Online Supplements to "Precommitments in Two-sided Market Competition"

The online supplements include five sections.

- Section A provides the detailed analysis of quantity precommitment competition.
- Section B analyzes the wage precommitment competition under market size uncertainty.
- Section C analyzes the price precommitment competition under market size uncertainty.
- Section D includes the remaining proof of lemmas and propositions in the main body.
- Section E provides the equilibrium outcome for each mode when the market sizes are asymmetric.

A. Quantity Precommitment Competition in KS Equivalency

We assume the market size is deterministic. Section A.1 examines the price and wage decisions in the second stage conditional on the capacity decisions of the first stage, and then analyzes the first stage capacity decision. Section A.2 derives the equilibrium of the quantity precommitment game.

A.1. Analysis

For any fixed $\boldsymbol{q}$, we study the second stage competition where firm $i \in \{1,2\}$ decides its price p_i and wage w_i simultaneously. For any fixed (p_j, w_j) where $j \neq i$, firm i faces the following optimization problem

$$\max_{p_i, w_i} \quad (p_i - w_i)\min\{d_i(p_i, p_j), s_i(w_i, w_j), q_i\}. \tag{A.1}$$

At optimality, we have $\Omega - p_i^* + \gamma p_j = z_i = w_i^* - \beta w_j$, because any excess demand or supply does not benefit firm i. Hence,

$$p_i^* = \Omega + \gamma p_j - z_i, \quad w_i^* = z_i + \beta w_j. \tag{A.2}$$

Thus, (A.1) can be written as

$$\max_{z_i} \quad (\Omega + \gamma p_j - 2z_i - \beta w_j)\min\{z_i, q_i\}. \tag{A.3}$$

Solving (A.3) yields that, if $q_i \geq \frac{\Omega+\gamma p_j - \beta w_j}{4}$, then $z_i^* = \frac{\Omega+\gamma p_j-\beta w_j}{4}$. Otherwise, $z_i^* = q_i$. To see this, if the capacity is above that threshold, then the supply and demand quantity should take the one that maximizes the profit as if there is no capacity constraint. Otherwise, the firms should set the price and wage such that both the demand and supply equal to the capacity.

Based on the values of $\boldsymbol{q}$, we have four cases.

Case (a): Suppose for any $i = 1, 2$,

$$q_i \geq \frac{\Omega + \gamma p_j - \beta w_j}{4}. \tag{A.4}$$

That is, both firms set a large capacity in the first stage such that the price and wage decisions in the second stage are not constrained. In this symmetric case, $z_i^* = \frac{\Omega+\gamma p_j - \beta w_j}{4}$. Using (A.2), we obtain the best-response function of firm i,

$$p_i^*(p_j, w_j) = \frac{3}{4}(\Omega + \gamma p_j) + \frac{1}{4}\beta w_j, \quad w_i^*(p_j, w_j) = \frac{1}{4}(\Omega + \gamma p_j) + \frac{3}{4}\beta w_j.$$

Solving the set of the above equations yields the equilibrium quantities for any $i = 1, 2$,

$$p_i^* = \frac{3 - 2\beta}{4 - 3\beta - 3\gamma + 2\beta\gamma}\Omega, \quad w_i^* = \frac{1}{4 - 3\beta - 3\gamma + 2\beta\gamma}\Omega, \quad z_i^* = \frac{1 - \beta}{4 - 3\beta - 3\gamma + 2\beta\gamma}\Omega. \tag{A.5}$$

Putting p_i^* and w_i^* back to (A.4) rewrites the initial condition as follows,

$$q_i \geq \frac{1 - \beta}{4 - 3\beta - 3\gamma + 2\beta\gamma}\Omega. \tag{A.6}$$

That is, if (A.6) holds, then according to (A.5), we obtain the equilibrium profit, denoted by $\pi_{i,a}^*$, as follows,

$$\pi_{i,a}^* = (p_i^* - w_i^*)z_i^* = \frac{2(1-\beta)^2}{(4 - 3\beta - 3\gamma + 2\beta\gamma)^2}\Omega^2.$$

Observe that the result in this case is exactly the same as the simultaneous price and wage competition prescribed in Section 3.1.

Case (b): Suppose for any $i = 1, 2$,

$$q_i < \frac{\Omega + \gamma p_j - \beta w_j}{4}. \tag{A.7}$$

That is, both firms set a small capacity in the first stage. In this symmetric case, $z_i^* = q_i$. Using (A.2), we obtain the best-response function of firm i,

$$p_i^* = \Omega + \gamma p_j - q_i, \quad w_i^* = q_i + \beta w_j.$$

Solving the set of the above equations yields the equilibrium quantities for any $i = 1, 2$,

$$p_i^* = \frac{(1+\gamma)\Omega - \gamma q_j - q_i}{1 - \gamma^2}, \quad w_i^* = \frac{q_i + \beta q_j}{1 - \beta^2}. \tag{A.8}$$

Putting p_i^* and w_i^* back to (A.7) rewrites the initial condition as follows,

$$(4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2)q_i + (\gamma - \gamma\beta^2 + \beta - \beta\gamma^2)q_j < (1+\gamma)(1-\beta^2)\Omega. \tag{A.9}$$

That is, if (A.9) holds, then according to (A.8), firm i's profit function can be written as follows,

$$\pi_i = (p_i^* - w_i^*)q_i = \Big\{\frac{(1+\gamma)\Omega - \gamma q_j - q_i}{1-\gamma^2} - \frac{q_i + \beta q_j}{1-\beta^2}\Big\}q_i.$$

Maximizing π_i yields firm i's best-response function

$$q_i^* = \frac{(1+\gamma)(1-\beta^2)\Omega - \{\gamma(1-\beta^2) + \beta(1-\gamma^2)\}q_j}{2(2-\beta^2-\gamma^2)}. \tag{A.10}$$

Solving the set of equations (A.10) gives the equilibrium quantities in this case

$$q_i^* = \frac{(1+\gamma)(1-\beta^2)\Omega}{4 - 2\beta^2 - 2\gamma^2 + \beta + \gamma - \beta^2\gamma - \beta\gamma^2}.$$

Therefore,

$$p_i^* = \frac{\Omega - q_i^*}{1-\gamma}, \quad w_i^* = \frac{q_i^*}{1-\beta}, \quad \pi_{i,b}^* = \frac{(1+\gamma)(1-\beta^2)(2-\beta^2-\gamma^2)}{(1-\gamma)(4 - 2\beta^2 - 2\gamma^2 + \beta + \gamma - \beta^2\gamma - \beta\gamma^2)^2}\Omega^2.$$

Observe that the result in this case is exactly the same as the quantity competition prescribed in Section 4.1.

Case (c): Suppose

$$q_1 < \frac{\Omega + \gamma p_2 - \beta w_2}{4}, \quad q_2 \geq \frac{\Omega + \gamma p_1 - \beta w_1}{4}. \tag{A.11}$$

That is, firm 1 sets a small capacity while firm 2 sets a large capacity in the first stage. In this asymmetric case, $z_1^* = q_1$ and $z_2^* = \frac{\Omega+\gamma p_1 - \beta w_1}{4}$. Using (A.2), we obtain the best-response function of firm i,

$$p_1^* = \Omega + \gamma p_2 - q_1, \qquad w_1^* = q_1 + \beta w_2,$$
$$p_2^* = \frac{3}{4}(\Omega + \gamma p_1) + \frac{1}{4}\beta w_1, \qquad w_2^* = \frac{1}{4}(\Omega + \gamma p_1) + \frac{3}{4}\beta w_1.$$

Solving the set of above equations yields the equilibrium quantities for any $i = 1, 2$,

$$p_1^* = \frac{(4 - 3\beta^2 + 3\gamma - 2\beta^2\gamma)\Omega - (4 - 3\beta^2 - \beta\gamma)q_1}{4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2}, \qquad w_1^* = \frac{\beta(1+\gamma)\Omega + (4 - 3\gamma^2 - \beta\gamma)q_1}{4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2}, \tag{A.12}$$
$$p_2^* = \frac{(3 + 3\gamma - 2\beta^2 - 2\beta^2\gamma)\Omega - (3\gamma - \beta - 2\beta^2\gamma)q_1}{4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2}, \qquad w_2^* = \frac{(1+\gamma)\Omega - (\gamma - 3\beta + 2\beta\gamma^2)q_1}{4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2}. \tag{A.13}$$

Putting (p_1^*, p_2^*) and (w_1^*, w_2^*) back to (A.11) rewrites the initial condition as follows,

$$q_1 < \frac{4 - 3\beta^2 + 3\gamma - 2\beta^2\gamma - \beta - \beta\gamma}{16 - 2\beta\gamma - 9\beta^2 - 9\gamma^2 + 4\beta^2\gamma^2}\Omega, \tag{A.14}$$
$$(\gamma - \gamma\beta^2 + \beta - \beta\gamma^2)q_1 + (4 - 3\beta^2 - 3\gamma^2 + 2\beta^2\gamma^2)q_2 \geq (1+\gamma)(1-\beta^2)\Omega. \tag{A.15}$$

That is, if (A.14) and (A.15) hold, then according to (A.12), firm 1's profit function can be written as follows,

$$\pi_1 = (p_1^* - w_1^*)q_1 = \frac{(4-3\beta^2+3\gamma-2\beta^2\gamma-\beta-\beta\gamma)\Omega-(8-3\beta^2-2\beta\gamma-3\gamma^2)q_1}{4-3\beta^2-3\gamma^2+2\beta^2\gamma^2}q_1.$$

Maximizing π_1 yields firm 1's equilibrium quantities,

$$q_1^* = \frac{4-3\beta^2+3\gamma-2\beta^2\gamma-\beta-\beta\gamma}{16-6\beta^2-6\gamma^2-4\beta\gamma}\Omega,$$
$$\pi_{1,c}^* = \frac{1}{2}\frac{(4-3\beta^2+3\gamma-2\beta^2\gamma-\beta-\beta\gamma)^2}{(4-3\beta^2-3\gamma^2+2\beta^2\gamma^2)(16-6\beta^2-6\gamma^2-4\beta\gamma)}\Omega^2.$$

One can check that q_1^* satisfies (A.14).

Next we derive $\pi_{2,c}^*$. Using (A.13) and (A.12), firm 2's profit can be obtained as follows,

$$\begin{aligned}\pi_{2,c}^* &= (p_2^*-w_2^*)z_2^* = (p_2^*-w_2^*)\frac{\Omega+\gamma p_1^*-\beta w_1^*}{4}\\ &= \frac{2\Omega^2}{(4-3\beta^2-3\gamma^2+2\beta^2\gamma^2)^2}\Big\{(1+\gamma-\beta^2-\beta^2\gamma)-(\beta+\gamma-\beta^2\gamma-\beta\gamma^2)\frac{4-3\beta^2+3\gamma-2\beta^2\gamma-\beta-\beta\gamma}{16-6\beta^2-6\gamma^2-4\beta\gamma}\Big\}^2.\end{aligned}$$

<u>Case (d):</u> Suppose

$$q_1 \geq \frac{\Omega+\gamma p_2-\beta w_2}{4}, \quad q_2 < \frac{\Omega+\gamma p_1-\beta w_1}{4}.$$

This case is symmetric to Case (c) and thus omitted.

A.2. Nash Equilibrium

The following Lemma shows the Nash equilibrium of the two-stage quantity precommitment competition.

LEMMA A.1. *In Nash equilibrium of the two-stage quantity precommitment competition, the resulting capacity is set in the first stage such that there does not exist any excess supply and demand in the second stage. That is, Case (b) provides the Nash equilibrium of the two-stage quantity precommitment competition:*

$$q_i^* = \frac{(1+\gamma)(1-\beta^2)}{4-2\beta^2-2\gamma^2+\beta+\gamma-\beta^2\gamma-\beta\gamma^2}\Omega, \tag{A.16}$$
$$p_i^* = \frac{3+\beta-\beta^2-2\gamma^2-\beta\gamma^2}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)}\Omega,$$
$$w_i^* = \frac{(1+\gamma)(1+\beta)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}\Omega,$$
$$\pi_i^* = \frac{(1+\gamma)(1-\beta^2)(2-\beta^2-\gamma^2)}{(1-\gamma)(4-2\beta^2-2\gamma^2+\beta+\gamma-\beta^2\gamma-\beta\gamma^2)^2}\Omega^2. \tag{A.17}$$

Proof of Lemma A.1. We expect to show among Cases (a)-(d), only Case (b) is the possible Nash equilibrium. We first focus on symmetric solutions, Case (a) and Case (b). Note that condition (A.6) in Case (a) is

$$q_i \geq \frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma}\Omega,$$

and condition (A.9) in Case (b) reduces to

$$q_i \leq \frac{1-\beta^2+\gamma-\beta^2\gamma}{4-3\beta^2-3\gamma^2+2\beta^2\gamma^2+\beta+\gamma-\beta^2\gamma-\beta\gamma^2}\Omega.$$

One can check

$$\frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma}\Omega = \frac{1-\beta^2+\gamma-\beta^2\gamma}{4-3\beta^2-3\gamma^2+2\beta^2\gamma^2+\beta+\gamma-\beta^2\gamma-\beta\gamma^2}\Omega,$$

and verify that $\pi^*_{i,a} \leq \pi^*_{i,b}$ for $i = 1, 2$. This establishes that Case (a) is not the equilibrium.

We then examine the asymmetric solutions. We first show that each $\boldsymbol{q}$ is covered by at least one of these four cases. First, the two lines in (A.9) intersect at the point $(\frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma}, \frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma})$. Second, one can show

$$\begin{aligned}
&\frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma}\Omega - \frac{4-3\beta^2+3\gamma-2\beta^2\gamma-\beta-\beta\gamma}{16-2\beta\gamma-9\beta^2-9\gamma^2+4\beta^2\gamma^2}\Omega \\
=&\frac{-12\gamma}{(4-3\beta-3\gamma+2\beta\gamma)(16-2\beta\gamma-9\beta^2-9\gamma^2+4\beta^2\gamma^2)} \\
\leq& 0.
\end{aligned}$$

Finally, the line in (A.15) coincides with the line of a larger slope in (A.9). The above three points ensure that each $\boldsymbol{q}$ is covered by at least one of these four cases.

Moreover, it can be verified that $\pi^*_{i,c} \leq \pi^*_{i,b}$ for $i = 1, 2$. Therefore, Case (c) is not the equilibrium. By symmetry, Case (d) is not the equilibrium, either. Hence, Case (b) is the only possible equilibrium. Therefore, the firms choose $\boldsymbol{q}$ in the first stage such that both the supply and demand will equal to $\boldsymbol{q}$ in the second stage. □

B. Wage Precommitment Competition under Market Size Uncertainty

This section assumes that Ω is a random variable distributed on $[\underline{\Omega}, \bar{\Omega}]$. We start with the analysis of the subgame equilibrium conditional on a fixed wage in the first stage and the realization of market size in Section B.1, and then derive the equilibrium of wage precommitment competition in Section B.2.

B.1. Subgame Equilibrium in the Second Stage

The following lemma characterizes the outcome of the pricing game in the second stage for any fixed precommitted wage $\boldsymbol{w}$ and realized market size x.

LEMMA B.1. *For any fixed $\boldsymbol{w}$ and x, the subgame equilibrium prices are given by*

(i) If

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1-\Big(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_2\le\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)x, \tag{B.1}$$

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2-(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_1\le\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)x, \tag{B.2}$$

the equilibrium prices are

$$p_1^*=p_1^d=\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x-(1-\beta\gamma)w_1+(\beta-\gamma)w_2\Big\}, \tag{B.3}$$

$$p_2^*=p_2^d=\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x-(1-\beta\gamma)w_2+(\beta-\gamma)w_1\Big\}. \tag{B.4}$$

(ii) If

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_1+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2\ge\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})x, \tag{B.5}$$

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_2+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1\ge\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})x, \tag{B.6}$$

the equilibrium prices are

$$p_1^*=p_1^m=\frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x+\frac{\gamma}{2}w_2+w_1\Big\}, \tag{B.7}$$

$$p_2^*=p_2^m=\frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x+\frac{\gamma}{2}w_1+w_2\Big\}. \tag{B.8}$$

(iii) If

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_1+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2\le\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})x, \tag{B.9}$$

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2-\Big(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_1\ge\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)x, \tag{B.10}$$

the equilibrium prices are

$$p_1^*=p_1^d=\frac{2}{2-\gamma^2}\Big\{(1+\frac{\gamma}{2})x-w_1+(\frac{\gamma}{2}+\beta)w_2\Big\}, \tag{B.11}$$

$$p_2^*=p_2^m=\frac{\gamma+1}{2-\gamma^2}x-\frac{\gamma}{2-\gamma^2}w_1+\frac{1+\beta\gamma}{2-\gamma^2}w_2. \tag{B.12}$$

(iv) If

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1-\Big(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_2\geq\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)x,$$
$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_2+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1\leq\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})x,$$

the equilibrium prices are

$$p_1^*=p_1^m=\frac{\gamma+1}{2-\gamma^2}x-\frac{\gamma}{2-\gamma^2}w_2+\frac{1+\beta\gamma}{2-\gamma^2}w_1,$$
$$p_2^*=p_2^d=\frac{2}{2-\gamma^2}\Big\{(1+\frac{\gamma}{2})x-w_2+(\frac{\gamma}{2}+\beta)w_1\Big\}.$$

Proof of Lemma B.1. For any fixed $\boldsymbol{w}$ and x, we derive the inventory-$d$epletion price for both platforms

$$p_1^d(p_2)=x+\gamma p_2-(w_1-\beta w_2),\quad p_2^d(p_1)=x+\gamma p_1-(w_2-\beta w_1).$$

By solving $\max_{p_i}(p_i-w_i)(x-p_i+\gamma p_j)$, we obtain the profit-$m$aximizing price for both platforms

$$p_1^m(p_2)=\frac{x+\gamma p_2+w_1}{2},\quad p_2^m(p_1)=\frac{x+\gamma p_1+w_2}{2}.$$

According to Hu and Zhou (2017), the best response for firm i given firm j's price p_j is

$$p_1^*(p_2)=\max\{p_1^d(p_2),p_1^m(p_2)\},\quad p_2^*(p_1)=\max\{p_2^d(p_1),p_2^m(p_1)\}.$$

We examine Case (i) first. We solve

$$p_1^*(p_2)=p_1^d(p_2),\quad p_2^*(p_1)=p_2^d(p_1) \tag{B.13}$$

and obtain

$$p_1^*=p_1^d=\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x-(1-\beta\gamma)w_1+(\beta-\gamma)w_2\Big\}, \tag{B.14}$$
$$p_2^*=p_2^d=\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x-(1-\beta\gamma)w_2+(\beta-\gamma)w_1\Big\}. \tag{B.15}$$

Then we solve

$$p_1^*(p_2)=p_1^m(p_2),\quad p_2^*(p_1)=p_2^d(p_1)$$

and obtain

$$p_1^*=\frac{1+\gamma}{2-\gamma^2}x-\frac{x}{2-\gamma^2}w_2+\frac{1+\beta\gamma}{2-\gamma^2}w_1.$$

We also solve

$$p_1^*(p_2) = p_1^d(p_2), \quad p_2^*(p_1) = p_2^m(p_1)$$

and obtain

$$p_2^* = \frac{1+\gamma}{2-\gamma^2}x - \frac{x}{2-\gamma^2}w_1 + \frac{1+\beta\gamma}{2-\gamma^2}w_2.$$

Given that $p_1^*(p_2) = \max\{p_1^d(p_2), p_1^m(p_2)\}$ and $p_2^*(p_1) = \max\{p_2^d(p_1), p_2^m(p_1)\}$, in order to ensure (B.13) holds, it must be the case that when firm i sets price p_i^d, firm j does not have the incentive to set price p_j^m, that is, $p_j^d(p_i^d) \geq p_j^m(p_i^d)$ for $i = 1, 2$. Hence,

$$\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x - (1-\beta\gamma)w_1 + (\beta-\gamma)w_2\Big\} \geq \frac{1+\gamma}{2-\gamma^2}x - \frac{x}{2-\gamma^2}w_2 + \frac{1+\beta\gamma}{2-\gamma^2}w_1,$$
$$\frac{1}{1-\gamma^2}\Big\{(1+\gamma)x - (1-\beta\gamma)w_2 + (\beta-\gamma)w_1\Big\} \geq \frac{1+\gamma}{2-\gamma^2}x - \frac{x}{2-\gamma^2}w_1 + \frac{1+\beta\gamma}{2-\gamma^2}w_2,$$

which yields the constraints (B.1) and (B.2).

Continuing in this fashion, we establish Cases (ii), (iii), and (iv). □

Before deriving the equilibrium of wage precommitment competition, we first derive the equilibrium of wage precommitment competition without demand uncertainty based on Lemma B.1, which can help us understand the scenario when the demand variance is sufficiently small. The following lemma implies that without demand uncertainty, the equilibrium wages are chosen in the first stage such that there will not be any excess quantity in the second-stage pricing game.

LEMMA B.2. *Suppose there is no market size uncertainty. In wage precommitment competition, the resulting equilibrium wages and prices are such that the supply is equal to the demand.*

Proof of Lemma B.2. We assume that the market size Ω is deterministic. We show Cases (ii), (iii), and (iv) in Lemma B.1 cannot be equilibrium. The approach is to show a unilateral change in wage could benefit at least one firm.

Case (ii): In this case, both firms have excess supply and choose the profit-maximizing prices. Suppose firm 1 reduces its wage by setting $\tilde{w}_1 = w_1 - e$ and firm 2 keeps its wage unchanged by setting $\tilde{w}_2 = w_2$. Note that e is chosen such that the constraints (B.5) and (B.6) still hold. In other words, reducing the wage does not affect both firms choosing the profit-maximizing prices in their quantity-constrained pricing game.

By (B.7) and (B.8), we have $\tilde{p}_1 = p_1 - \frac{2}{4-\gamma^2}e$, $\tilde{p}_2 = p_2 - \frac{\gamma}{2}\frac{2}{4-\gamma^2}e$, and $\tilde{d}_1(\tilde{p}_1, \tilde{p}_2) = \Omega - \tilde{p}_1 + \gamma\tilde{p}_2 = \Omega - p_1 + \gamma p_2 + \frac{2}{4-\gamma^2}(1 - \frac{\gamma^2}{2})e$. Hence,

$$\tilde{\pi}_1 = (\tilde{p}_1 - \tilde{w}_1)\tilde{d}_1(\tilde{p}_1, \tilde{p}_2)$$

$$= (p_1 - w_1 + e - \frac{2}{4-\gamma^2}e)\Big(d_1(p_1,p_2) + \frac{2}{4-\gamma^2}(1-\frac{\gamma^2}{2})e\Big)$$
$$> (p_1 - w_1)d_1(p_1,p_2) = \pi_1. \qquad [\text{by } 1 > \frac{2}{4-\gamma^2} \text{ and } 1 > \frac{\gamma^2}{2}]$$

Therefore, reducing w_1 can improve firm 1's profit.

Case (iii): In this case, firm 1's supply runs out and firm 2 has excess supply. We will show a unilateral change in w_2 can benefit both firms, which indicates that the firms will not choose the quantity in the first stage such that this case will occur in the second stage.

Subcase 1: Suppose $\gamma(\gamma+\beta) < 1$. Suppose firm 2 reduces its wage by setting $\tilde{w}_2 = w_2 - e$ and firm 1 keeps its wage unchanged by setting $\tilde{w}_1 = w_1$. Note that e is chosen such that the constraints (B.9) and (B.10) still hold. In other words, reducing the wage does not affect firm 1 choosing the supply-depletion price and firm 2 choosing the profit-maximizing price in their quantity-constrained pricing game.

By (B.11) and (B.12), we have $\tilde{p}_1 = p_1 - \frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)e$, $\tilde{p}_2 = p_2 - \frac{1+\beta\gamma}{2-\gamma^2}e$, and $\tilde{d}_2(\tilde{p}_1,\tilde{p}_2) = \Omega - \tilde{p}_2 + \gamma\tilde{p}_1 = \Omega - p_2 + \gamma p_1 + \frac{1+\beta\gamma}{2-\gamma^2}e - \frac{2\gamma}{2-\gamma^2}(\frac{\gamma}{2}+\beta)e$. Hence,

$$\begin{aligned}
\tilde{\pi}_2 &= (\tilde{p}_2 - \tilde{w}_2)\tilde{d}_2(\tilde{p}_1,\tilde{p}_2)\\
&= (p_2 - w_2 + e - \frac{1+\beta\gamma}{2-\gamma^2}e)\Big(d_2(p_1,p_2) + \frac{1+\beta\gamma}{2-\gamma^2}e - \frac{2\gamma}{2-\gamma^2}(\frac{\gamma}{2}+\beta)e\Big)\\
&> (p_2 - w_2)d_2(p_1,p_2) = \pi_2
\end{aligned}$$

where the last inequality holds because $1 > \frac{1+\beta\gamma}{2-\gamma^2}$ and $\frac{1+\beta\gamma}{2-\gamma^2} > \frac{2\gamma}{2-\gamma^2}(\frac{\gamma}{2}+\beta)$, both of which hold due to $\gamma(\gamma+\beta) < 1$. Therefore, reducing w_2 can improve firm 2's profit. Moreover, note that the supply quantity for firm 1 increases (because w_2 is reduced) and firm 1 keeps adopting the supply-depletion price, so due to the concavity of firm 1's profit function, we arrive that firm 1's profit increases as well.

Subcase 2: Suppose $\gamma(\gamma+\beta) > 1$. Suppose firm 2 increases its wage by setting $\tilde{w}_2 = w_2 + e$ and firm 1 keeps its wage unchanged by setting $\tilde{w}_1 = w_1$. By a similar process as Subcase 1, we could show that increasing w_2 can improve firm 2's profit.

Case (iv): Symmetric to Case (iii).

Combining the above three cases, we could see only Case (i) is the possible equilibrium. That is, the equilibrium wages are chosen in the first stage such that there will not be any excess quantity in the second-stage pricing game. □

B.2. Deriving the Equilibrium Wage in the First Stage

The previous section shows that for any fixed $\boldsymbol{w}$, when the market size is realized, the optimal price takes either the supply-depletion price or profit-maximizing price. Now we derive the equilibrium wage in the first stage. For the sake of tractability, we assume the two firms take symmetric actions. We have three cases with respect to the value of $\boldsymbol{w}$, detailed below. Suppose the realized market size is x.

Case 1: Low Wage. The wage is too low such that for any $x \in [\underline{\Omega}, \bar{\Omega}]$, both firms adopt the supply-depletion price at the subsequent stage.

According to Lemma B.1(i), the wage $\boldsymbol{w}$ is set such that

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1 - \Big(\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_2 \leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\underline{\Omega}, \tag{B.16}$$

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2 - (\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_1 \leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\underline{\Omega}. \tag{B.17}$$

Then the equilibrium prices are

$$p_i^* = p_i^d = \frac{1}{1-\gamma^2}\Big\{(1+\gamma)x - (1-\beta\gamma)w_i + (\beta-\gamma)w_j\Big\}. \tag{B.18}$$

Therefore, firm i's expected profit can be written as follows

$$\begin{aligned} E[\pi_i(w_1, w_2)] = E[(p_i - w_i)z_i] &= \int_x \left\{\frac{x}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i - \beta w_j)dF(x) \\ &= \left\{\frac{E[\Omega]}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i - \beta w_j). \end{aligned}$$

So, the firm's optimization problem can be written as

$$\max_{w_i} \left\{\frac{E[\Omega]}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i - \beta w_j) \tag{B.19}$$

$$\begin{aligned} s.t. \quad & \Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1 - \Big(\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_2 \leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\underline{\Omega}, \\ & \Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2 - (\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_1 \leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\underline{\Omega}. \end{aligned} \tag{B.20}$$

The first order condition gives

$$w_{i,c1}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\gamma^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2}E[\Omega]. \tag{B.21}$$

And the second order derivative is negative. Let

$$w_{ub}^1 = \frac{1}{3-2\beta-2\gamma+\beta\gamma}\underline{\Omega}. \tag{B.22}$$

The constraint (B.20) gives

$$w_i \leq w_{ub}^1.$$

Let $E[\pi^*_{wp,c1}]$ denote the optimal expected profit in Case 1. Clearly, if $w^*_{i,c1} < w^1_{ub}$, then $E[\pi^*_{wp,c1}] = E[\pi_{wp,c1}|_{w=w^*_{i,c1}}]$. Otherwise, $E[\pi^*_{wp,c1}] = E[\pi_{wp,c1}|_{w=w^1_{ub}}]$.

Case 2: Medium Wage. The wage is in a medium range such that there exists a threshold $\widehat{\Omega}(\boldsymbol{w})$ beyond which the firms would adopt the supply-depletion price and adopt the profit-maximizing price otherwise.

According to Lemma B.1(i) and (ii), the wage $\boldsymbol{w}$ is set such that

$$\begin{aligned}
\Big(\frac{2}{4-\gamma^2} + \frac{2}{2-\gamma^2}\Big)w_1 + \Big(\frac{\gamma}{4-\gamma^2} - \frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2 &\geq \Big(\frac{2}{2-\gamma^2} - \frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\underline{\Omega},\\
\Big(\frac{2}{4-\gamma^2} + \frac{2}{2-\gamma^2}\Big)w_2 + \Big(\frac{\gamma}{4-\gamma^2} - \frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1 &\geq \Big(\frac{2}{2-\gamma^2} - \frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\underline{\Omega},\\
\Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1 - \Big(\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_2 &\leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\bar{\Omega},\\
\Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2 - (\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_1 &\leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\bar{\Omega}.
\end{aligned}$$

Then the equilibrium prices are

$$p_i^* = \begin{cases} p_i^d = \frac{1}{1-\gamma^2}\Big\{(1+\gamma)x - (1-\beta\gamma)w_i + (\beta-\gamma)w_j\Big\}, & \text{if} \quad x \geq \widehat{\Omega}(\boldsymbol{w}),\\ p_i^m = \frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x + \frac{\gamma}{2}w_j + w_i\Big\}, & \text{if} \quad x < \widehat{\Omega}(\boldsymbol{w}). \end{cases}$$

Therefore, firm i's expected profit can be written

$$\begin{aligned}
E[\pi_i(w_i, w_j)] =& E[(p_i - w_i)z_i]\\
=& \int_{\widehat{\Omega}(\boldsymbol{w})}^{\bar{\Omega}} \left\{\frac{x}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i - \beta w_j)dF(x)\\
&+ \int_{\underline{\Omega}}^{\widehat{\Omega}(\boldsymbol{w})} \left\{\frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x + \frac{\gamma}{2}w_j + w_i\Big\} - w_i\right\}\left(x - (1-\gamma)\frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x + \frac{\gamma}{2}w_j + w_i\Big\}\right)dF(x).
\end{aligned}$$

So, the firm's optimization problem can be written as

$$\begin{aligned}
\max_{w_i} & \int_{\widehat{\Omega}(\boldsymbol{w})}^{\bar{\Omega}} \left\{\frac{x}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i - \beta w_j)dF(x)\\
&+ \int_{\underline{\Omega}}^{\widehat{\Omega}(\boldsymbol{w})} \Big(\frac{x}{2-\gamma} - \frac{2-\gamma^2}{4-\gamma^2}w_i + \frac{\gamma}{4-\gamma^2}w_j\Big)^2 dF(x)\\
s.t. \quad & \Big(\frac{2}{4-\gamma^2} + \frac{2}{2-\gamma^2}\Big)w_1 + \Big(\frac{\gamma}{4-\gamma^2} - \frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2 \geq \Big(\frac{2}{2-\gamma^2} - \frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\underline{\Omega}, \qquad \text{(B.23)}\\
& \Big(\frac{2}{4-\gamma^2} + \frac{2}{2-\gamma^2}\Big)w_2 + \Big(\frac{\gamma}{4-\gamma^2} - \frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1 \geq \Big(\frac{2}{2-\gamma^2} - \frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\underline{\Omega},\\
& \Big(\frac{1-\beta\gamma}{1-\gamma^2} + \frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1 - \Big(\frac{\gamma}{2-\gamma^2} + \frac{\beta-\gamma}{1-\gamma^2}\Big)w_2 \leq \Big(\frac{1}{1-\gamma} - \frac{\gamma+1}{2-\gamma^2}\Big)\bar{\Omega}, \qquad \text{(B.24)}
\end{aligned}$$

$$\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2-(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_1\leq\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)\bar{\Omega}.$$

Let w^*_{i,c_2} denote the solution derived from the first order condition. Also let

$$w^2_{lb}=\frac{1}{3-2\beta-2\gamma+\beta\gamma}\underline{\Omega},\quad w^2_{ub}=\frac{1}{3-2\beta-2\gamma+\beta\gamma}\bar{\Omega}.$$

The constraints (B.23) and (B.24) give

$$w^2_{lb}\leq w_i\leq w^2_{ub}.$$

Let $E[\pi^*_{wp,c2}]$ denote the optimal expected profit in Case 2. Clearly, if $w^2_{lb}\leq w^*_{i,c2}\leq w^2_{ub}$, then $E[\pi^*_{wp,c2}]=\max\{E[\pi_{wp,c2}|_{w=w^*_{i,c2}}],E[\pi_{wp,c2}|_{w=w^2_{lb}}],E[\pi_{wp,c2}|_{w=w^2_{ub}}]\}$. Otherwise, $E[\pi^*_{wp,c2}]=\max\{E[\pi_{wp,c2}|_{w=w^2_{lb}}],E[\pi_{wp,c2}|_{w=w^2_{ub}}]\}$.

The expression of w^*_{i,c_2} is hard to derive with a general distribution for market size Ω. Here, we assume a two-point distribution for Ω and derive the expression of w^*_{i,c_2}, which will be used in the proof of Proposition D.1 in Online Supplement D. We assume that Ω takes the value of Ω_H with probability q and the value of Ω_L with probability $1-q$. So, the firm's optimization problem can be written as

$$\begin{aligned}
&\max_{w_i} q\left\{\frac{\Omega_H}{1-\gamma}+\frac{\beta-\gamma}{1-\gamma^2}w_j-\frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i-\beta w_j)+(1-q)\Big(\frac{\Omega_L}{2-\gamma}-\frac{2-\gamma^2}{4-\gamma^2}w_i+\frac{\gamma}{4-\gamma^2}w_j\Big)^2\\
s.t.\quad &\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_1+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2\geq\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\Omega_L,\\
&\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_2+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1\geq\Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\Omega_L,\\
&\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_1-\Big(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_2\leq\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)\Omega_H,\\
&\Big(\frac{1-\beta\gamma}{1-\gamma^2}+\frac{1+\beta\gamma}{2-\gamma^2}\Big)w_2-(\frac{\gamma}{2-\gamma^2}+\frac{\beta-\gamma}{1-\gamma^2}\Big)w_1\leq\Big(\frac{1}{1-\gamma}-\frac{\gamma+1}{2-\gamma^2}\Big)\Omega_H.
\end{aligned}$$

The first order condition gives

$$w^*_{i,c_2}=\frac{q(2-\gamma)^2(2+\gamma)(1+\gamma)\Omega_H-2(1-q)(2-\gamma^2)(1-\gamma^2)\Omega_L}{q(4+\gamma-3\beta+\beta^2\gamma+\beta\gamma^2-2\gamma^2-2\beta\gamma)(2-\gamma)^2(2+\gamma)-2(1-q)(2-\gamma^2)(1-\gamma^2)(1-\gamma)}. \tag{B.25}$$

And

$$w^2_{lb}=\frac{1}{3-2\beta-2\gamma+\beta\gamma}\Omega_L,\quad w^2_{ub}=\frac{1}{3-2\beta-2\gamma+\beta\gamma}\Omega_H.$$

Case 3: High Wage. The wage is too high such that for any $x\in[\underline{\Omega},\bar{\Omega}]$, both firms adopt the profit-maximizing price at the subsequent stage.

According to Lemma B.1(ii), the wage $\boldsymbol{w}$ is set such that

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_1+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2 \geq \Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\bar{\Omega}, \tag{B.26}$$

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_2+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1 \geq \Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\bar{\Omega}. \tag{B.27}$$

Then the equilibrium prices are

$$p_i^* = p_i^m = \frac{2}{4-\gamma^2}\Big\{(1+\frac{\gamma}{2})x+\frac{\gamma}{2}w_j+w_i\Big\}.$$

Therefore, firm i's expected profit can be written

$$E[\pi_i(w_i,w_j)] = E[(p_i-w_i)z_i] = \int_x\Big(\frac{x}{2-\gamma}-\frac{2-\gamma^2}{4-\gamma^2}w_i+\frac{\gamma}{4-\gamma^2}w_j\Big)^2 dF(x).$$

So, firm i's optimization problem can be written as

$$\max_{w_i}\int_x\Big(\frac{x}{2-\gamma}-\frac{2-\gamma^2}{4-\gamma^2}w_i+\frac{\gamma}{4-\gamma^2}w_j\Big)^2 dF(x)$$

$$s.t.\quad \Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_1+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_2 \geq \Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\bar{\Omega}, \tag{B.28}$$

$$\Big(\frac{2}{4-\gamma^2}+\frac{2}{2-\gamma^2}\Big)w_2+\Big(\frac{\gamma}{4-\gamma^2}-\frac{2}{2-\gamma^2}(\frac{\gamma}{2}+\beta)\Big)w_1 \geq \Big(\frac{2}{2-\gamma^2}-\frac{2}{4-\gamma^2}\Big)(1+\frac{\gamma}{2})\bar{\Omega},$$

$$\frac{\underline{\Omega}}{2-\gamma}-\frac{2-\gamma^2}{4-\gamma^2}w_i+\frac{\gamma}{4-\gamma^2}w_j \geq 0, \tag{B.29}$$

where the last constraint is derived from $p_i \geq w_i$ for any $x \in [\underline{\Omega}, \bar{\Omega}]$.

The first order condition gives

$$w_{i,c3}^* = \frac{2+\gamma}{2-\gamma^2-\gamma}E[\Omega]. \tag{B.30}$$

Let

$$w_{lb}^3 = \frac{1}{3-2\beta-2\gamma+\beta\gamma}\bar{\Omega}, \quad w_{ub}^3 = \frac{2+\gamma}{2-\gamma^2-\gamma}\underline{\Omega}. \tag{B.31}$$

The constraints (B.28) and (B.29) give

$$w_{lb}^3 \leq w_i \leq w_{ub}^3.$$

Let $E[\pi_{wp,c3}^*]$ denote the optimal expected profit in Case 3. Clearly, $w_{i,c3}^* \geq w_{ub}^3$. Then, the convexity of the profit function implies that

$$E[\pi_{wp,c3}^*] = E[\pi_{wp,c3}|_{w=w_{lb}^3}]. \tag{B.32}$$

Summary: The expected equilibrium profit in wage precommitment competition is

$$E[\pi_{wp}^*] = \max\Big\{E[\pi_{wp,c1}^*], E[\pi_{wp,c2}^*], E[\pi_{wp,c3}^*]\Big\}.$$

C. Price Precommitment Competition under Market Size Uncertainty

This section assumes that Ω is a random variable distributed on $[\underline{\Omega}, \bar{\Omega}]$. We start with the analysis of the subgame equilibrium conditional on a fixed price in the first stage and the realization of market size in Section C.1, and then derive the equilibrium of price precommitment competition in Section C.2.

C.1. Subgame Equilibrium in the Second Stage

The following lemma characterizes the outcome of the wage game in the second stage for any fixed precommitted price and realized market size.

LEMMA C.1. *For any fixed $\boldsymbol{p}$ and x, the subgame equilibrium wages are given by*

(i) If

$$(6-2\beta^2)p_1 - (4\gamma - \beta^2\gamma + \beta)p_2 < (4-\beta^2)x, \tag{C.1}$$

$$(6-2\beta^2)p_2 - (4\gamma - \beta^2\gamma + \beta)p_1 < (4-\beta^2)x, \tag{C.2}$$

the equilibrium prices are

$$w_1^* = w_1^m = \frac{2p_1 + \beta p_2}{4-\beta^2}, \tag{C.3}$$

$$w_2^* = w_2^m = \frac{2p_2 + \beta p_1}{4-\beta^2}. \tag{C.4}$$

(ii) If

$$(3-2\beta^2-\beta\gamma)p_1 - (2\gamma - \beta - \beta^2\gamma)p_2 \geq (2+\beta-\beta^2)x,$$
$$(3-2\beta^2-\beta\gamma)p_2 - (2\gamma - \beta - \beta^2\gamma)p_1 \geq (2+\beta-\beta^2)x,$$

the equilibrium prices are

$$w_1^* = w_1^d = \frac{1}{1-\beta^2}\Big\{(1+\beta)x - (1-\beta\gamma)p_1 + (\gamma-\beta)p_2\Big\},$$
$$w_2^* = w_2^d = \frac{1}{1-\beta^2}\Big\{(1+\beta)x - (1-\beta\gamma)p_2 + (\gamma-\beta)p_1\Big\}.$$

(iii) If

$$(3-2\beta^2-\beta\gamma)p_1 - (2\gamma - \beta - \beta^2\gamma)p_2 < (2+\beta-\beta^2)x,$$
$$(6-2\beta^2)p_2 - (4\gamma - \beta^2\gamma + \beta)p_1 \geq (4-\beta^2)x,$$

the equilibrium prices are

$$w_1^* = \frac{1}{2-\beta^2}\Big\{\beta x + (1+\beta\gamma)p_1 - \beta p_2\Big\},$$
$$w_2^* = \frac{1}{2-\beta^2}\Big\{2x + (2\gamma+\beta)p_1 - 2p_2\Big\}.$$

(iv) If

$$(6-2\beta^2)p_1-(4\gamma-\beta^2\gamma+\beta)p_2 \geq (4-\beta^2)x,$$
$$(3-2\beta^2-\beta\gamma)p_2-(2\gamma-\beta-\beta^2\gamma)p_1 < (2+\beta-\beta^2)x,$$

the equilibrium prices are

$$w_1^* = \frac{1}{2-\beta^2}\Big\{2x+(2\gamma+\beta)p_2-2p_1\Big\},$$
$$w_2^* = \frac{1}{2-\beta^2}\Big\{\beta x+(1+\beta\gamma)p_2-\beta p_1\Big\}.$$

Proof of Lemma C.1. For any fixed $\boldsymbol{p}$ and x, the demand quantity for each firm is fixed. On the one hand, the firm could set its wage such that the supply quantity is smaller than the demand, that is, the firm faces such an optimization problem:

$$\max_{w_i}(p_i-w_i)(w_i-\beta w_j)$$
$$s.t. \quad w_i-\beta w_j \leq x-p_i+\gamma p_j.$$

Maximizing the profit function gives that if $p_i \leq \frac{2x+2\gamma p_j+\beta w_j}{3}$, then $w_i^* = \frac{p_i+\beta w_j}{2}$. Otherwise, $w_i^* = x-p_i+\gamma p_j+\beta w_j$. On the other hand, the firm could set its wage such that the supply quantity is greater than the demand, that is, the firm faces such an optimization problem:

$$\max_{w_i}(p_i-w_i)(x-p_i+\gamma p_j)$$
$$s.t. \quad w_i-\beta w_j \geq x-p_i+\gamma p_j.$$

Clearly, $w_i^* = x-p_i+\gamma p_j+\beta w_j$. Combining the above two scenarios yields that

$$w_i^* = \begin{cases} \frac{p_i+\beta w_j}{2}, & \text{if} \quad p_i \leq \frac{2x+2\gamma p_j+\beta w_j}{3}, \\ x-p_i+\gamma p_j+\beta w_j, & \text{otherwise.} \end{cases}$$

We examine Case (i) first. We obtain if

$$p_1 \leq \frac{2x+2\gamma p_2+\beta w_2}{3}, \quad p_2 \leq \frac{2x+2\gamma p_1+\beta w_1}{3}, \tag{C.5}$$

then

$$w_1^* = \frac{p_1+\beta w_2}{2}, \quad w_2^* = \frac{p_2+\beta w_1}{2}.$$

Solving the above set of equations yields that

$$w_1^* = \frac{2p_1+\beta p_2}{4-\beta^2}, \quad w_2^* = \frac{2p_2+\beta p_1}{4-\beta^2}.$$

Putting w_1^* and w_2^* back into (C.5) gives (C.1) and (C.2).

Continuing in this fashion, we establish Cases (ii)-(iv). $\square$

Before deriving the equilibrium of price precommitment competition, we first derive the equilibrium of price precommitment competition without demand uncertainty based on Lemma C.1, which can help us understand the scenario when the demand variance is sufficiently small. The following lemma implies that without demand uncertainty, the equilibrium prices are chosen in the first stage such that there will not be any excess quantity in the second-stage wage game.

LEMMA C.2. *Suppose there is no market size uncertainty. In price precommitment competition, the resulting equilibrium wages and prices are such that the supply is equal to the demand.*

The proof of Lemma C.2 is similar to that of Lemma B.2 and thus omitted.

C.2. Deriving Equilibrium Price in the First Stage

The previous section shows that for any fixed $\boldsymbol{p}$, when the market size is realized, the optimal wage takes either the demand-depletion wage or profit-maximizing wage. To simplify the analysis, we assume the two firms take symmetric actions. We have three cases with respect to the value of $\boldsymbol{p}$, detailed below. Suppose the realized market size is x.

Case 1: High Price. The price is too high such that for any $x \in [\underline{\Omega}, \bar{\Omega}]$, both firms adopt the demand-depletion wage at the subsequent stage.

According to Lemma C.1(ii), the price $\boldsymbol{p}$ is set such that

$$(3-2\beta^2-\beta\gamma)p_1-(2\gamma-\beta-\beta^2\gamma)p_2 \geq (2+\beta-\beta^2)\bar{\Omega}, \tag{C.6}$$

$$(3-2\beta^2-\beta\gamma)p_2-(2\gamma-\beta-\beta^2\gamma)p_1 \geq (2+\beta-\beta^2)\bar{\Omega}, \tag{C.7}$$

Then the equilibrium wages are

$$w_i^* = w_i^d = \frac{1}{1-\beta^2}\Big\{(1+\beta)x-(1-\beta\gamma)p_i+(\gamma-\beta)p_j\Big\}. \tag{C.8}$$

Therefore, firm i's expected profit can be written as follows

$$\begin{aligned}
&E[\pi_i(p_i,p_j)] = E[(p_i-w_i)z_i]\\
=&\int_x \left\{p_i-\frac{1}{1-\beta^2}\Big\{(1+\beta)x-(1-\beta\gamma)p_i+(\gamma-\beta)p_j\Big\}\right\}(x-p_i+\gamma p_j)dF(x)\\
=&\int_x \left\{\frac{1}{1-\beta^2}\Big\{-(1+\beta)x+(2-\beta^2-\beta\gamma)p_i-(\gamma-\beta)p_j\Big\}\right\}(x-p_i+\gamma p_j)dF(x).
\end{aligned}$$

So, the firm's optimization problem can be written as

$$\max_{p_i}\int_x \left\{\frac{1}{1-\beta^2}\Big\{-(1+\beta)x+(2-\beta^2-\beta\gamma)p_i-(\gamma-\beta)p_j\Big\}\right\}(x-p_i+\gamma p_j)dF(x) \tag{C.9}$$

$$s.t. \quad (3-2\beta^2-\beta\gamma)p_1-(2\gamma-\beta-\beta^2\gamma)p_2 \geq (2+\beta-\beta^2)\bar{\Omega}, \tag{C.10}$$

$$(3-2\beta^2-\beta\gamma)p_2-(2\gamma-\beta-\beta^2\gamma)p_1 \geq (2+\beta-\beta^2)\bar{\Omega}.$$

The first order condition gives

$$p^*_{i,c_1} = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],$$

and the second order derivative is negative. Let

$$p^1_{lb} = \frac{2+\beta-\beta^2}{3-2\gamma+\beta-\beta\gamma-2\beta^2+\beta^2\gamma}\bar{\Omega}.$$

The constraint (C.10) gives that

$$p_i \geq p^1_{lb}.$$

Let $E[\pi^*_{pw,c1}]$ denote the optimal expected profit in Case 1. Clearly, if $p^*_{i,c1} \geq p^1_{lb}$, then $E[\pi^*_{pw,c1}] = E[\pi_{pw,c1}|_{p=p^*_{i,c1}}]$. Otherwise, $E[\pi^*_{pw,c1}] = E[\pi_{pw,c1}|_{p=p^1_{lb}}]$.

Case 2: Medium Price. The price is in a medium range such that there exists a threshold $\widehat{\Omega}(\boldsymbol{p})$ beyond which the firms would adopt the profit-maximizing wage and adopt the demand-depletion wage otherwise.

According to Lemma C.1(i) and (ii), the price $\boldsymbol{p}$ is set such that

$$\begin{aligned}
(6-2\beta^2)p_1-(4\gamma-\beta^2\gamma+\beta)p_2 &< (4-\beta^2)\bar{\Omega},\\
(6-2\beta^2)p_2-(4\gamma-\beta^2\gamma+\beta)p_1 &< (4-\beta^2)\bar{\Omega},\\
(3-2\beta^2-\beta\gamma)p_1-(2\gamma-\beta-\beta^2\gamma)p_2 &\geq (2+\beta-\beta^2)\underline{\Omega},\\
(3-2\beta^2-\beta\gamma)p_2-(2\gamma-\beta-\beta^2\gamma)p_1 &\geq (2+\beta-\beta^2)\underline{\Omega}.
\end{aligned}$$

Then the equilibrium wages are

$$w^*_i = \begin{cases} w^d_i = \frac{1}{1-\beta^2}\Big\{(1+\beta)x-(1-\beta\gamma)p_i+(\beta-\gamma)p_j\Big\}, & \text{if} \quad x \leq \widehat{\Omega}(\boldsymbol{p}),\\ w^m_i = \frac{2p_i+\beta p_j}{4-\beta^2}, & \text{otherwise.}\end{cases}$$

Therefore, firm i's expected profit can be written

$$\begin{aligned}
E[\pi_i(p_i,p_j)] =& E[(p_i-w_i)z_i]\\
=& \int_{\underline{\Omega}}^{\widehat{\Omega}(\boldsymbol{p})} \left\{\frac{1}{1-\beta^2}\Big\{-(1+\beta)x+(2-\beta^2-\beta\gamma)p_i-(\gamma-\beta)p_j\Big\}\right\}(x-p_i+\gamma p_j)dF(x)\\
&+\int_{\widehat{\Omega}(\boldsymbol{p})}^{\bar{\Omega}}\Big(\frac{(2-\beta^2)p_i-\beta p_j}{4-\beta^2}\Big)^2 dF(x).
\end{aligned}$$

So, the firm's optimization problem can be written as

$$\max_{p_i}\int_{\underline{\Omega}}^{\widehat{\Omega}(\boldsymbol{p})} \left\{\frac{1}{1-\beta^2}\Big\{-(1+\beta)x+(2-\beta^2-\beta\gamma)p_i-(\gamma-\beta)p_j\Big\}\right\}(x-p_i+\gamma p_j)dF(x) \quad \text{(C.11)}$$

$$
\begin{aligned}
&\quad + \int_{\widehat{\Omega}(\boldsymbol{p})}^{\bar{\Omega}} \Big(\frac{(2-\beta^2)p_i - \beta p_j}{4-\beta^2}\Big)^2 dF(x) \\
s.t. \quad & (6-2\beta^2)p_1 - (4\gamma - \beta^2\gamma + \beta)p_2 < (4-\beta^2)\bar{\Omega}, \qquad \text{(C.12)} \\
& (6-2\beta^2)p_2 - (4\gamma - \beta^2\gamma + \beta)p_1 < (4-\beta^2)\bar{\Omega}, \\
& (3-2\beta^2-\beta\gamma)p_1 - (2\gamma - \beta - \beta^2\gamma)p_2 \geq (2+\beta-\beta^2)\underline{\Omega}, \qquad \text{(C.13)} \\
& (3-2\beta^2-\beta\gamma)p_2 - (2\gamma - \beta - \beta^2\gamma)p_1 \geq (2+\beta-\beta^2)\underline{\Omega}.
\end{aligned}
$$

Let p^*_{i,c_2} denote the solution derived from the first order condition. Also let

$$
p^2_{lb} = \frac{2+\beta-\beta^2}{3-2\gamma+\beta-\beta\gamma-2\beta^2+\beta^2\gamma}\underline{\Omega}, \quad p^2_{ub} = \frac{4-\beta^2}{6-\beta-4\gamma-2\beta^2+\beta^2\gamma}\bar{\Omega}.
$$

The constraints (C.12) and (C.13) give

$$
p^2_{lb} \leq p_i \leq p^2_{ub}.
$$

Let $E[\pi^*_{pw,c2}]$ denote the optimal expected profit in Case 2. Clearly, if $p^2_{lb} \leq p^*_{i,c2} \leq p^2_{ub}$, then $E[\pi^*_{pw,c2}] = \max\{E[\pi_{pw,c2}|_{p=p^*_{i,c2}}], E[\pi_{pw,c2}|_{p=p^2_{lb}}], E[\pi_{pw,c2}|_{p=p^2_{ub}}]\}$. Otherwise, $E[\pi^*_{pw,c2}] = \max\{E[\pi_{pw,c2}|_{p=p^2_{lb}}], E[\pi_{pw,c2}|_{p=p^2_{ub}}]\}$.

Case 3: Low Price. The price is too low such that for any $x \in [\underline{\Omega}, \bar{\Omega}]$, both firms adopt the profit-maximizing wage at the subsequent stage.

According to Lemma C.1(i), the price $\boldsymbol{p}$ is set such that

$$
\begin{aligned}
&(6-2\beta^2)p_1 - (4\gamma - \beta^2\gamma + \beta)p_2 \leq (4-\beta^2)\underline{\Omega}, \\
&(6-2\beta^2)p_2 - (4\gamma - \beta^2\gamma + \beta)p_1 \leq (4-\beta^2)\underline{\Omega}.
\end{aligned}
$$

Then the equilibrium wages are

$$
w^*_i = w^m_i = \frac{2p_i + \beta p_j}{4-\beta^2}.
$$

Therefore, firm i's expected profit can be written

$$
E[\pi_i(p_i,p_j)] = E[(p_i - w_i)z_i] = \int_x \Big(\frac{(2-\beta^2)p_i - \beta p_j}{4-\beta^2}\Big)^2 dF(x) = \Big(\frac{(2-\beta^2)p_i - \beta p_j}{4-\beta^2}\Big)^2.
$$

So, firm i's optimization problem can be written as

$$
\begin{aligned}
&\max_{p_i} \Big(\frac{(2-\beta^2)p_i - \beta p_j}{4-\beta^2}\Big)^2 \\
s.t. \quad & (6-2\beta^2)p_1 - (4\gamma - \beta^2\gamma + \beta)p_2 \leq (4-\beta^2)\underline{\Omega}, \qquad \text{(C.14)} \\
& (6-2\beta^2)p_2 - (4\gamma - \beta^2\gamma + \beta)p_1 \leq (4-\beta^2)\underline{\Omega}.
\end{aligned}
$$

For the sake of tractability, we consider symmetric actions. Letting $p_i = p_j$, then (C.14) gives that

$$p_i \le \frac{4-\beta^2}{6-\beta-4\gamma-2\beta^2+\beta^2\gamma}\underline{\Omega}.$$

Observe from the objective function that the optimal price is

$$p_i^* = \frac{4-\beta^2}{6-\beta-4\gamma-2\beta^2+\beta^2\gamma}\underline{\Omega}.$$

Putting $p_i^* = p_j^* = \frac{4-\beta^2}{6-\beta-4\gamma-2\beta^2+\beta^2\gamma}\underline{\Omega}$ into the profit function yields

$$E[\pi_{pw,c3}^*] = \frac{(1-\beta)^2(2-\beta)^2}{(6-\beta-4\gamma-2\beta^2+\beta^2\gamma)^2}\underline{\Omega}^2. \tag{C.15}$$

Summary: The expected equilibrium profit in price precommitment competition is

$$E[\pi_{pw}^*] = \max\left\{E[\pi_{pw,c1}^*], E[\pi_{pw,c2}^*], E[\pi_{pw,c3}^*]\right\}.$$

D. The Remaining Proof of Lemmas and Propositions in the Main Body

Before proving Lemma 4, we first analyze the subgame of the second stage conditional on the commission rate decisions of the first stage and the realization of the market size. We obtain the following lemma.

LEMMA D.1. *For any realized market size x and fixed commission rate $\boldsymbol{\alpha}$, it is optimal for platforms to set the equilibrium price such that the demand equals to the supply quantity.*

Proof of Lemma D.1. For any realized market size x and fixed commission rate $\boldsymbol{\alpha}$, firm 1's profit is

$$\pi_1(\boldsymbol{\alpha}) = (p_1-\alpha_1 p_1)\min\{x-p_1+\gamma p_2, \alpha_1 p_1-\beta\alpha_2 p_2\} = (1-\alpha_1)p_1\min\{x-p_1+\gamma p_2, \alpha_1 p_1-\beta\alpha_2 p_2\}$$

$$= \begin{cases} (1-\alpha_1)p_1(x-p_1+\gamma p_2), & \text{if } p_1 \ge \frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}, \\ (1-\alpha_1)p_1(\alpha_1 p_1-\beta\alpha_2 p_2), & \text{otherwise.} \end{cases}$$

We first consider the following optimization problem.

$$\max_{p_1} \quad (1-\alpha_1)p_1(x-p_1+\gamma p_2)$$
$$s.t. \quad p_1 \ge \frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}.$$

The first order condition (without constraint) gives $\frac{x+\gamma p_2}{2}$, which is smaller than $\frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}$. Therefore, by the concavity of the profit function, we have $p_1^* = \frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}$.

Then, we consider the other optimization problem.

$$\max_{p_1} \quad (1-\alpha_1)p_1(\alpha_1 p_1 - \beta\alpha_2 p_2)$$
$$s.t. \quad p_1 \le \frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}.$$

The first order condition gives $\frac{\beta\alpha_2 p_2}{2\alpha_1}$. Moreover, the second order derivative is positive, indicating that $\frac{\beta\alpha_2 p_2}{2\alpha_1}$ is the solution to minimize the objective function instead of maximizing. Note that $\pi_1 = 0$ when $p_1 = 0$, so it is not hard to see in this case the optimal solution $p_1^* = \frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}$.

To summarize, no matter in which case, the optimal price is $\frac{x+\gamma p_2+\beta\alpha_2 p_2}{1+\alpha_1}$, implying that the demand equals to the supply. □

Proof of Lemma 4. For any realized market size x, Lemma D.1 implies that

$$x - p_1^* + \gamma p_2 = \alpha_1 p_1^* - \beta\alpha_2 p_2, \quad x - p_2^* + \gamma p_1 = \alpha_2 p_2^* - \beta\alpha_1 p_1.$$

Solving this set of equations yields firm 1's optimal pricing decision for any fixed $\boldsymbol{\alpha}$,

$$p_1^* = \frac{1+\alpha_2+\gamma+\beta\alpha_2}{(1+\alpha_1)(1+\alpha_2)-(\gamma+\beta\alpha_1)(\gamma+\beta\alpha_2)}x. \tag{D.1}$$

Then firm 1's expected profit function can be written as follows,

$$E[\pi_1(\boldsymbol{\alpha})] = (1-\alpha_1)\frac{(1+\alpha_2+\gamma+\beta\alpha_2)\left\{\alpha_1(1+\alpha_2+\gamma+\beta\alpha_2)-\beta\alpha_2(1+\alpha_1+\gamma+\beta\alpha_1)\right\}}{\left\{(1+\alpha_1)(1+\alpha_2)-(\gamma+\beta\alpha_1)(\gamma+\beta\alpha_2)\right\}^2}E[\Omega^2]. \tag{D.2}$$

Maximizing the profit gives firm 1's best-response function,

$$\alpha_1^*(\alpha_2) = \frac{(1+\alpha_2+\gamma+\beta\alpha_2-\beta^2\alpha_2+\beta\gamma\alpha_2)(1+\alpha_2-\gamma^2-\gamma\beta\alpha_2)+2(1+\alpha_2-\beta\gamma-\beta^2\alpha_2)(\beta\alpha_2+\beta\gamma\alpha_2)}{2(1+\alpha_2+\gamma-\beta^2\alpha_2)(1+\alpha_2-\gamma^2-\gamma\beta\alpha_2)+(1+\alpha_2-\beta\gamma-\beta^2\alpha_2)(1+\alpha_2+\gamma+\beta\alpha_2-\beta^2\alpha_2+\beta\gamma\alpha_2)}.$$

Similarly, we obtain firm 2's best-response function $\alpha_2^*(\alpha_1)$. Solving the set of equations yields the equilibrium commission rate α^*. Putting α^* back to (D.1) and (D.2) gives the equilibrium price p_C^* and expected equilibrium profit $E[\pi_C^*]$. □

Proof of Lemma 5. For any price vector $\boldsymbol{p} \ge \mathbf{0}$, the demand function $d_i(\boldsymbol{p})$ is implicitly defined by the solution to the following utility maximization problem of a representative consumer:

$$\max_{d_i,d_j\ge 0} \frac{E[\Omega]}{1-\gamma}(d_1+d_2) - \frac{1}{2}\left(\frac{d_1^2}{1-\gamma^2} + 2\frac{\gamma}{1-\gamma^2}d_1 d_2 + \frac{d_2^2}{1-\gamma^2}\right) - p_1 d_1 - p_2 d_2. \tag{D.3}$$

Taking derivative to d_1 and d_2 yields $\frac{E[\Omega]}{1-\gamma} - \frac{d_1}{1-\gamma^2} - \frac{\gamma d_2}{1-\gamma^2} - p_1 = 0$ and $\frac{E[\Omega]}{1-\gamma} - \frac{d_2}{1-\gamma^2} - \frac{\gamma d_1}{1-\gamma^2} - p_2 = 0$. Solving the set of equations gives

$$d_1 = E[\Omega] - p_1 + \gamma p_2, \quad d_2 = E[\Omega] - p_2 + \gamma p_1. \tag{D.4}$$

which is exactly the same as our demand functions.

By (D.4), we obtain $p_1 = \frac{1}{1-\gamma^2}\{(1+\gamma)E[\Omega] - \gamma d_2 - d_1\}$ and $p_2 = \frac{1}{1-\gamma^2}\{(1+\gamma)E[\Omega] - \gamma d_1 - d_2\}$. Putting p_1 and p_2 back to (D.3) and simplifying yields consumer's surplus $\frac{1}{2}\frac{d_1^2}{1-\gamma^2} + \frac{1}{2}\frac{d_2^2}{1-\gamma^2} + \frac{\gamma}{1-\gamma^2}d_1 d_2$, which is increasing in both d_1 and d_2.

Similarly, for any wage vector $\boldsymbol{w} \geq \boldsymbol{0}$, the supply function $s_i(\boldsymbol{w})$ is implicitly defined by the solution to the following utility maximization problem of a representative driver (service provider):

$$\max_{s_i, s_j \geq 0} w_1 s_1 + w_2 s_2 - \frac{1}{2}\Big(\frac{s_1^2}{1-\beta^2} + \frac{2\beta}{1-\beta^2}s_1 s_2 + \frac{s_2^2}{1-\beta^2}\Big). \tag{D.5}$$

Taking derivative to s_1 and s_2 yields $\frac{s_1}{1-\beta^2} + \frac{\beta s_2}{1-\beta^2} - w_1 = 0$ and $\frac{s_2}{1-\beta^2} + \frac{\beta s_1}{1-\beta^2} - w_2 = 0$. Solving the set of equations gives

$$s_1 = w_1 - \beta w_2, \quad s_2 = w_2 - \beta w_1. \tag{D.6}$$

which is exactly the same as our supply functions.

By (D.6), we obtain $w_1 = \frac{1}{1-\beta^2}s_1 + \frac{\beta}{1-\beta^2}s_2$ and $w_2 = \frac{1}{1-\beta^2}s_2 + \frac{\beta}{1-\beta^2}s_1$. Putting w_1 and w_2 back to (D.5) and simplifying yields driver's surplus $\frac{1}{2}\frac{s_1^2}{1-\beta^2} + \frac{1}{2}\frac{s_2^2}{1-\beta^2} + \frac{\beta}{1-\beta^2}s_1 s_2$, which is increasing in both s_1 and s_2.

Next, we examine the social welfare, which is the summation of consumer surplus, driver surplus, and firm's profit. Note that in equilibrium, supply equals to demand. Hence, the profit function can be written as $\pi_i(\boldsymbol{w}, \boldsymbol{p}) = (p_i - w_i)z_i$. Thus, social welfare can be written as

$$\begin{aligned}
&\frac{E[\Omega]}{1-\gamma}(z_1+z_2) - \frac{1}{2}\Big(\frac{z_1^2}{1-\gamma^2} + 2\frac{\gamma}{1-\gamma^2}z_1 z_2 + \frac{z_2^2}{1-\gamma^2}\Big) - p_1 z_1 - p_2 z_2 \\
&+ w_1 z_1 + w_2 z_2 - \frac{1}{2}\Big(\frac{z_1^2}{1-\beta^2} + \frac{2\beta}{1-\beta^2}z_1 z_2 + \frac{z_2^2}{1-\beta^2}\Big) + (p_1 - w_1)z_1 + (p_2 - w_2)z_2 \\
=&\frac{E[\Omega]}{1-\gamma}(z_1+z_2) - \frac{1}{2}\Big(\frac{z_1^2}{1-\gamma^2} + 2\frac{\gamma}{1-\gamma^2}z_1 z_2 + \frac{z_2^2}{1-\gamma^2}\Big) - \frac{1}{2}\Big(\frac{z_1^2}{1-\beta^2} + \frac{2\beta}{1-\beta^2}z_1 z_2 + \frac{z_2^2}{1-\beta^2}\Big).
\end{aligned}$$

Taking derivative with respect to z_1 yields that

$$\begin{aligned}
&\frac{E[\Omega]}{1-\gamma} - \frac{z_1}{1-\gamma^2} - \frac{\gamma z_2}{1-\gamma^2} - \frac{z_1}{1-\beta^2} - \frac{\beta z_2}{1-\beta^2} \\
=&\frac{(1+\gamma)E[\Omega] - z_1 - \gamma z_2}{1-\gamma^2} - \frac{z_1 + \beta z_2}{1-\beta^2} \\
=&\frac{(1+\gamma)E[\Omega] - (E[\Omega] - p_1 + \gamma p_2) - \gamma(E[\Omega] - p_2 + \gamma p_1)}{1-\gamma^2} - \frac{z_1 + \beta z_2}{1-\beta^2} && \text{[by (D.4)]} \\
=&p_1 - \frac{z_1 + \beta z_2}{1-\beta^2} = p_1 - \frac{w_1 - \beta w_2 + \beta(w_2 - \beta w_1)}{1-\beta^2} && \text{[by (D.6)]} \\
=&p_1 - w_1 \geq 0.
\end{aligned}$$

Similarly, we can show the derivative with respect to z_2 is also non-negative. Hence, the social welfare also increases in matching quantity z_1 and z_2. □

Proof of Proposition 1. Note that if for any realized market size x, the comparison between p_X^* and p_Y^* (w_X^* and w_Y^*) depends on comparing γ and β, then it follows immediately that the comparison between $E[p_X^*]$ and $E[p_Y^*]$ ($E[w_X^*]$ and $E[w_Y^*]$) also depends on comparing γ and β.

Part 1: We first compare mode wp with mode P. For any realized market size x, as $\text{Var}(\Omega) \to 0$,

$$p_{wp}^* = \frac{(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)x-(1-\beta)(1+\gamma)E[\Omega]}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}$$
$$\to \frac{3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}E[\Omega],$$

and

$$p_P^* \to \frac{3-2\beta}{4-3\beta-3\gamma+2\beta\gamma}E[\Omega].$$

Therefore, $p_{wp}^* - p_P^*$ reduces to

$$(3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)(4-3\beta-3\gamma+2\beta\gamma)-(3-2\beta)(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)$$
$$=\gamma^2+\beta(-\gamma-2\gamma^2)+\beta^2(2\gamma+\gamma^2)-\beta^3\gamma=(\gamma-\beta)(1-\beta)^2\gamma.$$

Note also that

$$w_{wp}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}E[\Omega], \quad w_P^* \to \frac{1}{4-3\beta-3\gamma+2\beta\gamma}E[\Omega],$$

as $\text{Var}(\Omega) \to 0$. Therefore, $w_{wp}^* - w_P^*$ reduces to

$$(1+\gamma)(4-3\beta-3\gamma+2\beta\gamma)-(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)=-\gamma(\gamma-\beta)(1-\beta).$$

Finally, as $\text{Var}(\Omega) \to 0$,

$$E[\pi_P^*] \to \frac{2(1-\beta)^2}{(4-3\beta-3\gamma+2\beta\gamma)^2}(E[\Omega])^2.$$

One can verify that the comparison for the profit also reduce to comparing the competition intensities of the two sides.

Part 2: We then compare mode pw with mode P. For any realized market size x,

$$p_{pw}^* = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega], \quad p_P^* \to \frac{3-2\beta}{4-3\beta-3\gamma+2\beta\gamma}E[\Omega],$$

as $\text{Var}(\Omega) \to 0$. Therefore, $p_{pw}^* - p_P^*$ reduces to

$$(3+\beta-\beta^2-\beta\gamma)(4-3\beta-3\gamma+2\beta\gamma)-(3-2\beta)(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)$$
$$=-\beta\gamma+\beta^2+\beta^2\gamma-\beta^3=-\beta(\gamma-\beta)(1-\beta).$$

Note also that for any realized market size x, as $\mathrm{Var}(\Omega) \to 0$,

$$\begin{aligned} w^*_{pw} &= \frac{(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)x-(1-\gamma)(3+\beta-\beta^2-\beta\gamma)E[\Omega]}{(1-\beta)(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)} \\ &\to \frac{1+\beta}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega], \end{aligned}$$

and

$$w^*_P \to \frac{1}{4-3\beta-3\gamma+2\beta\gamma}E[\Omega].$$

Therefore, $w^*_{pw} - w^*_P$ reduces to

$$(1+\beta)(4-3\beta-3\gamma+2\beta\gamma)-(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2) = \beta(\gamma-\beta)(1-\gamma).$$

Finally, as $\mathrm{Var}(\Omega) \to 0$,

$$\begin{aligned} E[\pi^*_{pw}] &\to \frac{(1-\beta^2)(2-\beta^2-\beta\gamma)}{(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)^2}(E[\Omega])^2, \\ E[\pi^*_P] &\to \frac{2(1-\beta)^2}{(4-3\beta-3\gamma+2\beta\gamma)^2}(E[\Omega])^2. \end{aligned}$$

One can verify that the comparison for the profit also reduce to comparing the competition intensities of the two sides. □

Lemma D.2 is used in the main body for the discussion immediately following Proposition 2.

LEMMA D.2.

(i) If $\beta = 0$, then $\lim_{Var(\Omega)\to 0} p^*_P - p^*_{pw} = 0$.

(ii) If $\gamma = 0$, then $\lim_{Var(\Omega)\to 0} p^*_P - p^*_{wp} = 0$.

Proof of Lemma D.2. When $\beta = 0$, one can verify that

$$\lim_{Var(\Omega)\to 0} p^*_P - p^*_{pw} = \lim_{Var(\Omega)\to 0} \frac{3}{4-3\gamma}E[\Omega] - \frac{3}{4-3\gamma}E[\Omega] = 0.$$

When $\gamma = 0$, one can verify that

$$\lim_{Var(\Omega)\to 0} p^*_P - p^*_{pw} = \lim_{Var(\Omega)\to 0} \frac{3-2\beta}{4-3\beta}E[\Omega] - \frac{3-2\beta}{4-3\beta}E[\Omega] = 0.$$

This completes the proof. □

Lemma D.2 says that when the demand variance is sufficiently small, in the absence of competition in supply (resp., demand) side, simultaneous price and wage competition is equivalent to the price precommitment competition (resp., wage precommitment competition). This result is intuitive. For instance, in the absence of supply side competition (i.e., $\beta = 0$), the supply quantity is determined by each firm itself, so it makes no difference in simultaneous price and wage competition for the firm to decide the wage either together with the price or later. Therefore, simultaneous price and wage competition can be understood as a price competition in the first stage and followed by the wage determined automatically to match supply with the demand, which is equivalent to the prescribed price precommitment competition.

Proof of Proposition 2. Part (a): Note that if for any realized market size x, the comparison between p_X^* and p_Y^* (w_X^* and w_Y^*) depends on comparing γ and β, then it follows immediately that the comparison between $E[p_X^*]$ and $E[p_Y^*]$ ($E[w_X^*]$ and $E[w_Y^*]$) also depends on comparing γ and β.

We first compare mode wp with mode C. Recall that for any realized market size x, as $\mathrm{Var}(\Omega) \to 0$,

$$\begin{aligned} p_{wp}^* &= \frac{(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)x-(1-\beta)(1+\gamma)E[\Omega]}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)} \\ &\to \frac{3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}E[\Omega], \end{aligned}$$

and

$$p_C^* \to \frac{1}{1-\gamma+(1-\beta)\alpha^*}E[\Omega],$$

so

$$p_{wp}^* - p_C^* = \frac{3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}E[\Omega] - \frac{1}{1-\gamma+(1-\beta)\alpha^*}E[\Omega].$$

One can verify that $p_{wp}^* \geq p_C^*$ if $\gamma \geq \beta$ and $p_{wp}^* < p_C^*$ otherwise. The comparisons for the wage and profit also reduce to comparing the competition intensities of the two sides.

We then compare mode pw with mode C. Recall that for any realized market size x,

$$p_{pw}^* = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],$$

and

$$p_C^* \to \frac{1}{1-\gamma+(1-\beta)\alpha^*}E[\Omega],$$

as $\mathrm{Var}(\Omega) \to 0$. So

$$p_{pw}^* - p_C^* = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega] - \frac{1}{1-\gamma+(1-\beta)\alpha^*}E[\Omega].$$

One can verify that $p_{pw}^* \leq p_C^*$ if $\gamma \geq \beta$ and $p_{wp}^* > p_C^*$ otherwise. The comparisons for the wage and profit also reduce to comparing the competition intensities of the two sides.

Part (b): We compare mode P with model C. Recall that for any realized market size x,

$$p_P^* = \frac{3-2\beta}{4-3\beta-3\gamma+2\beta\gamma}x, \quad p_C^* = \frac{1}{1-\gamma+(1-\beta)\alpha^*}x,$$

so

$$p_P^* - p_C^* = \frac{3-2\beta}{4-3\beta-3\gamma+2\beta\gamma}x - \frac{1}{1-\gamma+(1-\beta)\alpha^*}x.$$

One can verify that $p_P^* \le p_C^*$ if $\gamma \ge \beta$ or γ is sufficiently small, and $p_P^* > p_C^*$ if $\gamma \le \beta$ and γ is sufficiently close to β. Similarly, the comparisons for the wage and profit also depend on the competition intensities of the two sides. This completes the proof. □

Proof of Lemma 6. By Theorem 2 in Hu and Zhou (2020), given the quantity output decision of the competition, it is optimal for firm 1 to set its price and wage such that its demand and supply are equal to the quantity output. That is, for any fixed (p_2, w_2) and realized market size x, there exists a $z_1^*(p_2, w_2)$ such that

$$x - p_1^* + \gamma p_2 = z_1^* = w_1^* - \beta w_2.$$

Similarly,

$$x - p_2^* + \gamma p_1 = z_2^* = w_2^* - \beta w_1.$$

Then we derive

$$\begin{aligned} p_1 &= \frac{1}{1-\gamma^2}[(1+\gamma)x - z_1 - \gamma z_2], \quad w_1 = \frac{1}{1-\beta^2}[z_1 + \beta z_2], \\ p_2 &= \frac{1}{1-\gamma^2}[(1+\gamma)x - z_2 - \gamma z_1], \quad w_2 = \frac{1}{1-\beta^2}[z_2 + \beta z_1]. \end{aligned} \tag{D.7}$$

Therefore, firm i's expected profit can be expressed as follows,

$$E[\pi_i(z_i, z_j)] = E[(p_i - w_i)z_i] = \int \left\{ \frac{x}{1-\gamma} - (\frac{1}{1-\gamma^2} + \frac{1}{1-\beta^2})z_i - (\frac{\gamma}{1-\gamma^2} + \frac{\beta}{1-\beta^2})z_j \right\} z_i dx. \tag{D.8}$$

We obtain firm i's best-response function as follows,

$$\begin{aligned} z_1^*(z_2) &= \frac{1}{2(\frac{1}{1-\gamma^2} + \frac{1}{1-\beta^2})} \left\{ \frac{E[\Omega]}{1-\gamma} - (\frac{\gamma}{1-\gamma^2} + \frac{\beta}{1-\beta^2})z_2 \right\}, \\ z_2^*(z_1) &= \frac{1}{2(\frac{1}{1-\gamma^2} + \frac{1}{1-\beta^2})} \left\{ \frac{E[\Omega]}{1-\gamma} - (\frac{\gamma}{1-\gamma^2} + \frac{\beta}{1-\beta^2})z_1 \right\}. \end{aligned}$$

Solving the above set of equations yields the equilibrium matching quantity z_Q^*. Putting z_Q^* into (D.7) gives the equilibrium price p_Q^* and wage w_Q^*. Finally, putting z_Q^* into (D.8) gives the equilibrium expected profit $E[\pi_Q^*]$. □

Proof of Proposition 3. According to Proposition 1 and Proposition 2(a), it suffices to show $p_Q^* \geq \max\{p_{wp}^*, p_{pw}^*\}$ and $E[\pi_Q^*] \geq \max\{E[\pi_{wp}^*], E[\pi_{pw}^*]\}$.

Part 1: We first compare mode Q with mode wp. Note that as $\mathrm{Var}(\Omega) \to 0$,

$$p_Q^* \to \frac{3+\beta-\beta^2-2\gamma^2-\beta\gamma^2}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)}E[\Omega],$$
$$p_{wp}^* \to \frac{3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}E[\Omega],$$

so $p_Q^* - p_{wp}^*$ reduces to

$$\begin{aligned}
&(3+\beta-\beta^2-2\gamma^2-\beta\gamma^2)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)\\
&-(3-2\beta-\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)\\
=&\Big\{12+3\gamma-14\gamma^2-2\gamma^3+4\gamma^4+\beta(-5-5\gamma+3\gamma^2+3\gamma^3)+\beta^2(-7+6\gamma^2-\gamma^4)+\beta^3(3+3\gamma-\gamma^2-\gamma^3)+\beta^4(-\gamma)\Big\}\\
&-\Big\{12+3\gamma-14\gamma^2-2\gamma^3+4\gamma^4+\beta(-5-6\gamma+2\gamma^2+3\gamma^3)+\beta^2(-8+8\gamma^2+\gamma^3-\gamma^4)\\
&\quad+\beta^3(4+5\gamma-\gamma^2-2\gamma^3)+\beta^4(-2\gamma-\gamma^2)\Big\}\\
=&\beta\Big\{\gamma+\gamma^2+\beta(1-\gamma^3)(1-\beta)-2\beta\gamma^2-2\beta^2\gamma+\beta^3\gamma+\beta^3\gamma^2\Big\}\\
=&\beta(1-\beta)\Big\{\beta(1-\gamma^3)+\gamma(1+\beta+\gamma-\beta\gamma-\beta^2\gamma-\beta^2)\Big\}\\
\geq&0.
\end{aligned}$$

Note also that

$$w_Q^* = \frac{(1+\gamma)(1+\beta)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}E[\Omega], \quad w_{wp}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],$$

so $w_Q^* - w_{wp}^*$ reduces to

$$\begin{aligned}
&(1+\beta)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)-(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)\\
=&-\beta^2(1-\gamma^2)-\beta\gamma(1-\beta^2)\leq 0
\end{aligned}$$

Moreover, one can verify that $E[\pi_Q^*] \geq E[\pi_{wp}^*]$.

Part 2: We then compare mode Q with mode pw. Note that as $\mathrm{Var}(\Omega) \to 0$,

$$p_Q^* \to \frac{3+\beta-\beta^2-2\gamma^2-\beta\gamma^2}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)}E[\Omega],$$
$$p_{pw}^* = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],$$

so $p_Q^* - p_{pw}^*$ reduces to

$$(3+\beta-\beta^2-2\gamma^2-\beta\gamma^2)(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)$$

$$
\begin{aligned}
&-(3+\beta-\beta^2-\beta\gamma)(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)\\
=&6\gamma+\gamma^2+\beta(\gamma-\gamma^3)+\beta^2(-2\gamma^2)+\beta^3(-\gamma+\gamma^3)+\beta^4\gamma^2\\
=&6\gamma+\gamma^2(1-\beta^2)^2+\beta\gamma(1-\gamma^2)(1-\beta^2)\geq 0.
\end{aligned}
$$

Note also that as $\mathrm{Var}(\Omega)\to 0$,

$$
\begin{aligned}
w_Q^* &= \frac{(1+\gamma)(1+\beta)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}E[\Omega],\\
w_{pw}^* &\to \frac{1+\beta}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],
\end{aligned}
$$

so $w_Q^*-w_{pw}^*$ reduces to

$$
\begin{aligned}
&(1+\gamma)(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)-(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)\\
=&-\gamma(1-\beta\gamma)(\beta+\gamma)\leq 0.
\end{aligned}
$$

Moreover, one can verify that $E[\pi_Q^*]\geq E[\pi_{pw}^*]$. □

Proof of Proposition 4. On the one hand, according to Lemma 6, when there is no market size uncertainty, the equilibrium of the single-stage quantity competition reduces to

$$
\begin{aligned}
p_Q^* &= \frac{3+\beta-\beta^2-2\gamma^2-\beta\gamma^2}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)}\Omega,\\
w_Q^* &= \frac{(1+\gamma)(1+\beta)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}\Omega,\\
z_Q^* &= \frac{(1+\gamma)(1-\beta^2)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}\Omega,\\
\pi_Q^* &= \frac{(2-\gamma^2-\beta^2)(1+\gamma)(1-\beta^2)}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)^2}\Omega^2.
\end{aligned}
$$

On the other hand, Lemma A.1 in Online Supplement A.2 gives the equilibrium of the two-stage quantity precommitment competition. Comparing the two equilibria establishes the result. □

Before proving Proposition 5, we first introduce Lemma D.3.

LEMMA D.3. *Suppose Ω is distributed on $[\underline{\Omega},\bar{\Omega}]$.*

(a) Suppose $\beta=0$. If γ is sufficiently small, then $E[\pi_{wp,c2}^*]\leq E[\pi_{wp,c3}^*]$.

(b) Suppose $\gamma=0$. If β is sufficiently small, then $E[\pi_{wp,c2}^*]\leq E[\pi_{wp,c3}^*]$.

Proof of Lemma D.3. Part (a): According to Cases 2 and 3 in Online Supplement B.2, it suffices to show for each $x\in[\underline{\Omega},\widehat{\Omega}(\boldsymbol{w})]$, the profit earned by setting the inventory-depletion price is lower than that by setting the profit-maximizing price, that is,

$$
\left\{\frac{x}{1-\gamma}+\frac{\beta-\gamma}{1-\gamma^2}w_j-\frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\right\}(w_i-\beta w_j)\leq\left(\frac{x}{2-\gamma}-\frac{2-\gamma^2}{4-\gamma^2}w_i+\frac{\gamma}{4-\gamma^2}w_j\right)^2. \tag{D.9}
$$

Since the firms take symmetric actions, we let $w_i = w_j$. Putting $\beta = 0$ into (D.9) yields

$$\frac{x-(2-\gamma)w_i}{1-\gamma}w_i \leq (\frac{x-(1-\gamma)w_i}{2-\gamma})^2.$$

One can check

$$\begin{aligned}
&\frac{x-(2-\gamma)w_i}{1-\gamma}w_i - (\frac{x-(1-\gamma)w_i}{2-\gamma})^2 \\
=&\frac{1}{(2-\gamma)^2(1-\gamma)}\Big\{-(1-\gamma)x^2 - [(1-\gamma)^3+(2-\gamma)^3]w_i^2 + [2(1-\gamma)^2+(2-\gamma)^2]xw_i\Big\} \\
\to& -\frac{1}{4}(x-3w_i)^2 \qquad [\text{as } \gamma \to 0] \\
<&0.
\end{aligned}$$

This establishes that $E[\pi^*_{wp,c2}] \leq E[\pi^*_{wp,c3}]$.

Part (b): Again, we expect to show

$$\Big\{\frac{x}{1-\gamma} + \frac{\beta-\gamma}{1-\gamma^2}w_j - \frac{2-\gamma^2-\beta\gamma}{1-\gamma^2}w_i\Big\}(w_i-\beta w_j) \leq \Big(\frac{x}{2-\gamma} - \frac{2-\gamma^2}{4-\gamma^2}w_i + \frac{\gamma}{4-\gamma^2}w_j\Big)^2. \tag{D.10}$$

Since the firms take symmetric actions, we let $w_i = w_j$. Putting $\gamma = 0$ into (D.10) yields

$$(x-(2-\beta)w_i)(1-\beta)w_i \leq \frac{1}{4}(x-w_i)^2.$$

One can check

$$\begin{aligned}
&(x-(2-\beta)w_i)(1-\beta)w_i - \frac{1}{4}(x-w_i)^2 \\
=&(\frac{3}{2}-\beta)xw_i - [(2-\beta)(1-\beta)+\frac{1}{4}]w_i^2 - \frac{1}{4}x^2 \\
\to& -(\frac{1}{2}x - \frac{3}{2}w_i)^2 \qquad [\text{as } \beta \to 0] \\
<&0.
\end{aligned}$$

This establishes that $E[\pi^*_{wp,c2}] \leq E[\pi^*_{wp,c3}]$. This completes the proof. □

Before proving Proposition 5, we also provide the conditions imposed on γ and Ω (distributed on $[\underline{\Omega}, \bar{\Omega}]$) for Proposition 5(a-i): γ is sufficiently small and Var(Ω) is not sufficiently large such that

$$\frac{1+\gamma}{4+\gamma-2\gamma^2}E[\Omega] \leq \frac{\underline{\Omega}}{3-2\gamma}, \tag{D.11}$$

$$\frac{(2-\gamma^2)(1+\gamma)}{(4+\gamma-2\gamma^2)^2(1-\gamma)}E[\Omega]^2 \leq \frac{2}{(4-3\gamma)^2}E[\Omega^2], \tag{D.12}$$

$$\frac{E[\Omega^2]}{(2-\gamma)^2} - \frac{2(2-\gamma^2-\gamma)\bar{\Omega}E[\Omega]}{(2-\gamma)(4-\gamma^2)(3-2\gamma)} + \Big(\frac{(2-\gamma^2-\gamma)\bar{\Omega}}{(4-\gamma^2)(3-2\gamma)}\Big)^2 \leq \frac{2}{(4-3\gamma)^2}E[\Omega^2]. \tag{D.13}$$

It is easy to verify that (D.11) and (D.12) hold if γ is sufficiently small and $\mathrm{Var}(\Omega)$ is not sufficiently large. Here, we illustrate (D.13) indeed holds when γ is sufficiently small and $\mathrm{Var}(\Omega)$ is not sufficiently large using an example of two-point distribution for Ω. We assume that Ω takes the values of Ω_H and Ω_L with equal probability. We have

$$\begin{aligned}
&\lim_{\gamma\to 0}\frac{E[\Omega^2]}{(2-\gamma)^2}-\frac{2(2-\gamma^2-\gamma)\bar{\Omega}E[\Omega]}{(2-\gamma)(4-\gamma^2)(3-2\gamma)}+\big(\frac{(2-\gamma^2-\gamma)\bar{\Omega}}{(4-\gamma^2)(3-2\gamma)}\big)^2-\frac{2}{(4-3\gamma)^2}E[\Omega^2]\\
=&\frac{1}{4}E[\Omega^2]-\frac{1}{6}\bar{\Omega}E[\Omega]+\frac{1}{36}\bar{\Omega}^2-\frac{1}{8}E[\Omega^2]=\frac{1}{8}E[\Omega^2]-\frac{1}{6}\bar{\Omega}E[\Omega]+\frac{1}{36}\bar{\Omega}^2\\
=&\frac{1}{4}(\frac{1}{2}\Omega_H^2+\frac{1}{2}\Omega_L^2)-\frac{1}{6}\Omega_H(\frac{1}{2}\Omega_H+\frac{1}{2}\Omega_L)+\frac{1}{36}\Omega_H^2=\frac{1}{16}\Big\{(\frac{1}{3}\Omega_H-\Omega_L)^2-\frac{2}{3}\Omega_H\Omega_L\Big\},
\end{aligned}$$

which is negative if $\Omega_H\le 5\Omega_L$ ($\mathrm{Var}(\Omega)$ is not sufficiently large).

Now we provide the conditions imposed on β and Ω (distributed on $[\underline{\Omega},\bar{\Omega}]$) for Proposition 5(b-i): β is sufficiently small and $\mathrm{Var}(\Omega)$ is not sufficiently large such that

$$\frac{1}{4-3\beta}E[\Omega]\le\frac{1}{3-2\beta}\underline{\Omega}, \tag{D.14}$$

$$\frac{1}{4}E[\Omega^2]-\frac{\bar{\Omega}E[\Omega]}{2(3-2\beta)}+\frac{1}{4}\frac{\bar{\Omega}^2}{(3-2\beta)^2}\le\frac{2(1-\beta)^2}{(4-3\beta)^2}E[\Omega^2]. \tag{D.15}$$

As above, one can verify that (D.14) and (D.15) indeed hold if β is sufficiently small and $\mathrm{Var}(\Omega)$ is not sufficiently large. Now, we are ready to prove Proposition 5.

Proof of Proposition 5. Part (a-i): We first derive the equilibrium profit in wage precommitment competition when $\beta=0$ by examining the three cases for mode wp in Online Supplement B.2. Assume γ is sufficiently small.

Case 1: Low Wage. Inequality (D.11) tells us that $w^*_{i,c1}\le w^1_{ub}$, hence, the equilibrium profit in Case 1 is as follows,

$$E[\pi^*_{wp,c1}]=E[\pi_{wp,c1}|_{w=w^*_{i,c1}}]=\frac{(2-\gamma^2)(1+\gamma)}{(4+\gamma-2\gamma^2)^2(1-\gamma)}E[\Omega]^2.$$

Case 2: Medium Wage. According to Lemma D.3(a), $E[\pi^*_{wp,c2}]\le E[\pi^*_{wp,c3}]$.

Case 3: High Wage. (B.32) indicates that

$$\begin{aligned}
E[\pi^*_{wp,c3}]&=\int_x(\frac{x}{2-\gamma}-\frac{2-\gamma^2-\gamma}{4-\gamma^2}w^3_{lb})^2dF(x)\\
&=\frac{E[\Omega^2]}{(2-\gamma)^2}-\frac{2(2-\gamma^2-\gamma)\bar{\Omega}E[\Omega]}{(2-\gamma)(4-\gamma^2)(3-2\gamma)}+\big(\frac{(2-\gamma^2-\gamma)\bar{\Omega}}{(4-\gamma^2)(3-2\gamma)}\big)^2.
\end{aligned}$$

Summarizing the above three cases, we have

$$E[\pi^*_{wp}]=\max\big\{E[\pi^*_{wp,c1}],E[\pi^*_{wp,c3}]\big\}.$$

On the other hand, (4) gives

$$E[\pi_P^*] = \frac{2}{(4-3\gamma)^2} E[\Omega^2].$$

Inequalities (D.12) and (D.13) show that $E[\pi_{wp,c1}^*] \le E[\pi_P^*]$ and $E[\pi_{wp,c3}^*] \le E[\pi_P^*]$, respectively. Hence, $E[\pi_{wp}^*] \le E[\pi_P^*]$.

Part (a-ii): We obtain

$$E[\pi_{wp}^*] \ge E[\pi_{wp,c1}|_{w=w_{ub}^1}] = \Big(\frac{E[\Omega]}{1-\gamma} - \frac{\underline{\Omega}}{1-\gamma}\frac{2-\gamma}{3-2\gamma}\Big)\frac{\underline{\Omega}}{3-2\gamma},$$
$$E[\pi_P^*] = \frac{2}{(4-3\gamma)^2} E[\Omega^2].$$

One can check $E[\pi_{wp}^*] > E[\pi_P^*]$ if γ is sufficiently large.

Part (b-i): We first derive the equilibrium profit in wage precommitment competition when $\gamma = 0$ by examining the three cases for mode wp in Online Supplement B.2. Assume β is sufficiently small.

Case 1: Low Wage. Inequality (D.14) tells us that $w_{i,c1}^* \le w_{ub}^1$, hence, the equilibrium profit in Case 1 is as follows,

$$E[\pi_{wp,c1}^*] = E[\pi_{wp,c1}|_{w=w_i^*}] = \frac{2(1-\beta)^2}{(4-3\beta)^2}(E[\Omega])^2.$$

Case 2: Medium Wage. According to Lemma D.3(b), $E[\pi_{wp,c2}^*] \le E[\pi_{wp,c3}^*]$.

Case 3: High Wage. (B.32) indicates that

$$E[\pi_{wp,c3}^*] = \int_x \Big(\frac{x}{2-\gamma} - \frac{2-\gamma^2-\gamma}{4-\gamma^2} w_{lb}^3\Big)^2 dF(x) = \frac{1}{4}E[\Omega^2] - \frac{\bar{\Omega}E[\Omega]}{2(3-2\beta)} + \frac{1}{4}\frac{\bar{\Omega}^2}{(3-2\beta)^2}.$$

Summarizing the above three cases, we have

$$E[\pi_{wp}^*] = \max\big\{E[\pi_{wp,c1}^*], E[\pi_{wp,c3}^*]\big\}.$$

On the other hand, (4) gives

$$E[\pi_P^*] = \frac{2(1-\beta)^2}{(4-3\beta)^2} E[\Omega^2].$$

Clearly, $E[\pi_{wp,c1}^*] \le E[\pi_P^*]$ because $(E[\Omega])^2 \le E[\Omega^2]$. Inequality (D.15) shows $E[\pi_{wp,c3}^*] \le E[\pi_P^*]$. Hence, $E[\pi_{wp}^*] \le E[\pi_P^*]$.

Part (b-ii): We obtain

$$E[\pi_{wp}^*] \ge E[\pi_{wp,c1}|_{w=w_{ub}^1}] = \frac{(3-2\beta)E[\Omega] - (2-\beta)\underline{\Omega}}{(3-2\beta)^2}(1-\beta)\underline{\Omega},$$
$$E[\pi_P^*] = \frac{2(1-\beta)^2}{(4-3\beta)^2} E[\Omega^2].$$

One can check that if β is sufficiently large such that

$$\frac{(3-2\beta)E[\Omega]-(2-\beta)\underline{\Omega}}{(3-2\beta)^2}\underline{\Omega} \geq \frac{2(1-\beta)}{(4-3\beta)^2}E[\Omega^2],$$

$E[\pi^*_{wp}] \geq E[\pi^*_P]$ holds immediately. This completes the proof. $\square$

PROPOSITION D.1. *Suppose Ω takes the values of Ω_H and Ω_L with probability q and $1-q$, respectively. Assume $\beta = \gamma$.*

(a) If Var(Ω) is not sufficiently large and β is sufficiently small such that $\Omega_H \leq \min\left\{\frac{2(1-q)(2-\beta^2)(1-\beta)(2-\beta)+2q(2-\beta)^3(2+\beta)}{q(2-\beta)^2(2+\beta)(3-\beta)}\Omega_L, \frac{q(3-\beta)+1-\beta}{q(3-\beta)}\Omega_L\right\}$, then $E[\pi^*_{wp}] \leq E[\pi^*_P]$.

(b) If β is sufficiently large, then $E[\pi^*_{wp}] \geq E[\pi^*_P]$.

Proof of Proposition D.1. Part (a): We first derive the equilibrium profit in wage precommitment competition when $\beta = \gamma$ by examining the three cases for mode wp in Online Supplement B.2.

Case 1: Letting $\beta = \gamma$ yields

$$w^*_{i,c1} = \frac{E[\Omega]}{2(2-\beta)(1-\beta)}, \quad w^1_{ub} = \frac{\Omega_L}{(3-\beta)(1-\beta)}.$$

Since $\Omega_H \leq \frac{q(3-\beta)+1-\beta}{q(3-\beta)}\Omega_L$, it follows immediately that $w^*_{i,c1} \leq w^1_{ub}$. Due to the concavity of the profit function, the equilibrium profit in Case 1 is as follows,

$$E[\pi^*_{c1}] = E[\pi|_{w=w^*_{i,c1}}] = \frac{(E[\Omega])^2}{2(2-\beta)^2}. \tag{D.16}$$

Note that

$$E[\pi^*_{c1}] \geq E[\pi|_{w=w^1_{ub}}] = \frac{\big((3-\beta)E[\Omega]-2\Omega_L\big)\Omega_L}{(3-\beta)^2(1-\beta)}. \tag{D.17}$$

Case 2: Letting $\beta = \gamma$ yields

$$w^*_{i,c2} = \frac{q(2+\beta)(2-\beta)^2\Omega_H - 2(1-q)(2-\beta^2)(1-\beta)\Omega_L}{2q(2+\beta)(2-\beta)^3(1-\beta) - 2(1-q)(2-\beta^2)(1-\beta)^2}, \quad w^2_{lb} = \frac{\Omega_L}{(3-\beta)(1-\beta)}.$$

Since $\Omega_H \leq \frac{2(1-q)(2-\beta^2)(1-\beta)(2-\beta)+2q(2-\beta)^3(2+\beta)}{q(2-\beta)^2(2+\beta)(3-\beta)}\Omega_L$, we have $w^*_{i,c2} \leq w^2_{lb}$. Due to the concavity of the profit function,

$$E[\pi^*_{c2}] = E[\pi|_{w=w^2_{lb}}] = \frac{\big((3-\beta)E[\Omega]-2\Omega_L\big)\Omega_L}{(3-\beta)^2(1-\beta)}. \tag{D.18}$$

Note that

$$E[\pi^*_{c2}] > E[\pi|_{w=w^2_{ub}}] = \frac{q(2-\beta)^2\Omega_H^2 + (1-q)[(3-\beta)\Omega_L - \Omega_H]^2}{(2-\beta)^2(3-\beta)^2}. \tag{D.19}$$

Case 3: Letting $\beta=\gamma$ yields

$$\begin{aligned} w_{lb}^3 &= \frac{\Omega_H}{(3-\beta)(1-\beta)}, \\ E[\pi_{c3}^*] &= \frac{q(2-\beta)^2\Omega_H^2+(1-q)[(3-\beta)\Omega_L-\Omega_H]^2}{(2-\beta)^2(3-\beta)^2}. \end{aligned} \tag{D.20}$$

Combining (D.16)-(D.20) gives $E[\pi_{c1}^*]\geq E[\pi_{c2}^*]\geq E[\pi_{c3}^*]$. Therefore,

$$E[\pi_{wp}^*]=\max\big\{E[\pi_{c1}^*],E[\pi_{c2}^*],E[\pi_{c3}^*]\big\}=E[\pi_{c1}^*]=\frac{(E[\Omega])^2}{2(2-\beta)^2}.$$

On the other hand, when $\beta=\gamma$, (4) gives

$$E[\pi_P^*]=\frac{E[\Omega^2]}{2(2-\beta)^2}.$$

Clearly, $E[\pi_{wp}^*]\leq E[\pi_P^*]$ because $(E[\Omega])^2\leq E[\Omega^2]$.

Part (b): Since $E[\pi_{wp}^*]$ takes the maximum of the three cases, we have

$$E[\pi_{wp}^*]\geq E[\pi_{c1}^*]\geq E[\pi|_{w=w_{ub}^1}]=\frac{\big((3-\beta)E[\Omega]-2\Omega_L\big)\Omega_L}{(3-\beta)^2(1-\beta)}.$$

Next, we compare $E[\pi_{wp}^*]$ and $E[\pi_P^*]$. One can check

$$\begin{aligned} &E[\pi_{wp}^*]-E[\pi_P^*] \\ \geq&\frac{\big((3-\beta)E[\Omega]-2\Omega_L\big)\Omega_L}{(3-\beta)^2(1-\beta)}-\frac{E[\Omega^2]}{2(2-\beta)^2} \\ =&\frac{1}{2(3-\beta)^2(1-\beta)(2-\beta)^2}\Big\{2(2-\beta)^2\Omega_L\big((3-\beta)E[\Omega]-2\Omega_L\big)-(3-\beta)^2(1-\beta)E[\Omega^2]\Big\} \\ \approx&\frac{1}{2(3-\beta)^2(1-\beta)(2-\beta)^2}\Big\{4\Omega_L\big(E[\Omega]-\Omega_L\big)\Big\}\geq 0, \end{aligned}$$

where the above approximation holds as $\beta\to 1$. □

Before proving Proposition 6, we first introduce Lemma D.4.

LEMMA D.4. *Suppose* Ω *is distributed on* $[\underline{\Omega},\bar{\Omega}]$.

(a) Suppose $\beta=0$. *Then* $E[\pi_{pw,c2}^*]\leq E[\pi_{pw,c3}^*]$.

(b) Suppose $\gamma=0$. *If* β *is sufficiently small, then* $E[\pi_{pw,c2}^*]\leq E[\pi_{pw,c3}^*]$.

Proof of Lemma D.4. In order to show $E[\pi_{pw,c2}^*]\leq E[\pi_{pw,c3}^*]$, according to Cases 2 and 3 in Online Supplement C.2, it suffices to show that for any $x\in[\underline{\Omega},\hat{\Omega}(p)]$, the profit earned by setting the demand-depletion wage is lower than that by setting the profit-maximizing wage, that is,

$$\frac{1}{1-\beta^2}\Big\{-(1+\beta)x+(2-\beta^2-\beta\gamma)p_i-(\gamma-\beta)p_j\Big\}(x-p_i+\gamma p_j)\leq\Big[\frac{(2-\beta^2)p_i-\beta p_j}{4-\beta^2}\Big]^2. \tag{D.21}$$

Part (a): Putting $\beta = 0$ into (D.21) yields

$$(-x + 2p_i - \gamma p_j)(x - p_i + \gamma p_j) \le \frac{p_i^2}{4}. \tag{D.22}$$

Since the firms take symmetric actions, we let $p_i = p_j$, and thus (D.22) further reduces to

$$[-x + (2-\gamma)p_i][x - (1-\gamma)p_i] \le \frac{p_i^2}{4}.$$

The first and second order derivatives with respect to γ indicate that the left hand side achieves its maximum when $\gamma = \frac{-2x+3p_i}{2p_i}$. Putting $\gamma = \frac{-2x+3p_i}{2p_i}$ into the left hand side yields that

$$[-x + (2-\gamma)p_i][x - (1-\gamma)p_i] = \frac{p_i^2}{4}.$$

This completes the proof of Part (a).

Part (b): Putting $\gamma = 0$ into (D.21) yields

$$\frac{1}{1-\beta^2}\Big\{-(1+\beta)x + (2-\beta^2)p_i + \beta p_j\Big\}(x - p_i) < \Big[\frac{(2-\beta^2)p_i - \beta p_j}{4-\beta^2}\Big]^2. \tag{D.23}$$

Since the firms take symmetric actions, we let $p_i = p_j$, and thus (D.23) further reduces to

$$\frac{1}{1-\beta^2}\Big\{-(1+\beta)x + (2-\beta^2+\beta)p_i\Big\}(x - p_i) < \Big[\frac{(2-\beta^2-\beta)p_i}{4-\beta^2}\Big]^2.$$

One can check

$$\begin{aligned}
&\frac{1}{1-\beta^2}\Big\{-(1+\beta)x + (2-\beta^2+\beta)p_i\Big\}(x - p_i) - \Big[\frac{(2-\beta^2-\beta)p_i}{4-\beta^2}\Big]^2 \\
=&(-x+2p_i)(x-p_i) - \frac{p_i^2}{4} \qquad [\text{when } \beta \to 0] \\
\le&0.
\end{aligned}$$

The last inequality holds because when $x = \frac{3}{2}p_i$, $(-x+2p_i)(x-p_i) - \frac{p_i^2}{4}$ achieves the maximum which is 0. This completes the proof of Part (b). □

Proposition D.2 is an extended version of Proposition 6 and includes the specific conditions on the magnitude of the variance of demand uncertainty. Assume that Ω is distributed on $[\underline{\Omega}, \bar{\Omega}]$.

Proposition D.2. *(a) Suppose $\beta = 0$ and $\frac{3}{4-3\gamma}E[\Omega] \ge \frac{2}{3-2\gamma}\bar{\Omega}$. Then, $E[\pi_{pw}^*] \le E[\pi_P^*]$.*

(b) Suppose $\gamma = 0$.

(i) If β is sufficiently small and $\frac{3+\beta-\beta^2}{4+\beta-2\beta^2}E[\Omega] \ge \frac{2+\beta-\beta^2}{3+\beta-2\beta^2}\bar{\Omega}$, then $E[\pi_{pw}^*] \le E[\pi_P^*]$.

(ii) If β is sufficiently large and $Var(\Omega) \le \big(E[\Omega] - \frac{\bar{\Omega}}{2}\big)\bar{\Omega}$, then $E[\pi_{pw}^*] > E[\pi_P^*]$.

Proof of Proposition D.2. Part (a): We first derive the equilibrium profit in price precommitment competition when $\beta = 0$ by examining the three cases for mode *pw* in Online Supplement C.2.

Case 1: High price. Inequality $\frac{3}{4-3\gamma}E[\Omega] \geq \frac{2}{3-2\gamma}\bar{\Omega}$ implies that $p^*_{i,c1} \geq p^1_{lb}$, so the equilibrium profit in Case 1 is

$$E[\pi^*_{pw,c1}] = E[\pi_{pw,c1}|_{p=p^*_{i,c1}}] = \frac{3(6-8\gamma+3\gamma^2)}{(4-3\gamma)^2}(E[\Omega])^2 - E[\Omega^2].$$

Case 2: Medium price. According to Lemma D.4(a), $E[\pi^*_{pw,c2}] \leq E[\pi^*_{pw,c3}]$.

Case 3: Low price. (C.15) shows

$$E[\pi^*_{pw,c3}] = \frac{1}{(3-2\gamma)^2}\underline{\Omega}^2.$$

Summarizing the three cases, we have $E[\pi^*_{pw}] = \max\left\{E[\pi^*_{pw,c1}], E[\pi^*_{pw,c3}]\right\}$.

On the other hand, (4) gives

$$E[\pi^*_P] = \frac{2}{(4-3\gamma)^2}E[\Omega^2].$$

Next, we compare $E[\pi^*_{pw}]$ with $E[\pi^*_P]$. One can verify

$$\frac{1}{(3-2\gamma)^2} - \frac{2}{(4-3\gamma)^2} = \frac{-(2-\gamma^2)}{(3-2\gamma)^2(4-3\gamma)^2} < 0,$$

so $E[\pi^*_{pw,c3}] \leq E[\pi^*_P]$. Also,

$$\begin{aligned}
&\frac{3(6-8\gamma+3\gamma^2)}{(4-3\gamma)^2}(E[\Omega])^2 - E[\Omega^2] - \frac{2}{(4-3\gamma)^2}E[\Omega^2] \\
=&\frac{3(6-8\gamma+3\gamma^2)}{(4-3\gamma)^2}(E[\Omega])^2 - \frac{2+(4-3\gamma)^2}{(4-3\gamma)^2}E[\Omega^2] \\
=&\frac{2+(4-3\gamma)^2}{(4-3\gamma)^2}\left\{(E[\Omega])^2 - E[\Omega^2]\right\} \\
\leq&0,
\end{aligned}$$

where the last inequality holds because $(E[\Omega])^2 \leq E[\Omega^2]$. Therefore, $E[\pi^*_{pw,c1}] \leq E[\pi^*_P]$. This completes the proof of Part (a).

Part (b-i): We first derive the equilibrium profit in price precommitment competition when $\gamma = 0$ by examining the three cases for mode *pw* in Online Supplement C.2. Assume β is sufficiently small.

Case 1: High price. Inequality $\frac{3+\beta-\beta^2}{4+\beta-2\beta^2}E[\Omega] \geq \frac{2+\beta-\beta^2}{3+\beta-2\beta^2}\bar{\Omega}$ implies that $p^*_{i,c1} \geq p^1_{lb}$, so the equilibrium profit in Case 1 is

$$E[\pi^*_{pw,c1}] = E[\pi_{pw,c1}|_{p=p^*_{i,c1}}] = \frac{(3+\beta-\beta^2)(6+6\beta-4\beta^2-3\beta^3+\beta^4)}{(1-\beta^2)(4+\beta-2\beta^2)^2}(E[\Omega])^2 - \frac{1}{1-\beta}E[\Omega^2].$$

Case 2: Medium price. According to Lemma D.4(b), $E[\pi^*_{pw,c2}] \le E[\pi^*_{pw,c3}]$.

Case 3: Low price. (C.15) shows

$$E[\pi^*_{pw,c3}] = \frac{(1-\beta)^2(2-\beta)^2}{(6-\beta-2\beta^2)^2}\underline{\Omega}^2.$$

Summarizing the three cases, we have $E[\pi^*_{pw}] = \max\left\{E[\pi^*_{pw,c1}], E[\pi^*_{pw,c3}]\right\}$.

On the other hand, (4) gives

$$E[\pi^*_P] = \frac{2(1-\beta)^2}{(4-3\beta)^2}E[\Omega^2].$$

Next, we compare $E[\pi^*_{pw}]$ with $E[\pi^*_P]$. One can verify

$$\frac{2-\beta}{6-\beta-2\beta^2} - \frac{\sqrt{2}}{4-3\beta} = \frac{1}{(6-\beta-2\beta^2)(4-3\beta)}\left\{8-6\sqrt{2}-(10-\sqrt{2})\beta+(3+2\sqrt{2})\beta^2\right\} \le 0,$$

where the last inequality holds because the term in the bracket decreases in β and achieves the maximum (which is negative) when $\beta = 0$. Therefore, $E[\pi^*_{pw,c3}] \le E[\pi^*_P]$.

One can also check

$$\begin{aligned}
&E[\pi^*_{pw,c1}] - E[\pi^*_P]\\
=&\frac{(3+\beta-\beta^2)(6+6\beta-4\beta^2-3\beta^3+\beta^4)}{(1-\beta^2)(4+\beta-2\beta^2)^2}(E[\Omega])^2 - \frac{1}{1-\beta}E[\Omega^2] - \frac{2(1-\beta)^2}{(4-3\beta)^2}E[\Omega^2]\\
\approx&\frac{9}{8}\left\{(E[\Omega])^2 - E[\Omega^2]\right\} < 0,
\end{aligned}$$

where the above approximation holds as $\beta \to 0$. Hence, $E[\pi^*_{pw}] \le E[\pi^*_P]$. This completes the proof of Part (b-i).

Part (b-ii): We have

$$\begin{aligned}
&E[\pi^*_{pw}] \ge E[\pi^*_{pw,c1}] \ge E[\pi_{pw,c1}|_{p=p^1_{lb}}]\\
=&\frac{2+\beta-\beta^2}{(1-\beta^2)(3+\beta-2\beta^2)^2}\left\{(3+2\beta-\beta^2)(3+\beta-2\beta^2)E[\Omega]\bar{\Omega} - (2+\beta-\beta^2)\bar{\Omega}^2\right\} - \frac{1}{1-\beta}E[\Omega^2],
\end{aligned}$$

while

$$E[\pi^*_P] = \frac{2(1-\beta)^2}{(4-3\beta)^2}E[\Omega^2].$$

One can check

$$\begin{aligned}
&E[\pi^*_{pw}] - E[\pi^*_P]\\
\ge&\frac{2+\beta-\beta^2}{(1-\beta^2)(3+\beta-2\beta^2)^2}\left\{(3+2\beta-\beta^2)(3+\beta-2\beta^2)E[\Omega]\bar{\Omega} - (2+\beta-\beta^2)\bar{\Omega}^2\right\} - \frac{1}{1-\beta}E[\Omega^2] - \frac{2(1-\beta)^2}{(4-3\beta)^2}E[\Omega^2]\\
=&\frac{1}{1-\beta}\left[\frac{2+\beta-\beta^2}{(1+\beta)(3+\beta-2\beta^2)}\left\{(3+2\beta-\beta^2)(3+\beta-2\beta^2)E[\Omega]\bar{\Omega} - (2+\beta-\beta^2)\bar{\Omega}^2\right\} - E[\Omega^2] - \frac{2(1-\beta)^3}{(4-3\beta)^2}E[\Omega^2]\right]
\end{aligned}$$

$$\approx\frac{1}{1-\beta}\Big[2E[\Omega]\bar{\Omega}-\frac{1}{2}\bar{\Omega}^2-E[\Omega^2]\Big]$$
$$=\frac{1}{1-\beta}\Big[2E[\Omega]\bar{\Omega}-\frac{1}{2}\bar{\Omega}^2-(E[\Omega])^2-Var(\Omega)\Big]$$
$$>\frac{1}{1-\beta}\Big[E[\Omega]\bar{\Omega}-\frac{1}{2}\bar{\Omega}^2-Var(\Omega)\Big]$$
$$\geq 0,$$

where the above approximation holds as $\beta \to 1$ and the last inequality holds because $\mathrm{Var}(\Omega) \leq \big(E[\Omega]-\frac{\bar{\Omega}}{2}\big)\bar{\Omega}$. This completes the proof of Part (b-ii). $\square$

E. Asymmetric Results

This section presents the analytical equilibrium outcome for each mode when the market sizes are asymmetric. First, for simultaneous price and wage competition, the unique equilibrium of price and wage for platform i and the resulting matching quantity and profit are as follows,

$$p^*_{P,i}=\frac{6(2-\beta^2)\Omega_i+(9\gamma+\beta-4\beta^2\gamma)\Omega_j}{16-2\beta\gamma-9\beta^2+4\beta^2\gamma^2-9\gamma^2},\qquad w^*_{P,i}=\frac{(4+2\beta\gamma)\Omega_i+(3\beta+3\gamma)\Omega_j}{(4+2\beta\gamma)^2-(3\beta+3\gamma)^2},$$

$$z^*_{P,i}=\frac{(4-3\beta^2-\beta\gamma)\Omega_i+(3\gamma-\beta-2\beta^2\gamma)\Omega_j}{16-2\beta\gamma-9\beta^2+4\beta^2\gamma^2-9\gamma^2},\quad \pi^*_{P,i}=2\Big[\frac{(4-3\beta^2-\beta\gamma)\Omega_i+(3\gamma-\beta-2\beta^2\gamma)\Omega_j}{16-2\beta\gamma-9\beta^2+4\beta^2\gamma^2-9\gamma^2}\Big]^2.$$

Second, for wage precommitment competition, the unique equilibrium of price and wage for platform i and the resulting matching quantity and profit are as follows,

$$p^*_{wp,i}=\frac{2(2-\beta\gamma-\gamma^2)(1-\gamma^2)\big[3\Omega_i+(2\gamma+\beta)\Omega_j\big]}{(1-\gamma^2)\big[4(2-\beta\gamma-\gamma^2)^2-(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)^2\big]}$$
$$-\frac{(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)\big[(2\beta+\gamma-\beta^2\gamma-2\beta\gamma^2)\Omega_i+(1+\beta\gamma-\beta^2\gamma^2-\beta\gamma^3)\Omega_j\big]}{(1-\gamma^2)\big[4(2-\beta\gamma-\gamma^2)^2-(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)^2\big]},$$

$$w^*_{wp,i}=\frac{2(2-\beta\gamma-\gamma^2)(\Omega_i+\gamma\Omega_j)+(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)(\Omega_j+\gamma\Omega_i)}{4(2-\beta\gamma-\gamma^2)^2-(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)^2},$$

$$z^*_{wp,i}=\frac{(4-2\beta\gamma-3\gamma^2-3\beta^2+\beta\gamma^3+\beta^3\gamma+2\beta^2\gamma^2)\Omega_i+(3\gamma-\beta-2\beta^2\gamma-2\gamma^3+\beta^3\gamma^2+\beta^2\gamma^3)\Omega_j}{4(2-\beta\gamma-\gamma^2)^2-(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)^2},$$

$$\pi^*_{wp,i}=\frac{2-\beta\gamma-\gamma^2}{1-\gamma^2}\Big[\frac{(4-2\beta\gamma-3\gamma^2-3\beta^2+\beta\gamma^3+\beta^3\gamma+2\beta^2\gamma^2)\Omega_i+(3\gamma-\beta-2\beta^2\gamma-2\gamma^3+\beta^3\gamma^2+\beta^2\gamma^3)\Omega_j}{4(2-\beta\gamma-\gamma^2)^2-(3\beta-\gamma-\beta^2\gamma-\beta\gamma^2)^2}\Big]^2.$$

Third, for price precommitment competition, the unique equilibrium of price and wage for platform i and the resulting matching quantity and profit are as follows,

$$p^*_{pw,i}=\frac{2(2-\beta^2-\beta\gamma)\big[(3-\beta^2-\beta\gamma)\Omega_i+\beta\Omega_j\big]+(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)\big[(3-\beta^2-\beta\gamma)\Omega_j+\beta\Omega_i\big]}{4(2-\beta^2-\beta\gamma)^2-(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)^2},$$

$$w^*_{pw,i}=\frac{2(2-\beta^2-\beta\gamma)(\Omega_i+\beta\Omega_j)+(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)(\beta\Omega_i+\Omega_j)}{4(2-\beta^2-\beta\gamma)^2-(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)^2},$$

$$z^*_{pw,i} = \frac{(1-\beta^2)\big[2(2-\beta^2-\beta\gamma)\Omega_i + (3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)\Omega_j\big]}{4(2-\beta^2-\beta\gamma)^2-(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)^2},$$

$$\pi^*_{pw,i} = (1-\beta^2)(2-\beta^2-\beta\gamma)\Big[\frac{2(2-\beta^2-\beta\gamma)\Omega_i + (3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)\Omega_j}{4(2-\beta^2-\beta\gamma)^2-(3\gamma-\beta-\beta^2\gamma-\beta\gamma^2)^2}\Big]^2.$$

Last, for quantity competition, the unique equilibrium of price and wage for platform i and the resulting matching quantity and profit are as follows,

$$p^*_{Q,i} = \frac{2(2-\beta^2-\gamma^2)(1-\gamma^2)\big[(3-\beta^2)\Omega_i + 2\gamma\Omega_j\big] - (\beta+\gamma)(1-\beta\gamma)\big[(\beta-\gamma)(1+\beta\gamma)\Omega_i - (1-\beta^2-\beta\gamma+\beta\gamma^3)\Omega_j\big]}{(1-\gamma^2)\big[4(2-\beta^2-\gamma^2)^2-(\beta+\gamma)^2(1-\beta\gamma)^2\big]},$$

$$w^*_{Q,i} = \frac{2(2-\beta^2-\gamma^2)\big[(1+\beta\gamma)\Omega_i + (\beta+\gamma)\Omega_j\big] - (\beta+\gamma)(1-\beta\gamma)\big[(\beta+\gamma)\Omega_i + (1+\beta\gamma)\Omega_j\big]}{4(2-\beta^2-\gamma^2)^2-(\beta+\gamma)^2(1-\beta\gamma)^2},$$

$$z^*_{Q,i} = \frac{(1-\beta^2)\big[2(2-\beta^2-\gamma^2)(\Omega_i+\gamma\Omega_j) - (\beta+\gamma)(1-\beta\gamma)(\Omega_j+\gamma\Omega_i)\big]}{4(2-\beta^2-\gamma^2)^2-(\beta+\gamma)^2(1-\beta\gamma)^2},$$

$$\pi^*_{Q,i} = \frac{(2-\beta^2-\gamma^2)(1-\beta^2)}{1-\gamma^2}\Big[\frac{2(2-\beta^2-\gamma^2)(\Omega_i+\gamma\Omega_j) - (\beta+\gamma)(1-\beta\gamma)(\Omega_j+\gamma\Omega_i)}{4(2-\beta^2-\gamma^2)^2-(\beta+\gamma)^2(1-\beta\gamma)^2}\Big]^2.$$