \mathrm{CS}(0)$ and $\mathrm{CS}_{\mathrm{discl}} = \mathrm{CS}(1)$ ($\mathrm{CS}(\gamma)$ is defined in expression 8). Using expressions (6) and (8), we can derive

$$\begin{aligned}
\lambda\left(U_w(1,\lambda) - U_d(1,\lambda)\right) &= \frac{c\lambda^2\left(\left((\alpha-1)^2 - \alpha^2\sigma^2\right)\mu - \lambda\alpha\left(-\alpha^2\sigma^2 + \sigma(\alpha-1) + (\alpha-1)^2\right)\right)}{\sigma(\mu-\alpha\lambda)^2\left(\sigma\mu - (1+\alpha(\sigma-1))\lambda\right)}, \\
\Delta S = \mathrm{CS}_{\mathrm{withhold}} - \mathrm{CS}_{\mathrm{discl}} &= \frac{c\alpha\lambda^2(1-\alpha)(\sigma-1)\left(\mu-\alpha(1-\sigma)\lambda\right)}{\sigma\mu(\mu-\alpha\lambda)\left(\sigma\mu - \lambda(1+\alpha(\sigma-1))\right)} = \frac{c\alpha(1-\alpha)(\sigma-1)}{\sigma\mu}\frac{\lambda^2}{\Theta(\lambda)},
\end{aligned}$$

where $\Theta(\lambda) = \frac{(\mu-\alpha\lambda)(\sigma\mu-\lambda(1+\alpha(\sigma-1)))}{\mu-\alpha(1-\sigma)\lambda}$. Then we have:

$$\lambda\left(U_w(1,\lambda) - U_d(1,\lambda)\right) - \left(\mathrm{CS}_{\mathrm{withhold}} - \mathrm{CS}_{\mathrm{discl}}\right) = \frac{c\lambda^2}{\sigma\mu(\mu-\alpha\lambda)^2\left(\sigma\mu - (1+\alpha(\sigma-1))\lambda\right)} \cdot \Xi_{P_d}(\lambda)$$

where:

$$\Xi_{P_d}(\lambda) = \alpha^3(1-\alpha)(\sigma-1)^2\lambda^2 + \alpha\mu\left(\alpha^2(2\sigma^2 - 3\sigma + 1) + \alpha(2\sigma - \sigma^2) + \sigma - 1\right)\lambda - \mu^2\left(\alpha^2\sigma^2 - \alpha^2\sigma + \alpha(\sigma+1) - 1\right)$$

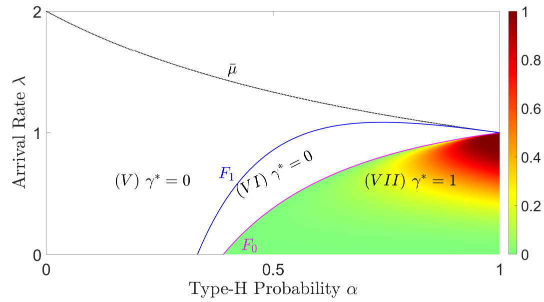


Figure 6 Monetary incentive needed to induce a customer to adopt the socially optimal information disclosure strategy (P_s), as a function of the arrival rate λ and the probability α of a type-H request, when $\sigma>1$ (LPT discipline). With a SPT discipline, the monetary incentive P_s is equivalent to P_d, the amount needed to induce full information disclosure (see Panel (a) in Figure 5).

When $\sigma>1$, from the fact that $\alpha^3(1-\alpha)(\sigma-1)^2>0$, $\Xi_{P_d}(\bar{\mu})=\frac{\mu^2(\alpha-1)^2\left(\alpha\sigma^2+1-\alpha\right)}{(\alpha\sigma-\alpha+1)^2}>0$, and $\Xi_{P_d}(G_1(\alpha))=\frac{-\alpha^2\sigma^2\mu^2(\alpha-1)^2(\sigma-1)\left(\alpha\sigma^2-\alpha+1\right)}{(\alpha^2\sigma^2-\alpha^2-\alpha\sigma+2\alpha+\sigma-1)^2}<0$, we thus conclude that $\Xi_{P_d}(\lambda)$ has a unique root ξ_{P_d} in $(G_1(\alpha),\bar{\mu})$ and $\Xi_{P_d}(\lambda)\le 0 \Leftrightarrow \lambda\le\xi_{P_d}$. Therefore, when $\sigma>1$, we have $P_d=\mathrm{CS}_{\text{withhold}}-\mathrm{CS}_{\text{discl}}$ if $\lambda\le\xi_{P_d}$, or $\lambda\left(U_w(1,\lambda)-U_d(1,\lambda)\right)$ if $\lambda>\xi_{P_d}$.

When $\sigma<1$, we have $\lambda\left(U_w(1,\lambda)-U_d(1,\lambda)\right)>\mathrm{CS}_{\text{withhold}}-\mathrm{CS}_{\text{discl}}$ for $\lambda\in(G_1(\alpha),\bar{\mu})$, because $U_w(1,\lambda)-U_d(1,\lambda)>0$ from Proposition 1 and $\mathrm{CS}_{\text{withhold}}-\mathrm{CS}_{\text{discl}}<0$ from Lemma 4. Thus, $P_d=\lambda\left(U_w(1,\lambda)-U_d(1,\lambda)\right)$. Figure 5 illustrates behavior of this function.

A.6. Proof of Proposition 4 (Page 20)

From Corollary 1, $\gamma^{\text{SO}}=1$ if $\sigma<1$, and $\gamma^{\text{SO}}=0$ o/w. Monetary incentive P_s should be such that γ^{SO} becomes equilibrium. If γ^{SO} is an equilibrium, it is Pareto dominant one. When $\sigma<1$, thus $P_s^{\sigma<1}=\lambda\left(U_w(1,\lambda)-U_d(1,\lambda)\right)$, and when $\sigma>1$, $P_s^{\sigma>1}=\lambda\left(U_d(0,\lambda)-U_w(0,\lambda)\right)$. The exact expressions have the following form:

$$P_s^{\sigma<1}=\frac{c\lambda^2\left(\left((\alpha-1)^2-\alpha^2\sigma^2\right)\mu-\lambda\alpha\left(-\alpha^2\sigma^2+\sigma(\alpha-1)+(\alpha-1)^2\right)\right)}{\sigma(\mu-\alpha\lambda)^2\left(\sigma\mu-(1+\alpha(\sigma-1))\lambda\right)},$$

$$P_s^{\sigma>1}=\frac{c\lambda^2\left(\sigma\left(\alpha^2\sigma^2-\alpha(1-\alpha)\sigma-(2\alpha^2-3\alpha+1)\right)\mu-\alpha\lambda\left(\alpha\sigma^3+(\alpha-1)^2\sigma^2-(2\alpha^2-3\alpha+1)\sigma+(\alpha-1)^2\right)\right)}{\sigma\mu\left(\sigma\mu-(1+\alpha(\sigma-1))\lambda\right)^2}.$$

Note that P_s has only to be paid when $P_s>0$, o/w when $P_s<0$, γ^{SO} is already an equilibrium. Next technical lemma characterizes properties of P_s and Figure 6 illustrates behavior of this function.

Lemma 5. *If $\sigma<1$, then $P_s^{\sigma<1}$ increases in $\lambda\in(0,\bar{\mu}]$ and decreases in $\alpha\in[0,1]$. Furthermore, $P_s\sigma<1=0$, if either (i) $\alpha\le\frac{1}{\sigma+1}$ and $\lambda=0$, or (ii) $\alpha>\frac{1}{\sigma+1}$ and $\lambda=G_1(\alpha)$. If $\sigma>1$, then $P_s^{\sigma>1}$ is concave wrt $\lambda\in(0,G_0(\alpha)]$ for $\forall\alpha\in\left(\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)},1\right]$, and also increases in α for $\forall\lambda\in(0,\bar{\mu})$. Finally, $P_s^{\sigma>1}=0$, if either (i) $\alpha\le\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$ and $\lambda=0$, or (ii) $\alpha>\frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$ and $\lambda=G_0(\alpha)$.*

A.7. Proof of Theorem 2 (Page 22)

The following technical lemma derives properties of function $\Delta S=\mathrm{CS}_{\text{withhold}}-\mathrm{CS}_{\text{discl}}$ earlier defined in the Proof of Proposition 3.

Lemma 6. *$\Delta S = \mathrm{CS}_{\text{withhold}} - \mathrm{CS}_{\text{discl}} = \frac{c\alpha(1-\alpha)(\sigma-1)}{\sigma\mu}\frac{\lambda^2}{\Theta(\lambda)}$, where $\Theta(\lambda) = \frac{(\mu-\alpha\lambda)(\sigma\mu-\lambda(1+\alpha(\sigma-1)))}{(\mu-\alpha(1-\sigma)\lambda)}$. Furthermore,*

1. *if $\sigma < 1$ then $\Delta S \le 0$ decreases in λ; $\lim_{\lambda\to\bar{\mu}} \Delta S = -\infty$; $\Delta S < 0$ when $\lambda = G_1(\alpha)$ and $\alpha > \frac{1}{\sigma+1}$; $\Delta S = 0$ when $\lambda = 0$ and $\alpha \le \frac{1}{\sigma+1}$;*
2. *if $\sigma > 1$ then $\Delta S > 0$ increases in λ and $\lim_{\lambda\nearrow G_0(\alpha)} \Delta S > 0$.*

Case $\sigma < 1$: we have $\gamma^{\mathrm{SO}} = 1$. From the proof of Proposition 4, we have $P_s = 0$, if $\alpha > \frac{1}{\sigma+1}$ and $\lambda = G_1(\alpha)$; also, $\gamma^* = 0$ for $\alpha > \frac{1}{\sigma^{3/2}+1} > \frac{1}{\sigma+1}$. Thus, $\mathrm{CS}_{\text{social}} - \mathrm{CS}_{\text{control}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right) - \mathrm{CS}\left(\gamma^*\right) = \mathrm{CS}\left(1\right) - \mathrm{CS}\left(0\right) = -\Delta S$. From the derived properties of ΔS above, we have $-\Delta S$ increases in λ and $-\Delta S > 0$ when $\alpha > \frac{1}{\sigma^{3/2}+1}$ and $\lambda = G_1$. Hence, there exists a ξ_1 such that $P_s < \mathrm{CS}_{\text{social}} - \mathrm{CS}_{\text{control}}$ for $\forall \lambda \in [G_1(\alpha), G_1(\alpha)+\xi_1]$.
Case $\sigma > 1$: we have $\gamma^{\mathrm{SO}} = 0$. From the proof of Proposition 4, we have $P_s = 0$, if $\alpha > \frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$ and $\lambda = G_0(\alpha)$; also, $\gamma^* = 1$ for $\alpha > \frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$. Thus, $\mathrm{CS}_{\text{social}} - \mathrm{CS}_{\text{control}} = \mathrm{CS}\left(\gamma^{\mathrm{SO}}\right) - \mathrm{CS}\left(\gamma^*\right) = \mathrm{CS}\left(0\right) - \mathrm{CS}\left(1\right) = \Delta S$. From the derived properties of ΔS above, we have ΔS increasing in λ and $\lim_{\lambda\nearrow G_0(\alpha)} \Delta S > 0$ when $\alpha > \frac{\sigma-3+\sqrt{5\sigma^2-2\sigma+1}}{2(\sigma^2+\sigma-2)}$ and $\lambda = G_0(\alpha)$. Hence, there exists a ξ_2 such that $P_s < \mathrm{CS}_{\text{social}} - \mathrm{CS}_{\text{control}}$ for $\forall \lambda \in [G_0(\alpha)-\xi_2, G_0(\alpha)]$.

Appendix B: Alternative Model Formulations

B.1. Multiple Request Types and Priority Classes (Page 25)

We can generalize our model to a setting with M types of service requests. Customers face a class-i request with an exogenous probability $\alpha_i \in [0,1]$, for $i = 1,\dots,M$, and $\sum_{i=1}^M \alpha_i = 1$. The service time of a class-i request follows an i.i.d. exponential distribution with mean $B_i = 1/(\sigma_i\mu)$; and the service rate of type-i request is $\mu_i = \sigma_i\mu$ for $i = 1,\dots,M$. Without loss of generality, we let $\sigma_1 = 1$ and, under the SPT (resp., LPT) policy, order these request classes by their mean service time from short to long, i.e., $\sigma_i > \sigma_j$ if $i < j$ (resp., long to short, i.e., $\sigma_i < \sigma_j$ if $i < j$). We then can derive the average service time of all requests as $\sum_{i=1}^M \alpha_i/(\sigma_i\mu)$, the capacity of the service facility as $\bar{\mu} = \left(\sum_{i=1}^M \alpha_i/(\sigma_i\mu)\right)^{-1}$, and the request class with the longest (resp., shortest) average service time shorter (resp., longer) than the average service time of all requests as

$$m = \max\left\{ i \in \{1,\dots,M\} \mid \sigma_i > \left(\sum_{j=1}^M \frac{\alpha_j}{\sigma_j}\right)^{-1} \right\} \tag{9}$$

under SPT policy (resp., $m = \max\left\{ i \mid \sigma_i < \left(\sum_{j=1}^M \alpha_j/\sigma_j\right)^{-1}\right\}$ under LPT policy). The service provider will prioritize requests in the following order: (i) "bucket"-H: $1,\dots,m$ requests, (ii) "bucket"-N: requests from information-withholding customers, and (iii) "bucket"-L: $m+1,\dots,M$ requests. Within these three "buckets," service requests are treated according to the priority policy that the service provider uses.

Note that the service rate of requests from withholding customers is $\bar{\mu}$. Then, from (7.15) of Adan and Resing (2015), the residual service time of bucket-N requests is

$$R_N = \frac{E\left(B_N^2\right)}{2B_N} = \frac{\sum_{i=1}^M \alpha_i/\left(\sigma_i^2\mu\right)}{\sum_{i=1}^M \alpha_i/\sigma_i}.$$

We have $M+1$ priority classes with the characteristics as specified in Table 1.

Type	Arrival Rate	Exp. Service Time	Exp. Residual Service Time	Workload
1	$\alpha_1\gamma\lambda$	$B_1 = 1/(\sigma_1\mu)$	$R_1 = 1/(\sigma_1\mu)$	$\rho_1 = \alpha_1\gamma\lambda/(\sigma_1\mu)$
$\vdots$				
m	$\alpha_m\gamma\lambda$	$B_m = 1/(\sigma_m\mu)$	$R_m = 1/(\sigma_m\mu)$	$\rho_m = \alpha_m\gamma\lambda/(\sigma_m\mu)$
N	$(1-\gamma)\lambda$	$B_N = \sum_{i=1}^{M}\alpha_i/(\sigma_i\mu)$	$R_N = \frac{\sum_{i=1}^{M}\alpha_i/(\sigma_i^2\mu)}{\sum_{i=1}^{M}\alpha_i/\sigma_i}$	$\rho_N = (1-\gamma)\lambda\sum_{i=1}^{M}\alpha_i/(\sigma_i\mu)$
$m+1$	$\alpha_{m+1}\gamma\lambda$	$B_{m+1} = 1/(\sigma_{m+1}\mu)$	$R_{m+1} = 1/(\sigma_{m+1}\mu)$	$\rho_{m+1} = \alpha_{m+1}\gamma\lambda/(\sigma_{m+1}\mu)$
$\vdots$				
M	$\alpha_M\gamma\lambda$	$B_M = 1/(\sigma_M\mu)$	$R_M = 1/(\sigma_M\mu)$	$\rho_M = \alpha_M\gamma\lambda/(\sigma_M\mu)$

Table 1 Summary of characteristics of different customer classes.

Type	Exp. Waiting Time
1	$W_1 = \frac{\rho_1 R_1}{1-\rho_1} + B_1 = \frac{1}{\mu\sigma_1 - \lambda\gamma\alpha_1}$
$\vdots$	
m	$W_m = \frac{\sum_{i=1}^{m}\rho_i R_i}{\left(1-\sum_{i=1}^{m}\rho_i\right)\left(1-\sum_{i=1}^{m-1}\rho_i\right)} + \frac{B_m}{1-\sum_{i=1}^{m-1}\rho_i}$
N	$W_N = \frac{\sum_{i=1}^{m}\rho_i R_i + \rho_N R_N}{\left(1-\sum_{i=1}^{m}\rho_i - \rho_N\right)\left(1-\sum_{i=1}^{m}\rho_i\right)} + \frac{B_N}{1-\sum_{i=1}^{m}\rho_i}$
$m+1$	$W_{m+1} = \frac{\sum_{i=1}^{m+1}\rho_i R_i + \rho_N R_N}{\left(1-\sum_{i=1}^{m+1}\rho_i - \rho_N\right)\left(1-\sum_{i=1}^{m}\rho_i - \rho_N\right)} + \frac{B_{m+1}}{1-\sum_{i=1}^{m}\rho_i - \rho_N}$
$\vdots$	
M	$W_M = \frac{\sum_{i=1}^{M}\rho_i R_i + \rho_N R_N}{\left(1-\sum_{i=1}^{M}\rho_i - \rho_N\right)\left(1-\sum_{i=1}^{M-1}\rho_i - \rho_N\right)} + \frac{B_M}{1-\sum_{i=1}^{M-1}\rho_i - \rho_N}$

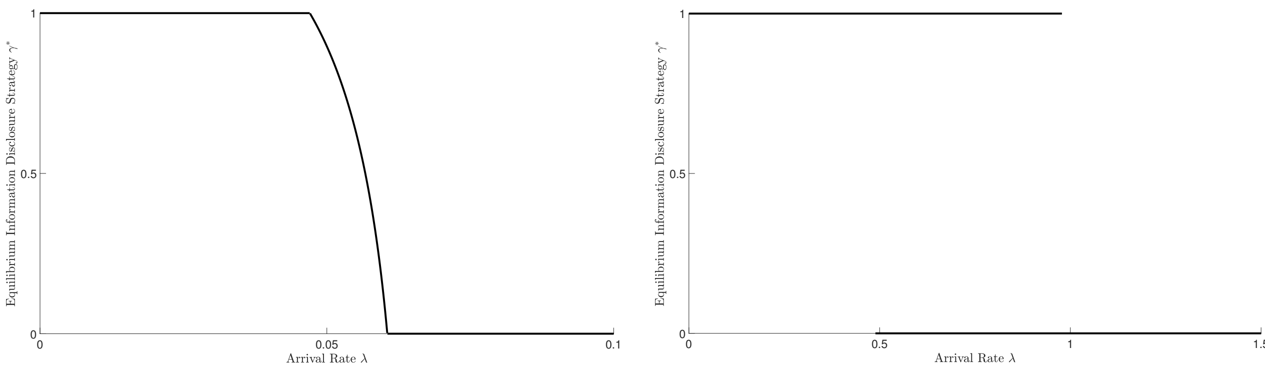
Table 2 Expected Wait Times of Different Customer Classes

Then, by Ch. 9.2 of Adan and Resing (2015), we can derive the expected wait times of all classes W_i, for $i = 1, \ldots m, N, m+1, \ldots, M$ as specified in Table 2. A customer's service request is class-i w.p. α_i. The expected utility of a disclosing customer is $U_d = R - c\sum_{i=1}^{M}\alpha_i W_i$. If a customer does not disclose information, this customer's request will stay in bucket-N. The expected utility of a withholding customer is then $U_w = R - cW_N$. Disclosing customers are prioritized (over withholding customers) to bucket-H with probability $\sum_{i=1}^{m}\alpha_i$ and deprioritized with probability $\sum_{i=m+1}^{M}\alpha_i$. The expected wait times of bucket-H, bucket-N, and bucket-L requests are $\sum_{i=1}^{m}\alpha_i W_i/\sum_{i=1}^{m}\alpha_i$, W_N, and $\sum_{i=m+1}^{M}\alpha_i W_i/\sum_{i=m+1}^{M}\alpha_i$, respectively. Overall, we have customer's expected utility as:

$$U = \begin{cases} U_d \equiv R - c\sum_{i=1}^{M}\alpha_i W_i & \text{if customer discloses information;} \\ U_w \equiv R - cW_N & \text{if customer withholds information.} \end{cases} \tag{10}$$

Substituting W_i and W_N in U_d and U_w, we obtain U_d and U_w as functions of the disclosure probability γ and effective arrival rate λ in the population. Similar to Proposition 1, we can derive customers' equilibrium information disclosure strategy γ by investigating $U_d(\gamma,\lambda) - U_w(\gamma,\lambda)$. Similarly to the base model, full information disclosure $\gamma = 1$ is an equilibrium if $U_d(1,\lambda) - U_w(1,\lambda) > 0$ ($\gamma = 0$ and $\gamma \in (0,1)$ cases can be derived similarly). Even though we expect similar insights to hold under this general model as compared to those of the base model, the analytical analysis is prohibitive due to complexity of the expressions for waiting times W_i (as per Table 2) and threshold m. We thus resort to the numerical analysis to verify the main insights of the base model.

For the particular numerical experiment that we report, we assume $\sigma_i = 1/i$ under SPT policy (resp., $\sigma_i = i$ under LPT policy) and that α_i follows a Binomial distribution with parameters $M = 10$ and p: $\alpha_i = \binom{M-1}{i-1} p^{i-1} (1-p)^{M-i}$ for $i = 1, \ldots, M$. Figure 7 illustrates the equilibrium information disclosure strategy γ^* as a function of the arrival rate λ and the priority discipline (i.e., SPT or LPT). We observe that, when given control over their information, customers may choose information disclosure strategies that deviate from the socially optimal one (i.e., $\gamma^o = 1$ under SPT and $\gamma^o = 0$ under LPT), which will lead to inferior social welfare. This result confirms the insights from our base model: there exist regimes under which privacy regulation in the form of giving customers full control over information may backfire.



(a) SPT discipline, $p = 0.45$.
(b) LPT discipline, $p = 0.2$.

Figure 7 Customers' equilibrium information disclosure strategy γ^* as a function of the arrival rate λ.

Finally, Figure 8 shows that the threshold m is an increasing function of p. This is intuitive: when p increases, the probability mass of the Binomial distribution moves to larger i, so the threshold m also increases. More generally, the threshold naturally increases when the probability mass is shifted towards higher request types (i.e., there is more weight on higher α_i).

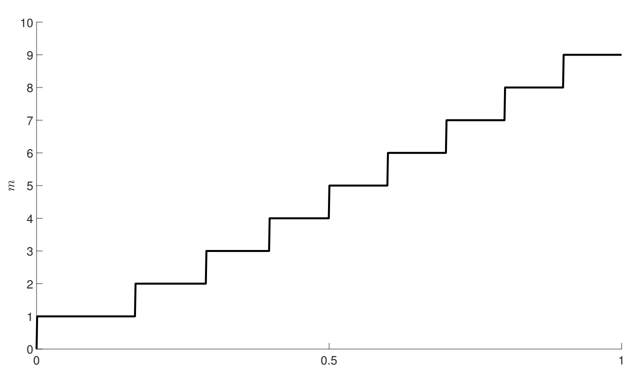
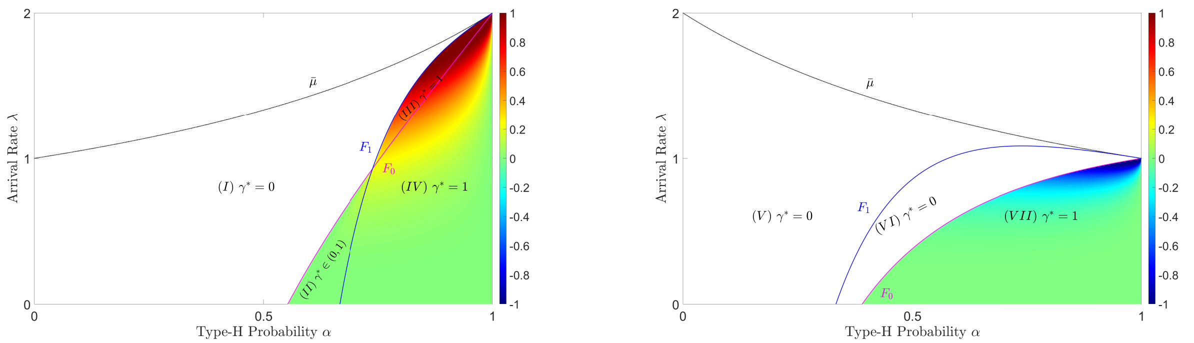


Figure 8 Threshold m as a function of p, $M = 10$.



(a) SPT discipline, $\sigma<1$.

(b) LPT discipline, $\sigma>1$.

Figure 9 Difference between the total customer surplus under customers' full control of information and under no information disclosure, $\mathrm{CS}_{\text{control}}-\mathrm{CS}_{\text{withhold}}$, as a function of the arrival rate λ and the probability α of a type-H request. In all of our colored graphs, for better contrast we plot the normalized value of x: $\operatorname{sign}(x)\left(1-e^{x\cdot\operatorname{sign}(-x)}\right)$.

B.2. No Disclosure Benchmark (Page 26)

The following result is immediate from Lemma 4.

THEOREM 3. *Whether self-control over personal information is less or more beneficial to society than is no information disclosure depends on the priority rule adopted by the service provider. If the provider prioritizes short jobs (SPT discipline, $\sigma<1$), then self-control over information leads to weakly higher total customer surplus (i.e., $\mathrm{CS}_{\text{control}}\geq\mathrm{CS}_{\text{withhold}}$); otherwise, if the service provider prioritizes long jobs (LPT discipline, $\sigma>1$), then $\mathrm{CS}_{\text{control}}\leq\mathrm{CS}_{\text{withhold}}$. The inequalities are strict if given control over personal information some customers choose to disclose it (i.e., $\gamma^*>0$).*

When the service provider's priority policy is aligned with collective incentives (i.e., under the SPT policy) and it can only be implemented if the provider obtains customers' personal information, a stricter privacy regulation that forbids information collection hurts customers' total surplus. On the contrary, if the service provider's priority policy is misaligned with the interests of society (i.e., under the LPT policy), proscribing information collection increases total customer surplus. Figure 9 shows that the magnitude of the difference in customers' total surplus under self-control and no disclosure is higher when the system is more congested.

B.3. Heterogeneous Waiting Costs (Page 27)

The next lemma gives the expected individual utility of disclosed and undisclosed customers under throughput λ.

LEMMA 7. *Given the arrival rate λ and the population's disclosure probability γ, an individual customer's expected utility from disclosing information $U_d(\gamma,\lambda)$, and from withholding information $U_w(\gamma,\lambda)$ are*

$$\begin{aligned}U_d(\gamma,\lambda)&=R-\left(\frac{c_H\alpha}{\mu-\gamma\alpha\lambda}+\frac{c_L(1-\alpha)\mu}{(\mu-\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))}\right),\\ U_w(\gamma,\lambda)&=R-\frac{(\alpha c_H+(1-\alpha)c_L)\mu}{(\mu-\gamma\alpha\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))}.\end{aligned}\tag{11}$$

Proof of Lemma 7 Due to the memoryless property of exponential distribution, the expected service time B_i and residual service time R_i of class i requests are identical, $B_i=R_i=1/\mu$, for $i=H$, N, or L.

Then, we have effectively three priority classes with type-i workload: $\rho_H = \alpha\gamma\lambda/\mu$, $\rho_N = (1-\gamma)\lambda/\mu$, and $\rho_L = (1-\alpha)\gamma\lambda/\mu$. From Chap 9.2 of Adan and Resing (2015), we obtain the expected waiting time of each request type:

Type	Exp. Waiting Time
H	$W_H = \frac{\rho_H R_H}{1-\rho_H} + B_H = \frac{1}{\mu-\gamma\alpha\lambda}$
N	$W_N = \frac{\rho_H R_H + \rho_N R_N}{(1-\rho_H-\rho_N)(1-\rho_H)} + \frac{B_N}{1-\rho_H} = \frac{\mu}{(\mu-\gamma\alpha\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))}$
L	$W_L = \frac{\rho_H R_H + \rho_N R_N + \rho_L R_L}{(1-\rho_H-\rho_N-\rho_L)(1-\rho_H-\rho_N)} + \frac{B_L}{1-\rho_H-\rho_N} = \frac{\mu}{(\mu-\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))}$

If customers disclose information, they will have type-H request w.p. α, or type-L request w.p. $1-\alpha$. Then, the expected utility of disclosing customers is $U_d = R - (c_H\alpha W_H + c_L(1-\alpha)W_L)$. If customers withhold information, they will have type-N requests. The expected utility of withholding customers is $U_w = R - (\alpha c_H + (1-\alpha)c_L)W_N$. Substituting W_H, W_N, and W_L in U_d and U_w, we obtain (11). $\square$

Proposition 5. *There exists a symmetric equilibrium information disclosure strategy γ^* such that*

$$\gamma^* = \begin{cases} 0 & \text{if } \lambda \geq \max\{H_0, H_1\}, \\ 1 & \text{if } \lambda \leq \min\{H_0, H_1\}, \\ \frac{(1-\alpha)\mu c_L - \alpha(\mu-\lambda)c_H}{\alpha(1-\alpha)(\mu c_L - (\mu-\lambda)c_H)} \in (0,1) & \text{if } H_1 < \lambda < H_0, \\ 0 \text{ or } 1 & \text{otherwise}, \end{cases} \tag{12}$$

where $H_0 = \mu\left(1 - \frac{(1-\alpha)c_L}{\alpha c_H}\right)$ *and* $H_1 = \mu\left(1 - \frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$.

Proof of Proposition 5 In this case, the sign of $U_d(\gamma,\lambda) - U_w(\gamma,\lambda)$ determines customers' equilibrium behavior:

$$U_d(\gamma,\lambda) - U_w(\gamma,\lambda) = \frac{\lambda}{(\mu-\lambda)(\mu-\gamma\alpha\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))} \cdot \Xi(\gamma), \tag{13}$$

where the denominator of the first multiplier of Expression (13) is positive and $\Xi(\gamma) \equiv \alpha(1-\alpha)(\mu c_L - (\mu-\lambda)c_H)\gamma + (\alpha(\mu-\lambda)c_H - (1-\alpha)\mu c_L)$ is a linear function of γ.

When $\alpha \leq \frac{1}{2} \Leftrightarrow \frac{(1-\alpha)^2}{\alpha^2} \geq \frac{1-\alpha}{\alpha} \geq 1$,

- If $\mu c_L - (\mu-\lambda)c_H < 0 \Leftrightarrow \frac{c_H}{c_L} > \frac{\mu}{\mu-\lambda} \Leftrightarrow \lambda < \mu\left(1-\frac{c_L}{c_H}\right)$, $\Xi(\gamma)$ is a decreasing function of γ.
  - —If $\Xi(1) \geq 0 \Leftrightarrow \frac{c_H}{c_L} \geq \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} \Leftrightarrow \lambda \leq \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$, then all-share (i.e., $\gamma^*=1$) is the only equilibrium.
  - —If $\Xi(0) > 0$ and $\Xi(1) < 0 \Leftrightarrow \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} < \frac{c_H}{c_L} < \frac{(1-\alpha)\mu}{\alpha(\mu-\lambda)} \Leftrightarrow \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right) < \lambda < \mu\left(1-\frac{(1-\alpha)c_L}{\alpha c_H}\right)$, then there is a non-trivial equilibrium $\gamma^* = \frac{(1-\alpha)\mu c_L - \alpha(\mu-\lambda)c_H}{\alpha(1-\alpha)(\mu c_L-(\mu-\lambda)c_H)} \in (0,1)$.
  - —If $\Xi(0) \leq 0 \Leftrightarrow \frac{c_H}{c_L} \leq \frac{(1-\alpha)\mu}{\alpha(\mu-\lambda)} \Leftrightarrow \mu\left(1-\frac{(1-\alpha)c_L}{\alpha c_H}\right) \leq \lambda < \mu\left(1-\frac{c_L}{c_H}\right)$, then all-hide (i.e., $\gamma^*=0$) is the only equilibrium.
- If $\mu c_L - (\mu-\lambda)c_H \geq 0 \Leftrightarrow \frac{c_H}{c_L} \leq \frac{\mu}{\mu-\lambda} \Leftrightarrow \lambda \geq \mu\left(1-\frac{c_L}{c_H}\right)$, $\Xi(\gamma)$ is an increasing function of γ. We have $\Xi(1) \leq 0 \Leftrightarrow \frac{c_H}{c_L} \leq \frac{\mu}{\mu-\lambda} \leq \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} \Leftrightarrow \lambda \geq \mu\left(1-\frac{c_L}{c_H}\right) \geq \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$, then withholding (i.e., $\gamma^*=0$) is the only equilibrium.

When $\alpha > \frac{1}{2} \Leftrightarrow \frac{(1-\alpha)^2}{\alpha^2} < \frac{1-\alpha}{\alpha} < 1$,

- If $\mu c_L - (\mu-\lambda)c_H \leq 0 \Leftrightarrow \frac{c_H}{c_L} \geq \frac{\mu}{\mu-\lambda} \Leftrightarrow \lambda \leq \mu\left(1-\frac{c_L}{c_H}\right)$, $\Xi(\gamma)$ is a decreasing function of γ. We have $\Xi(1) \geq 0 \Leftrightarrow \frac{c_H}{c_L} \geq \frac{\mu}{\mu-\lambda} > \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} \Leftrightarrow \lambda \leq \mu\left(1-\frac{c_L}{c_H}\right) < \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$, then all-share (i.e., $\gamma^*=1$) is the only equilibrium.

- $\mu c_L - (\mu-\lambda) c_H > 0 \Leftrightarrow \frac{c_H}{c_L} < \frac{\mu}{\mu-\lambda} \Leftrightarrow \lambda > \mu\left(1-\frac{c_L}{c_H}\right)$, $\Xi(\gamma)$ is an increasing function of γ.
  - —If $\Xi(0) \geq 0 \Leftrightarrow \frac{c_H}{c_L} \geq \frac{(1-\alpha)\mu}{\alpha(\mu-\lambda)} \Leftrightarrow \lambda \leq \mu\left(1-\frac{(1-\alpha)c_L}{\alpha c_H}\right)$, then all-share (i.e., $\gamma^*=1$) is the only equilibrium.
  - —If $\Xi(0) < 0$ and $\Xi(1) > 0 \Leftrightarrow \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} < \frac{c_H}{c_L} < \frac{(1-\alpha)\mu}{\alpha(\mu-\lambda)} \Leftrightarrow \mu\left(1-\frac{(1-\alpha)c_L}{\alpha c_H}\right) < \lambda < \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$, then there are two stable equilibria $\gamma^*=0$ and 1 (i.e., follow the crowd behavior).
  - —If $\Xi(1) \leq 0 \Leftrightarrow \frac{c_H}{c_L} \leq \frac{(1-\alpha)^2\mu}{\alpha^2(\mu-\lambda)} \Leftrightarrow \lambda \geq \mu\left(1-\frac{(1-\alpha)^2 c_L}{\alpha^2 c_H}\right)$, then all-hide (i.e., $\gamma^*=0$) is the only equilibrium.

Thus, customers' equilibrium behavior is given in (12). □

The customer surplus can be derived as the product of the effective joining rate and the expected utility of customers:

$$\begin{aligned} CS(\gamma) &= \lambda\left(\gamma U_d(\gamma,\lambda) + (1-\gamma) U_w(\gamma,\lambda)\right) \\ &= \lambda\left(R - \left(\frac{\gamma c_H \alpha}{\mu-\gamma\alpha\lambda} + \frac{\gamma c_L (1-\alpha)\mu}{(\mu-\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))} + \frac{(1-\gamma)(\alpha c_H + (1-\alpha)c_L)\mu}{(\mu-\gamma\alpha\lambda)(\mu-\lambda(\alpha+(1-\gamma)(1-\alpha)))}\right)\right) \end{aligned}$$

By the property of the $c\mu$ rule, it will be socially optimal for a social planner to prioritize customers with higher waiting cost, or equivalently to not to prioritize customers with lower expected waiting cost. Thus, we have the following proposition.

Lemma 8. *Customer surplus $CS(\gamma)$ is an increasing function of γ.*

Proof of Lemma 8 Using (11), we can rewrite $CS(\gamma)$ as

$$CS(\gamma) = \lambda\left(R - \left(\frac{(\lambda-\mu)c_H + \mu c_L}{\lambda(\mu-\lambda)} + \frac{\mu(2\alpha-1)^2(c_H-c_L)}{\lambda\left(\mu(1-\alpha)^3 + \alpha^3(\mu-\lambda)\right) + (c_H-c_L)\Gamma_{hc}(\gamma)}\right)\right)$$

where $\Gamma_{hc}(\gamma) \equiv \frac{\alpha(1-\alpha)(2\alpha-1)\lambda^2}{(c_H-c_L)}\gamma + \frac{\alpha(1-\alpha)(\mu-\alpha\lambda)^2\lambda}{\gamma(c_H-c_L)(2\alpha-1)\lambda-(\alpha c_H+(1-\alpha)c_L)\lambda+(\lambda-\mu)c_H+\mu c_L}$. We next prove that $\Gamma_{hc}(\gamma)$ decreases in γ by showing that its first derivative

$$\frac{\partial\Gamma_{hc}(\gamma)}{\partial\gamma} = \frac{\alpha(1-\alpha)(2\alpha-1)^2\lambda^3(1-\gamma)}{(c_H-c_L)(\mu-\lambda+\alpha\lambda+\lambda\gamma-2\alpha\lambda\gamma)^2}(2\mu-\lambda+(1-2\alpha)\lambda\gamma) > 0.$$

If $\alpha \leq 1/2$, $2\mu-\lambda+(1-2\alpha)\lambda\gamma$ is an increasing function of γ, so $2\mu-\lambda+(1-2\alpha)\lambda\gamma \geq 2\mu-\lambda > 0$; and if $\alpha > 1/2$, $2\mu-\lambda+(1-2\alpha)\lambda\gamma$ is a decreasing function of γ, so $2\mu-\lambda+(1-2\alpha)\lambda\gamma \geq 2\mu-2\alpha\lambda > 0$. In the above cases, we have $\partial\Gamma_{hc}(\gamma)/\partial\gamma > 0$. Thus, $CS(\gamma)$ increases in γ. □

From Lemma 8, the following proposition is immediate.

Proposition 6. *If the service provider prioritizes jobs with higher waiting cost, then self-control over information leads to weakly lower total customer surplus (i.e., $CS_{control} \leq CS_{discl}$). The inequalities are strict if given control over personal information, some customers choose to withhold it (i.e., $\gamma^* < 1$).*

We have $CS_{withhold} - CS_{discl} = CS(0) - CS(1) = -\frac{\alpha(1-\alpha)(c_H-c_L)\lambda^2}{(\mu-\lambda)(\mu-\alpha\lambda)} < 0$. Customers choose $\gamma^*=1$ in equilibrium only when $U_w(1,\lambda) - U_d(1,\lambda) \leq 0$. When $\lambda(U_w(1,\lambda) - U_d(1,\lambda)) < 0 \Leftrightarrow \lambda \leq \mu\left(1-\frac{(\alpha-1)^2 c_L}{\alpha^2 c_H}\right)$, disclosing is an equilibrium and is socially optimal, so no need to induce customer to do anything. Thus, we need to use monetary incentive to induce customers to disclose information, which is also the socially optimal information disclosure behavior, only when $\lambda(U_w(1,\lambda) - U_d(1,\lambda)) > 0 \Leftrightarrow \lambda > \mu\left(1-\frac{(\alpha-1)^2 c_L}{\alpha^2 c_H}\right)$:

$$P_d = P_s = \frac{\lambda^2\left((\alpha-1)^2\mu c_L - \alpha^2(\mu-\lambda)c_H\right)}{(\mu-\lambda)(\mu-\alpha\lambda)^2}. \quad (14)$$

B.4. Strategic Balking and Control over Information (Page 28)

Recall from Section 3.2 that the service reward has a lower bound: $R \geq c(1+\alpha(\sigma-1))/\sigma\mu$. Let Λ denote the total arrival rate of customers. The effective joining rate $\lambda = q\Lambda$, which is defined by the joining probability q, can be no higher than a *join-up-to level* $\bar{\lambda}^*$. If the total arrival rate to the service facility is below this level (i.e., if $\Lambda < \bar{\lambda}^*$) then customers will join with probability $q = 1$ (and so $\lambda = \Lambda$); they expect a non-negative surplus. But if the total arrival rate is above this level ($\Lambda > \bar{\lambda}^*$), then customers will join with probability $q < 1$. Hence the effective joining rate remains at this join-up-to level (i.e., $\lambda = \bar{\lambda}^*$) and all customers receive zero surplus. In other words, the join-up-to level represents the upper bound on the service provider's throughput when customers strategically choose to join or balk.

We emphasize that all the results and conclusions derived in Sections 4 and 5 continue to hold under this alternative setting of lower service reward values. So here the customers' equilibrium information disclosure strategy, our comparison of total customer surplus under customers' self-control over information with that under the full or the socially optimal information disclosure strategy, and the monetary incentives needed to induce particular information disclosure behavior, are all independent of the service reward R and remain structurally the same. The goal of this alternative model is to develop additional insights into how customers' control of their personal information affects the service provider by changing the system's join-up-to level and thus its maximum throughput. The following technical lemma derives the equilibrium join-up-to level.

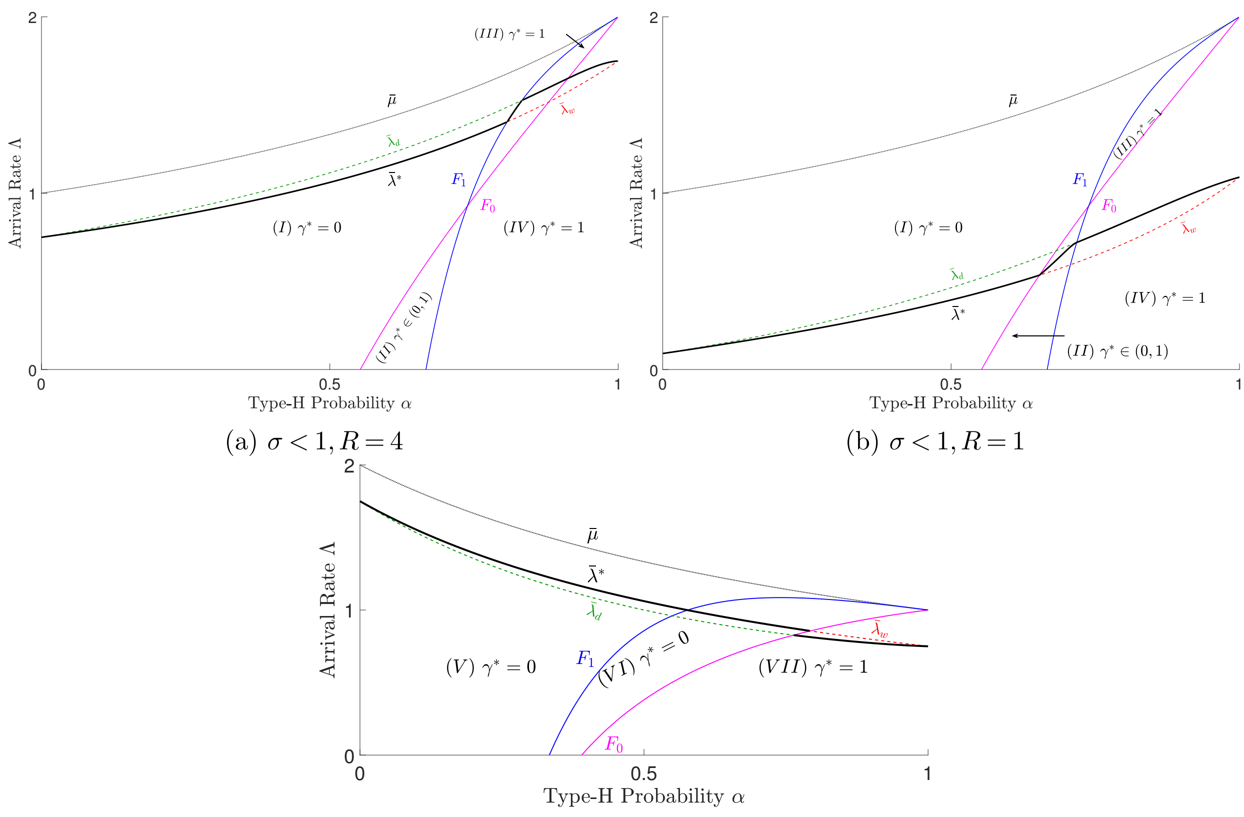
Lemma 9. *The equilibrium join-up-to level of the service system $\bar{\lambda}^*$ is given as follows:*

$$\bar{\lambda}^*(\alpha) = \begin{cases} \gamma^* \min\left\{\bar{\lambda}_d(\alpha), F_1^{-1}(\alpha)\right\} + (1-\gamma^*)\bar{\lambda}_w(\alpha) & \textit{if } \sigma < 1 \textit{ and } \gamma^* \in \{0,1\}, \\ \bar{\lambda}_m : U_d(\gamma^*(\bar{\lambda}_m), \bar{\lambda}_m) = 0 & \textit{if } \sigma < 1 \textit{ and } \gamma^* \in (0,1), \\ \gamma^* \bar{\lambda}_d(\alpha) + (1-\gamma^*)\bar{\lambda}_w(\alpha) & \textit{if } \sigma > 1 \textit{ and } \gamma^* \in \{0,1\}. \end{cases} \tag{15}$$

Here γ^* is the Pareto-dominant information disclosure strategy characterized in Section 4.1, and the functions $\bar{\lambda}_m$, $F_1^{-1}(\alpha)$, $\bar{\lambda}_w(\alpha)$, and $\bar{\lambda}_d(\alpha)$ are characterized in the proof.

This lemma characterizes the equilibrium join-up-to level in the service system, $\bar{\lambda}^*(\alpha)$ as a function of $\bar{\lambda}_d(\alpha)$ and $\bar{\lambda}_w(\alpha)$—join-up-to levels of customers who all disclose (d) or all withhold (w) information, correspondingly. Note that $\bar{\lambda}_d(\alpha)$ and $\bar{\lambda}_w(\alpha)$ characterize join-up-to levels in hypothetical systems in which *all* customers either disclosure or withhold information, respectively. Figure 10 plots the join-up-to levels $\bar{\lambda}_d(\alpha)$ and $\bar{\lambda}_w(\alpha)$ as well as the equilibrium join-up-to level $\bar{\lambda}^*(\alpha)$. We can see that, if the service provider adopts an SPT policy ($\sigma < 1$, Panels (a) and (b) of Figure 10) and if all customers disclose information ($\gamma^* = 1$, in regions III and IV), then the joining rate of customers may be upper bounded not only by the join-up-to level $\bar{\lambda}_d(\alpha)$ but also by $F_1^{-1}(\alpha)$ (the overlap region of the blue and the black curves on panel (a) of the Figure). Suppose, for instance, that the service reward is high enough that $\bar{\lambda}_w$ and $\bar{\lambda}_d$ cross region III (Panel (a) of Figure 10); then there exists an intermediate range of α for which $F_1^{-1}(\alpha) \in \left[\bar{\lambda}_w(\alpha), \bar{\lambda}_d(\alpha)\right]$. Here the customers have no incentive to join at the rate that exceeds $F_1^{-1}(\alpha)$ because then they would all prefer withholding information. At the same time, the join-up-to level when all customers withhold information, $\bar{\lambda}_w(\alpha)$, is below $F_1^{-1}(\alpha)$. So even if the total arrival rate Λ exceeds the system capacity $\bar{\mu}$, in this case customers will join only at rate $F_1^{-1}(\alpha)$ and each customer has a *positive* individual surplus. That result is driven by customers having control over their personal information. As far as we can tell, this outcome has not been reported in "unobservable queue" models, where customers usually obtain zero surplus at the join-up-to level. In addition, if the service reward is relatively small—so that $\bar{\lambda}_w$ and $\bar{\lambda}_d$ cross region II (Panel (b) of Figure 10)—then there is an intermediate range of α within which $\bar{\lambda}^* \in \left[\left(\bar{\lambda}_w(\alpha), \bar{\lambda}_d(\alpha)\right)\right]$. Here, at the join-up-to level $\bar{\lambda}^*$, customers use a nontrivial information disclosure strategy $\gamma^* \in (0,1)$ and expect zero surpluses.

An examination of Figure 10(c) reveals that, when a service provider employs the LPT priority rule ($\sigma > 1$), there exists an interval on which customers follow a *two-tier* join-up-to level: (i) customers use $\bar{\lambda}_d(\alpha)$ as their join-up-to level if the total arrival rate Λ to the system is lower than some threshold $F_0^{-1}(\alpha)$; and (ii) the maximum throughput is $\bar{\lambda}_w(\alpha)$ if that arrival rate Λ is higher than $F_0^{-1}(\alpha)$ (this is happening in the region where for each α we observe two levels of black line $\bar{\lambda}^*$). This result reflects customers' use of different equilibrium information disclosure strategies depending on whether Λ is below or above $F_0^{-1}(\alpha)$. In all other regions, customers' join-up-to level is less intricate: they join at a rate no higher than $\bar{\lambda}_d(\alpha)$ (resp., $\bar{\lambda}_w(\alpha)$) if it is in their interest for all to disclose information, $\gamma^* = 1$ (resp., to withhold information, $\gamma^* = 0$).



(a) $\sigma < 1, R = 4$ (b) $\sigma < 1, R = 1$ (c) $\sigma > 1, R = 4$

Figure 10 Join-up-to level $\bar{\lambda}^*$ as a function of the probability α of a type-H request: SPT discipline in Panels (a) and (b); LPT discipline in Panel (c).

Next, we investigate the effect of customers' control over information on the join-up-to level, or the service provider's maximum throughput, which directly affects its potential profitability. More specifically, we compare the equilibrium join-up-to level when customers have control over their personal information, $\bar{\lambda}^*(\alpha)$, with the join-up-to level in the benchmark case of full information disclosure, $\bar{\lambda}_d(\alpha)$.

THEOREM 4. *In comparison with full information disclosure, customer control of information can only benefit the service provider by increasing the join-up-to level, if it employs the long-processing-time-first priority discipline (LPT, $\sigma > 1$). In all other cases, customer control of information weakly reduces the join-up-to level at the service facility and hence also the service provider's maximum throughput. Formally: if $\sigma > 1$ then $\bar{\lambda}^*(\alpha) \geq \bar{\lambda}_d(\alpha)$; otherwise, $\bar{\lambda}^*(\alpha) \leq \bar{\lambda}_d(\alpha)$.*

Proof This theorem is straightforward from Lemmas 9 and 10. □

The join-up-to level at the service facility is closely related to the average wait time experienced by customers. Recall from Theorem 1 that a service provider's adoption of the LPT priority discipline lengthens customers' expected wait time. It follows that allowing customers to control their personal information may shorten their expected wait time under a given joining rate, which raises the join-up-to level and benefits the service provider (see also Lemma 9). In contrast, a service provider that adopts the SPT priority discipline reduces expected wait time and so the customer control of information may lower the join-up-to level , by increasing the expected wait time under *the same throughput*. A lower join-up level would, of course, reduce the service provider's profitability.[12]

References

Adan I, Resing J (2015) *Queueing Theory*. https://www.win.tue.nl/iadan/queueing.pdf.

[12] Assuming that customers make join/balk decisions after observing their request type does not change the result of Theorem 4 structurally. First, the join-up-to level of information-withholding customers $\bar{\lambda}_w(\alpha)$ doesn't change—the service provider has no information to prioritize their requests, hence such customers have no reason to behave differently. Second, the join-up-to level of information-disclosing customers $\bar{\lambda}_d(\alpha)$ will increase if $\sigma < 1$ and decrease otherwise—customers with type-L requests when balking increase the proportion of type-H requests in the throughput, which increases $\bar{\lambda}_d(\alpha)$ under SPT discipline and decreases it otherwise.