Online Appendix to
"Precommitments in Two-sided Market Competition"

The online appendix consists of four sections.

- Section A includes the comparisons of modes with a relatively high demand uncertainty that are not presented in the main body.
- Section B extends the analysis from symmetric platforms to asymmetric platforms.
- Section C incorporates the friction in the process of matching supply with demand.
- Section D includes proofs of lemmas and propositions in the main text.

A. Comparisons of Modes

Comparison of Modes wp and C. The following corollary is an immediate result of Proposition 5 and Corollary 1.

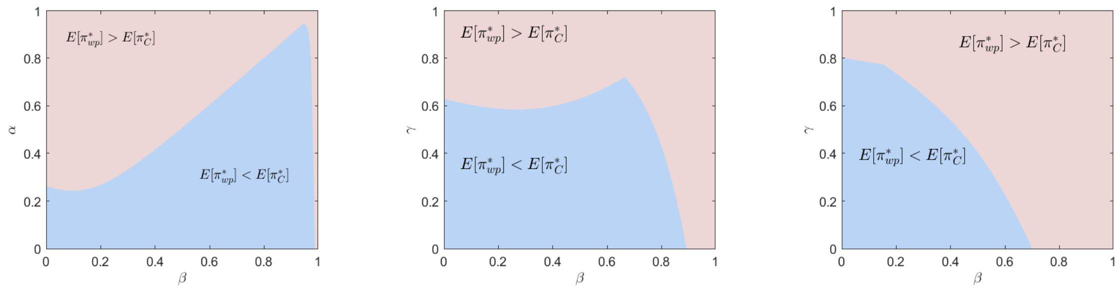
Corollary 2. *Suppose the variance of Ω is not sufficiently large. Then $E[\pi^*_{wp}] \le E[\pi^*_C]$ if either of the following conditions holds:*

(a) $\beta = 0$ and γ is sufficiently small;

(b) $\gamma = 0$ and β is sufficiently small.

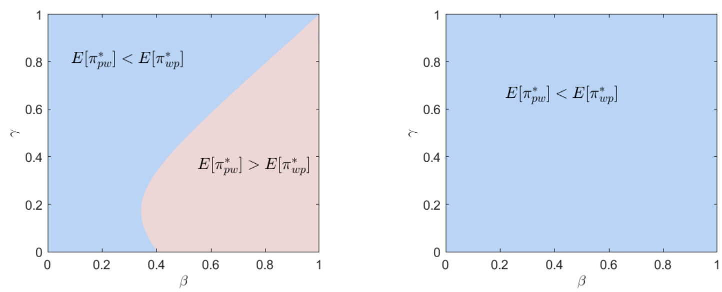
Proposition 5(a-i) and (b-i) specify the conditions under which mode wp performs worse than mode P, while Corollary 1 shows that mode P is worse than C either when $\beta = 0$ or when $\gamma = 0$. Therefore, Corollary 2 holds immediately. Figure A.1 indicates that the comparison between modes wp and C is very similar to that between wp and P, although the region where C is better is larger than that where P is better in Figure 2. Recall that demand variability benefits the firms under modes P and C on the same scale, so the comparison between P and C is fixed as the variability increases, which explains the similarity between Figures 2 and A.1. Moreover, Figure 1 shows that C is better than P in most of the parameter space with a sufficiently low uncertainty. Hence, it is not surprising that the region where C is better than wp is larger than that where P is better than wp.

Comparison of Modes wp and pw. Since the price precommitment fails to do well in alleviating the competition on the demand side, it is no surprise that the region where wp does better keeps expanding as the variability increases (see Figure A.2). When the variability is large enough (e.g., $\Omega_H = 4.5$ and $\Omega_L = 3.5$), mode pw is completely dominated by mode wp.



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$ (b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$ (c) $\Omega_H = 5, \Omega_L = 3, q = 0.5$

Figure A.1 Comparison between $E[\pi^*_{wp}]$ and $E[\pi^*_C]$ under market size uncertainty



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$ (b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$

Figure A.2 Comparison between $E[\pi^*_{wp}]$ and $E[\pi^*_{pw}]$ under market size uncertainty

B. Asymmetric Platforms

To facilitate navigating the results in the extensions, we provide the following Table 2 to summarize the comparison results among different competition modes for the model extensions with asymmetric platforms and matching friction. To simplify the analysis, we assume that the market size is deterministic.

B.1. Asymmetry in market sizes

We first assume that the two platforms face different potential market sizes with everything else the same as in the deterministic model. That is, the demand functions become $d_i(\boldsymbol{p}) = d_i(p_i, p_j) = [\Omega_i - p_i + \gamma p_j]^+$, $j \neq i$, $i = 1, 2$, where $\Omega_1 \neq \Omega_2$. Without loss of generality, we assume $\Omega_1 \geq \Omega_2$. Following the same procedure as in the deterministic model, we can characterize the equilibrium for each competition mode and then compare those equilibria whenever they exist.

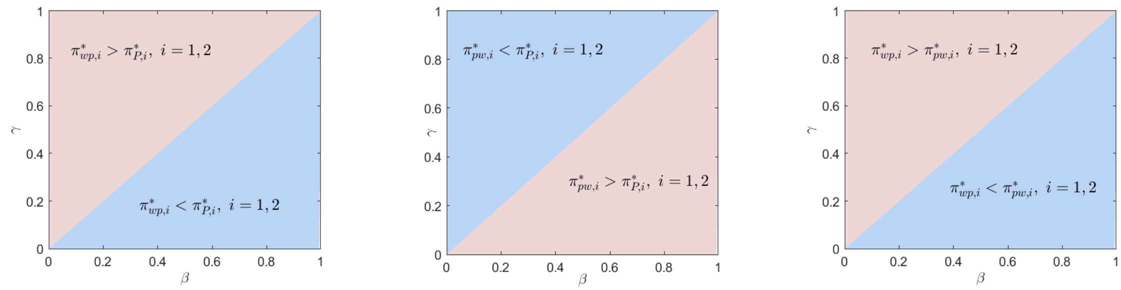
Observation 1 (Comparison of Modes P, wp, pw Under Asymmetric Market Sizes). Suppose asymmetric potential market sizes and the equilibria exist. If the demand side is more competitive than the supply side (i.e., $\gamma \geq \beta$), the preference ranking by the platform is: $wp \succeq P \succeq pw$. Otherwise, the ranking is reversed.

Model	Asymmetric Ω: $\Omega_1 > \Omega_2$	Asymmetric γ and β	Matching Friction
P vs. wp vs. pw	$wp \succeq P \succeq pw$ if $\gamma > \beta$; $wp \preceq P \preceq pw$ if $\gamma \leq \beta$.	if $\min\{\gamma_1, \gamma_2\} > \max\{\beta_1, \beta_2\}$, then $wp \succeq P \succeq pw$; if $\max\{\gamma_1, \gamma_2\} \leq \min\{\beta_1, \beta_2\}$, then $wp \preceq P \preceq pw$.	$wp \succeq P \succeq pw$ if $\gamma > \beta$; $wp \preceq P \preceq pw$ if $\gamma \leq \beta$.
C vs. wp vs. pw	$wp \succeq C \succeq pw$ if $\gamma > \beta$; $wp \preceq C \preceq pw$ if $\gamma \leq \beta$.	if $\min\{\gamma_1, \gamma_2\} > \max\{\beta_1, \beta_2\}$, then $wp \succeq C \succeq pw$; if $\max\{\gamma_1, \gamma_2\} \leq \min\{\beta_1, \beta_2\}$, then $wp \preceq C \preceq pw$.	$wp \succeq C \succeq pw$ if $\gamma > \beta$; $wp \preceq C \preceq pw$ if $\gamma \leq \beta$.
C vs. P	$C \succeq P$ unless γ is slightly lower than β.	$C \succeq P$ if $\gamma_i > \beta_i$, $i = 1, 2$; $C \preceq P$ if γ_i is slightly lower than β_i, $i = 1, 2$.	$C \succeq P$ unless γ is slightly lower than β.
Q vs. Others	$Q \succeq \max\{P, wp, C\}$; If $\Omega_1 \gg \Omega_2$, $\beta \to 1$, $\gamma \to 0$, then $Q \preceq pw$ for firm 1, and $Q \succeq pw$ for firm 2.	$Q \succeq \max\{P, wp, pw, C\}$.	$Q \succeq \max\{P, wp, pw, C\}$.

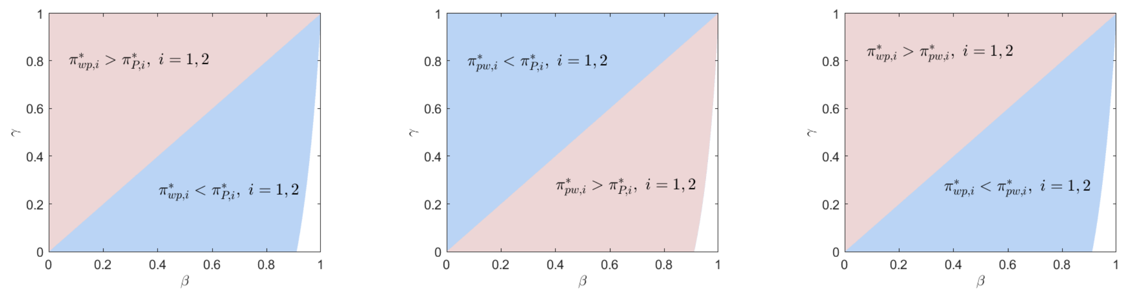
Table 2 Result Summary of the Comparisons for Asymmetric Platforms and Matching Friction

Observation 1 extends Proposition 1 (without demand uncertainty) from symmetric market sizes to asymmetric market sizes. Figure B.1 numerically displays the comparison among wage precommitment competition, simultaneous price and wage competition, and price precommitment competition. There is a region where the equilibrium does not exist when β is sufficiently large, and as the asymmetry increases (i.e., Ω_1 is increasingly greater than Ω_2), this region grows. This is because, facing a larger market size, platform 1 has an incentive to increase its supply. When β is sufficiently large, the competition on the supply side is so intense that both platforms need to offer very high wages to achieve their targeted level of supply, making it possible that the wage is higher than its price for platform 2 who faces a smaller market size. Consequently, platform 2 may be driven out of the market and the competition does not exist anymore. The larger the asymmetry, the larger the region. But as long as those equilibria exist, our result from the deterministic model that the comparison among wp, P, and pw depends on the comparison of the competition intensities of the two sides continues to hold.

Here, we comment on how to establish Observation 1. We first analytically derive the equilibrium profit for each mode (the profit expression for each mode is given in Online Supplement E). Then, we compare the equilibrium profits between any two modes. As one may notice, each profit expression includes four parameters, β, γ, Ω_1, and Ω_2. This makes it very challenging to analytically compare equilibrium profits as functions of four parameters. We normalize Ω_2 to 1 (without loss of generality) and vary Ω_1 by increasing it from 1 with a step size of 0.001. Then, for each fixed Ω_1, we enumerate β and γ both with a step size of 0.001, and check the ranking between the profit expressions. Finally, we do not observe any exceptions that Observation 1 fails to hold. The following Observation 2 is obtained similarly.



(a) wp vs. P (b) pw vs. P (c) wp vs. pw

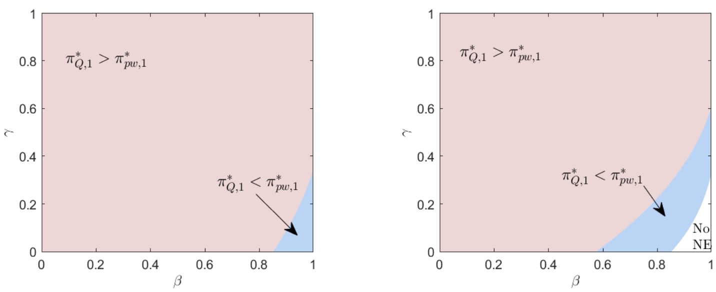


(d) wp vs. P (e) pw vs. P (f) wp vs. pw

Figure B.1 **Comparison among wp, P, and pw with asymmetric market sizes. Figures 2(a)-(c) assume $(\Omega_1, \Omega_2) = (4.1, 3.9)$, while Figures 2(d)-(f) assume $(\Omega_1, \Omega_2) = (5, 3)$.**

Observation 2 (Comparison of Mode Q with Others Under Asymmetric Market Sizes). Suppose asymmetric potential market sizes and the equilibria exist. (a) The platforms prefer Q over wp and P. (b) When the difference between Ω_1 and Ω_2 is sufficiently large, if β is sufficiently large and γ is sufficiently small, platform 1 prefers pw over Q, while platform 2 always prefers Q over pw.

Observation 2 shows that the insight from the deterministic model that quantity competition is the most profitable largely holds. Surprisingly, with asymmetric market sizes, when the demand side competition is not very intense and the supply side competition is sufficiently intense, price precommitment competition can enable the platform with a larger market size to obtain a higher profit than in quantity competition. Figure B.2 displays the comparison between price precommitment competition and quantity competition with asymmetric market sizes. In the lower-right corner, price precommitment competition earns platform 1 a higher profit, and as the asymmetry becomes more prominent, this region is growing.



(a) $\Omega_1 = 5, \Omega_2 = 3$ (b) $\Omega_1 = 6, \Omega_2 = 2$

Figure B.2 Comparison between pw and Q with asymmetric market sizes.

Finally, we numerically compare mode C with other modes. We provide a representative example to show the robustness of Proposition 2. Table 3(a) sets $(\Omega_1, \Omega_2, \gamma, \beta) = (5, 3, 0.7, 0.1)$ where the demand side is more competitive than the supply side and shows the equilibrium for each mode. We observe that $\pi^*_{wp,i} \geq \pi^*_{C,i} \geq \pi^*_{pw,i}$ for each $i = 1, 2$, implying that similar to simultaneous price and wage competition, the precommitment to the commission is less profitable than the direct precommitment on the less competitive side, but more profitable than the direct precommitment on the more competitive side, which is consistent with Proposition 2(a). Second, $\pi^*_{P,i} \leq \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment performs better than no commitment when one side is sufficiently more competitive than the other, which is consistent with Proposition 2 (b-i). Finally, $\pi^*_{C,i} \leq \pi^*_{Q,i}$ for each $i = 1, 2$, confirming that the precommitment to the matching quantity is more profitable than the commission precommitment.

Table 3(b) sets $(\Omega_1, \Omega_2, \gamma, \beta) = (5, 3, 0.45, 0.5)$ and compares mode C with mode P. It shows $\pi^*_{P,i} \geq \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough, which is consistent with Proposition 2(b-ii).

B.2. Asymmetry in β

This subsection assumes $\gamma_1 = \gamma_2$ in the demand system $d_i(\boldsymbol{p}) = [\Omega - p_i + \gamma_i p_j]^+, j \neq i, i = 1, 2$, but $\beta_1 \neq \beta_2$ in the supply system $s_i(\boldsymbol{w}) = [w_i - \beta_i w_j]^+, j \neq i, i = 1, 2$. That is, platforms are symmetric on the demand side but asymmetric on the supply side. Note that a large value of β_i means that platform i's supply is easily attracted away by the other platform, so $\beta_i > \beta_j$ implies that platform i is less attractive than platform j in the supply market.

Figure B.3 compares the three modes P, wp, and pw by fixing $\gamma_1 = \gamma_2 = 0.5$ and varying the values of β_1 and β_2 from 0.15 to 0.85 with a step size of 0.05. Although Figure B.3 shows the profit comparison for platform 2, the comparison for platform 1 can be obtained by reversing the

Mode	P	wp	pw	C	Q
p_1	6.9261	7.7264	6.8589	7.3984	7.8505
p_2	5.9475	6.7101	5.8838	6.4185	6.8289
w_1	2.4518	2.1621	2.4763	2.2935	2.1175
w_2	2.1459	1.9146	2.1651	1.9897	1.8782
z_1	2.2372	1.9707	2.2598	2.0945	1.9297
z_2	1.9008	1.6984	1.9174	1.7604	1.6664
π_1	10.0098	10.9653	9.9036	10.6924	11.0631
π_2	7.2259	8.1446	7.1304	7.7963	8.2501

(a) $(\gamma, \beta) = (0.7, 0.1)$

Mode	P	C
p_1	5.5479	5.5455
p_2	4.4521	4.4486
w_1	2.637	2.6397
w_2	2.363	2.3667
z_1	1.4555	1.4563
z_2	1.0445	1.0468
π_1	4.2368	4.2319
π_2	2.182	2.1795

(b) $(\gamma, \beta) = (0.45, 0.5)$

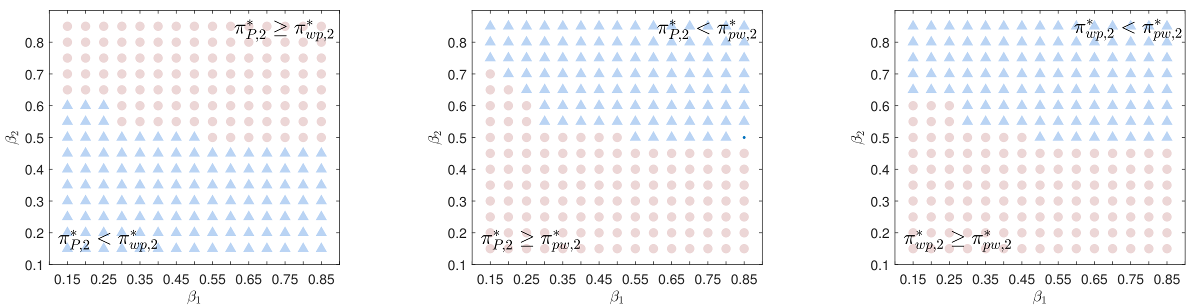
Table 3 Asymmetry in Market Sizes with $(\Omega_1, \Omega_2) = (5, 3)$

values of β_1 and β_2. For example, platform 1's profit comparison when $(\beta_1, \beta_2) = (0.15, 0.85)$ is the same as platform 2's profit comparison when $(\beta_1, \beta_2) = (0.85, 0.15)$. We focus on the comparison between modes wp and pw while their comparison with mode P is similar. In the upper-right region of Figure B.3(c), both β_1 and β_2 are greater than γ, indicating that the supply side is more competitive than the demand side. As expected, the price precommitment alleviates the competition and brings a higher profit for both platforms than the wage precommitment. Similarly, in the lower-left region where the demand side is more competitive than the supply side, the wage precommitment brings a higher profit for both platforms than the price precommitment. This is consistent with Proposition 1.

Moreover, in the lower-right region of Figure B.3(c) where $\beta_1 > \gamma$ and $\beta_2 < \gamma$, although it is not clear which of the two sides is more competitive, platform 1 is less attractive than platform 2 on the supply side. We find that the wage precommitment brings a higher profit to platform 2 than the price precommitment. This is because platform 2 has an advantage over platform 1 on the supply side, so the precommitment on the supply side enables platform 2 to take advantage of platform 1, thus increasing platform 2's profit. The upper-left region of of Figure B.3(c) can be interpreted similarly. In summary, a precommitment on the platform's advantageous (disadvantageous) side can bring a higher (lower) profit to that platform than a precommitment on the other side.

B.3. Asymmetry in both γ and β

This subsection analyzes the case in which the two platforms are asymmetric in the substitution factors on both the demand and supply sides. That is, the demand and supply systems take a general form: $d_i(\boldsymbol{p}) = [\Omega - p_i + \gamma_i p_j]^+, j \neq i, i = 1, 2,$ $s_i(\boldsymbol{w}) = [w_i - \beta_i w_j]^+, j \neq i, i = 1, 2.$ Note that a large value of γ_i means that platform i has a strong capability to attract demand from the other platform, that is, platform i is more attractive on the demand side. Similarly, a large value of β_i means platform i is less attractive on the supply side. Due to the complicated nature of this



(a) P vs. wp (b) P vs. pw (c) wp vs. pw

Figure B.3 Comparisons among P, wp, and pw for asymmetry in β.

general case, we provide several representative examples of the parameter set to investigate the comparisons among different modes.

In Tables 4 and 5, $\max\{\gamma_1,\gamma_2\} < \min\{\beta_1,\beta_2\}$, which means that the demand side is less competitive than the supply side. We find $\pi^*_{wp,i} \leq \pi^*_{P,i} \leq \pi^*_{pw,i}$ for each $i = 1,2$, indicating that a precommitment on the demand side (the less competitive side) alleviates the competition and generates a higher profit, which is consistent with Proposition 1. Second, $\pi^*_{wp,i} \leq \pi^*_{C,i} \leq \pi^*_{pw,i}$ for each $i = 1,2$, implying that similar to simultaneous price and wage competition, the commission precommitment is less profitable than the direct precommitment on the less competitive side, but more profitable than the direct precommitment on the more competitive side, which is consistent with Proposition 2(a). Third, $\pi^*_{Q,i}$ for each $i = 1,2$ is the highest among different modes, implying that the precommitment to the matching quantity is the most profitable compared to the precommitment to the wage, price, or commission, which is consistent with Proposition 3. In Tables 6 and 7, $\min\{\gamma_1,\gamma_2\} > \max\{\beta_1,\beta_2\}$, which means that the demand side is more competitive than the supply side. Again, we obtain results consistent with those from the deterministic model. Finally, in Tables 4-7, $\pi^*_{P,i} < \pi^*_{C,i}$ for each $i = 1,2$, implying that the commission precommitment performs better than no commitment when one side is sufficiently more competitive than the other. Table 8 shows $\pi^*_{P,i} > \pi^*_{C,i}$ for each $i = 1,2$, implying that the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough. These results are consistent with Proposition 2(b).

In Tables 4 and 6, $\gamma_1 < \gamma_2$, indicating that platform 1 is less attractive than platform 2 on the demand side, and $\beta_1 > \beta_2$, indicating that platform 1 is less attractive than platform 2 on the supply side. That is, platform 1 is less advantageous than platform 2 on both demand and supply sides. Not surprisingly, platform 1's profit is substantially lower than that of platform 2. In Tables 5

Mode	P	wp	pw	C	Q
p_1	0.98	0.9899	0.99	0.99	1.01
p_2	1.0925	1.083	1.12	1.1	1.1335
w_1	0.62	0.63	0.59	0.613	0.5532
w_2	0.5915	0.61	0.55	0.583	0.5243
z_1	0.1764	0.1725	0.1775	0.1758	0.16
z_2	0.2505	0.2635	0.2255	0.2459	0.22
π_1	0.0635	0.0621	0.071	0.0663	0.0731
π_2	0.1249	0.1246	0.1285	0.1271	0.134

Table 4 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.15, 0.35, 0.75, 0.55)$

Mode	P	wp	pw	C	Q
p_1	0.964	0.9678	0.98	0.9719	0.9921
p_2	1.1079	1.0987	1.13	1.1136	1.1472
w_1	0.559	0.56	0.522	0.5443	0.4936
w_2	0.649	0.66	0.604	0.6348	0.57
z_1	0.20205	0.197	0.1898	0.1951	0.1801
z_2	0.22975	0.24	0.2125	0.2266	0.1998
π_1	0.0818	0.0803	0.08693	0.0835	0.0898
π_2	0.1054	0.1053	0.1118	0.1085	0.1153

Table 5 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.15, 0.35, 0.55, 0.75)$

Mode	P	wp	pw	C	Q
p_1	1.46	1.554	1.42	1.503	1.5966
p_2	1.5945	1.681	1.57	1.6428	1.7574
w_1	0.62	0.57	0.65	0.6014	0.553
w_2	0.5935	0.57	0.592	0.575	0.523
z_1	0.4123	0.3705	0.4428	0.4001	0.37
z_2	0.5005	0.4845	0.4945	0.4848	0.44
π_1	0.3463	0.3646	0.341	0.3608	0.3861
π_2	0.501	0.5383	0.4836	0.5177	0.5432

Table 6 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.55, 0.75, 0.35, 0.15)$

Mode	P	wp	pw	C	Q
p_1	1.43	1.522	1.41	1.49	1.5813
p_2	1.6016	1.71	1.57	1.6668	1.766
w_1	0.54	0.51	0.5558	0.5216	0.4781
w_2	0.6599	0.61	0.682	0.6334	0.5873
z_1	0.441	0.4185	0.4535	0.4266	0.39
z_2	0.4709	0.4315	0.4875	0.4508	0.42
π_1	0.3925	0.4235	0.3874	0.4131	0.4302
π_2	0.4434	0.4747	0.4329	0.4659	0.495

Table 7 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.55, 0.75, 0.15, 0.35)$

Mode	P	C
p_1	1.4	1.3956
p_2	1.44	1.4321
w_1	0.75	0.7536
w_2	0.78	0.7877
z_1	0.321	0.3204
z_2	0.33	0.3355
π_1	0.2087	0.2057
π_2	0.2178	0.2162

Table 8 Asymmetry in both γ and β with $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.5, 0.55, 0.55, 0.6)$

and 7, $\gamma_1 < \gamma_2$, indicating that platform 1 is still less attractive than platform 2 on the demand side, but $\beta_1 < \beta_2$, indicating that platform 1 is more attractive than platform 2 on the supply side. Surprisingly, platform 1's profit is still fairly lower than that of platform 2, implying that the demand-side advantage can be more critical to the platform than the supply side.

C. Matching Friction

Now we extend our deterministic model to incorporate the friction in the process of matching supply with demand. In view of Bernstein et al. (2021), we capture customer's wait time in the steady state of the matching process before getting matched as follows: $W_i = \begin{cases} (\frac{d_i}{s_i})^2 & \text{if } d_i < s_i, \\ \infty & \text{otherwise.} \end{cases}$ If $s_i \le d_i$, the process becomes unstable, in which case customer's wait time is defined as ∞. Only if $d_i < s_i$, the system can serve all the demand and reach a steady state, and the more supply, the less wait time for customers. Thus the demand system can be enriched as follows: $d_i(\boldsymbol{p}) = d_i(p_i, p_j) = [\Omega - (p_i + hW_i) + \gamma(p_j + hW_j)]^+, j \ne i, i = 1, 2.$

Since the demand is always smaller than supply in equilibrium, not all drivers will be utilized all the time. Hence, the effective wage expected by each driver is $w_i \frac{d_i}{s_i}$, where $\frac{d_i}{s_i}$ is the utilization rate of drivers. Thus the supply system becomes $s_i(\boldsymbol{w}) = s_i(w_i, w_j) = \left[w_i \frac{d_i}{s_i} - \beta w_j \frac{d_j}{s_j}\right]^+, j \ne i,\, i = 1, 2.$

We note that unlike mode Q of the deterministic model in which supply is equal to demand and equal to the committed matching quantity, mode Q here assumes that the platforms commit to both demand and supply quantities, because in this extension demand is always smaller than supply; otherwise, the system would not be stable.

Due to the intractable nature of this extension, we resort to a numerical study to check the robustness of our findings. We provide several representative examples of the parameter set to investigate the comparisons among different competition modes.

In Tables 9-12,[10] we allow $h \in \{0, 0.1, 0.2, 0.3\}$, $\gamma \in \{0.6, 0.7\}$, and $\beta \in \{0.1, 0.2\}$, where the demand side is more competitive than the supply side. In every single table of the 16 combinations

[10] The value of w in mode C is calculated by multiplying the equilibrium price and equilibrium commission rate.

of the above parameter set, first, we find that $\pi^*_{wp} \geq \pi^*_P \geq \pi^*_{pw}$, implying that a precommitment on the supply side (the less competitive side) alleviates the competition and generates a higher profit than no precommitment, which, in turn, has a higher profit than the precommitment on the demand side (the more competitive side). This result is consistent with Proposition 1(a), indicating that taking consumers' wait time into consideration seems not to alter the effects of precommitments in various devices. Here we examine the underlying rationale in detail. Unlike the deterministic model, the wage precommitment cannot determine the supply quantity in the presence of matching friction; however, compared with mode P, a relatively lower wage still leads to a relatively lower supply quantity, even though the supply quantity is also affected by the utilization rate (but the effect of the utilization rate on the supply quantity is less prominent than that of the wage). This observation is confirmed by every single table in Tables 9-12. Suppose for a contradiction that a relatively lower wage leads to a relatively higher supply quantity, which means the utilization rate should be unreasonably high, which in turn implies a higher demand quantity. Given that the high utilization rate plays a less significant role in the demand system, a higher demand quantity requires a lower price, and thus a lower profit margin, which is not optimal. In summary, the incorporation of customers' wait time makes the system of matching demand and supply more complicated, but its effect on the demand or supply is secondary compared with that of the price or wage. Therefore, although the wage precommitment cannot determine the supply quantity directly, the reduced power of the precommitment can still alleviate the competition intensity in the second stage, leading to a less intense outcome than mode P. The comparison between modes P and pw can be interpreted similarly.

Second, $\pi^*_{wp} \geq \pi^*_C \geq \pi^*_{pw}$, implying that the commission precommitment is less profitable than the precommitment on the less competitive side, but more profitable than the precommitment on the more competitive side, which is consistent with Proposition 2(a). Again, although the presence of customers' wait time weakens the power of the precommitments, the commission precommitment is still not as direct and effective as the wage precommitment. Moreover, $\pi^*_C \geq \pi^*_P$, implying that when one side is more competitive than the other, the commission precommitment can be better than no precommitment at all. On the other hand, as shown in Table 13, the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough. These results are consistent with Proposition 2(b).

Finally, π^*_Q is the highest among various modes, implying that the precommitment to the demand and supply quantities is still the most effective compared to the precommitment to wage, price, or commission, which is consistent with Proposition 3, because, compared with other precommitments, the precommitment to both supply and demand quantities is more direct and suffers the least impact from the matching friction. With the ordering of the two-sided intensities reversed, Tables 14-17 with $\gamma < \beta$ also display consistent results.

Mode	P	wp	pw	C	Q
p	1.4	1.72	1.4	1.54	1.49
w	0.52	0.41	0.53	0.4928	0.4511
d	0.44	0.312	0.44	0.384	0.404
s	0.454	0.3395	0.4579	0.4125	0.405
π	0.3872	0.4087	0.3828	0.4021	0.4197

(a) $h = 0$

Mode	P	wp	pw	C	Q
p	1.37	1.54	1.35	1.43	1.4305
w	0.5	0.46	0.51	0.4719	0.4333
d	0.4151	0.3501	0.4231	0.3911	0.388
s	0.432	0.3809	0.4406	0.4077	0.389
π	0.3612	0.3782	0.3554	0.3747	0.3869

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p	1.32	1.47	1.3	1.42	1.37
w	0.48	0.44	0.49	0.4544	0.4156
d	0.3983	0.3428	0.4063	0.3613	0.372
s	0.4147	0.3683	0.4234	0.3844	0.373
π	0.3345	0.3531	0.3291	0.3489	0.3555

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.29	1.4	1.24	1.41	1.3142
w	0.45	0.42	0.45	0.4521	0.3967
d	0.3734	0.334	0.3888	0.3366	0.355
s	0.3888	0.3553	0.3969	0.3695	0.356
π	0.3137	0.3273	0.3071	0.3227	0.3257

(d) $h = 0.3$

Table 9 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.6, 0.1)$

Mode	P	wp	pw	C	Q
p	1.51	1.74	1.45	1.67	1.5175
w	0.56	0.43	0.57	0.501	0.4938
d	0.396	0.304	0.42	0.332	0.393
s	0.4211	0.3234	0.4378	0.3647	0.394
π	0.3762	0.3982	0.3696	0.3881	0.4023

(a) $h = 0$

Mode	P	wp	pw	C	Q
p	1.48	1.67	1.39	1.61	1.458
w	0.54	0.44	0.54	0.4991	0.4738
d	0.3734	0.2981	0.4064	0.3236	0.377
s	0.4018	0.3238	0.419	0.3594	0.378
π	0.351	0.3667	0.3454	0.3595	0.3711

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p	1.37	1.56	1.32	1.55	1.3961
w	0.54	0.48	0.54	0.5425	0.455
d	0.3813	0.3112	0.3983	0.3208	0.362
s	0.4061	0.3456	0.4147	0.3732	0.363
π	0.3165	0.3361	0.3107	0.3232	0.3407

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.29	1.45	1.247	1.41	1.4094
w	0.48	0.47	0.49	0.4935	0.3988
d	0.3688	0.3184	0.3768	0.3344	0.317
s	0.3763	0.3459	0.3842	0.3632	0.318
π	0.2987	0.3121	0.2939	0.3065	0.3204

(d) $h = 0.3$

Table 10 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.6, 0.2)$

Note that Tables 9-12 allow $h = 0$, which is an extension of the deterministic model's supply system to account for the drivers' utilization while keeping the demand system unchanged as being linear. We find that the main results in Propositions 1-3 still hold in this extension. Compared

Mode	P	wp	pw	C	Q
p^*	1.64	1.87	1.62	1.83	1.8367
w^*	0.6	0.54	0.62	0.5673	0.5011
d^*	0.508	0.439	0.514	0.451	0.449
s^*	0.5238	0.4617	0.5357	0.4799	0.45
π^*	0.5283	0.5839	0.514	0.5695	0.5997

(a) $h = 0$

Mode	P	wp	pw	C	Q
p^*	1.56	1.84	1.53	1.79	1.7805
w^*	0.57	0.53	0.58	0.537	0.4867
d^*	0.5026	0.4215	0.5116	0.4359	0.436
s^*	0.5079	0.4484	0.5168	0.4591	0.437
π^*	0.4976	0.5522	0.486	0.5462	0.5641

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p	1.52	1.81	1.49	1.74	1.7276
w	0.55	0.52	0.56	0.522	0.4711
d	0.4852	0.4051	0.4942	0.4239	0.422
s	0.49	0.4352	0.499	0.4463	0.423
π	0.4706	0.5226	0.4596	0.5162	0.5302

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.51	1.71	1.47	1.66	1.6715
w	0.56	0.5	0.56	0.5312	0.4567
d	0.4641	0.4058	0.4743	0.4225	0.409
s	0.4838	0.4275	0.4889	0.4494	0.41
π	0.4409	0.491	0.4316	0.4769	0.4969

(d) $h = 0.3$

Table 11 **Matching Friction with** $(\Omega, \gamma, \beta) = (1, 0.7, 0.1)$

Mode	P	wp	pw	C	Q
p^*	1.73	1.98	1.61	1.85	1.87
w^*	0.68	0.6	0.66	0.629	0.5513
d^*	0.481	0.406	0.517	0.445	0.439
s^*	0.5114	0.4416	0.5227	0.473	0.44
π^*	0.5051	0.5603	0.4912	0.5433	0.579

(a) $h = 0$

Mode	P	wp	pw	C	Q
p^*	1.7	1.97	1.61	1.82	1.8138
w^*	0.67	0.58	0.65	0.6188	0.535
d^*	0.4641	0.3842	0.4888	0.4281	0.426
s^*	0.4985	0.4222	0.5044	0.4604	0.427
π^*	0.478	0.5339	0.4692	0.5142	0.5448

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	1.6	1.94	1.59	1.79	1.7576
w^*	0.63	0.57	0.65	0.6086	0.5188
d^*	0.4647	0.3694	0.4689	0.4122	0.413
s^*	0.4838	0.4104	0.494	0.4479	0.414
π^*	0.4508	0.5061	0.4407	0.4869	0.5117

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.51	1.77	1.47	1.78	1.7048
w	0.6	0.58	0.6	0.623	0.5013
d	0.4606	0.3928	0.4708	0.3947	0.399
s	0.4704	0.4269	0.4752	0.4436	0.4
π	0.4191	0.4675	0.4096	0.4566	0.4802

(d) $h = 0.3$

Table 12 **Matching Friction with** $(\Omega, \gamma, \beta) = (1, 0.7, 0.2)$

Mode	P	C
p^*	1.38	1.32
w^*	0.57	0.6204
d^*	0.241	0.274
s^*	0.2622	0.2916
π^*	0.1952	0.1917

(a) $h = 0$

Mode	P	C
p^*	1.26	1.19
w^*	0.57	0.595
d^*	0.2574	0.2915
s^*	0.2708	0.2945
π^*	0.1776	0.1735

(b) $h = 0.1$

Table 13 **Matching Friction with** $(\Omega, \gamma, \beta) = (1, 0.45, 0.5)$

with the deterministic model, the equilibrium wage increases because of the consideration of the utilization rate. Moreover, since the demand needs to be smaller than the supply, a higher price is charged to regulate the demand. Hence, due to a smaller demand and a higher wage, the platforms

Mode	P	wp	pw	C	Q
p^*	0.85	0.87	0.83	0.84	0.8312
w^*	0.44	0.43	0.42	0.4368	0.4125
d^*	0.1555	0.1425	0.1678	0.1611	0.163
s^*	0.1654	0.1565	0.1663	0.1677	0.164
π^*	0.0637	0.0627	0.0676	0.0649	0.0682

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.78	0.86	0.76	0.81	0.7505
w^*	0.43	0.39	0.39	0.405	0.3725
d^*	0.1456	0.105	0.1466	0.1284	0.147
s^*	0.1582	0.128	0.1513	0.1442	0.148
π^*	0.051	0.0493	0.0543	0.052	0.0556

(b) $h = 0.2$

Table 14 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.1, 0.6)$

Mode	P	wp	pw	C	Q
p^*	1.01	1.03	1	0.96	0.9261
w^*	0.38	0.32	0.31	0.4704	0.455
d^*	0.1258	0.1083	0.1216	0.1628	0.18
s^*	0.1383	0.1178	0.1228	0.175	0.181
π^*	0.0792	0.0769	0.0839	0.0797	0.0848

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.95	0.98	0.92	0.96	0.8462
w^*	0.43	0.33	0.38	0.384	0.4175
d^*	0.1244	0.0977	0.1286	0.1137	0.165
s^*	0.1462	0.1135	0.1398	0.1321	0.166
π^*	0.0647	0.0635	0.0694	0.0655	0.0707

(b) $h = 0.2$

Table 15 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.2, 0.6)$

Mode	P	wp	pw	C	Q
p^*	0.9	0.93	0.89	0.88	0.8626
w^*	0.5	0.47	0.42	0.5016	0.4567
d^*	0.1187	0.0995	0.1161	0.1302	0.135
s^*	0.1335	0.1184	0.121	0.1399	0.136
π^*	0.0475	0.0458	0.0545	0.0492	0.0548

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.84	0.89	0.82	0.79	0.7788
w^*	0.5	0.49	0.41	0.4898	0.4134
d^*	0.1109	0.0895	0.1063	0.123	0.122
s^*	0.129	0.1147	0.1144	0.138	0.123
π^*	0.0377	0.0358	0.0436	0.039	0.0446

(b) $h = 0.2$

Table 16 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.1, 0.7)$

Mode	P	wp	pw	C	Q
p^*	1.01	1.07	1.01	1.06	0.9626
w^*	0.56	0.43	0.49	0.4028	0.51
d^*	0.13	0.0889	0.1243	0.0914	0.151
s^*	0.1478	0.1071	0.1352	0.1051	0.152
π^*	0.0585	0.0569	0.0646	0.0601	0.0683

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.97	1.04	0.97	0.96	0.88
w^*	0.55	0.32	0.33	0.5376	0.4667
d^*	0.1338	0.063	0.0856	0.1164	0.138
s^*	0.137	0.0778	0.0921	0.1371	0.139
π^*	0.0478	0.0454	0.0548	0.0492	0.0571

(b) $h = 0.2$

Table 17 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.2, 0.7)$

admit a lower profit than in the deterministic model. Lastly, when h increases from 0 to 0.3, the profit decreases, because the wait time reduces consumers' utility. Meanwhile, the price also decreases, but the demand does not necessarily increase, because the wait time also hurts the demand.

In addition to the robustness check, we would like to point out several differences from the deterministic model. First, in the deterministic model, a precommitment to either wage or price can determine the supply or demand quantity, constraining the subsequent competition on the other side. However, due to the existence of consumers' waiting, the precommitment to either wage or price cannot determine the supply or demand quantity, thereby limiting the power of the precommitment to alleviate the competition on the other side. But such a reduced power of the precommitment still plays a critical role in the competitions, and thus our main results remain robust after considering the wait time. Second, in the deterministic model, a higher equilibrium price corresponds to a higher equilibrium profit and a lower equilibrium wage and matching quantity (thus lower consumer surplus and service provider surplus). This may not be true in the presence of consumers' waiting. (i) Since the equilibrium demand is not equal to the supply, a higher price is not necessarily coupled with a lower wage; i.e., see modes wp and Q in Tables 9(b)-(d). (ii) Wait time also makes the relationship between price and demand more involved. On the one hand, a higher price may drive some consumers out of the market. On the other hand, a smaller amount of potential consumers leads to a lower wait time, which can boost the demand to some extent. Although the overall relationship between price and demand does not change, the wait time acts in the opposite direction to what the price does. Similarly, although a higher wage leads to more service providers, the utilization rate of each service provider is lower. That is, although there are more service providers, each service provider earns less, and the surplus of each individual service provider is reduced.

D. Proof of Lemmas and Propositions in the Main Body

Proof of Lemma 1. Suppose the realized market size is x. For any fixed (p_2, w_2), platform 1's best choice is to render the demand equal to the supply quantity. Otherwise, the platform could always improve its profit by either raising the price or lowering the wage. Let z_1 denote the matching quantity, then

$$x - p_1^* + \gamma p_2 = z_1 = w_1^* - \beta w_2.$$

Hence,

$$p_1^* = x + \gamma p_2 - z_1, \quad w_1^* = z_1 + \beta w_2. \tag{D.1}$$

Platform 1 faces such an optimization problem:

$$\begin{aligned} \max_{z_1} \quad & (x + \gamma p_2 - 2z_1 - \beta w_2) z_1 \\ s.t. \quad & z_1 \leq \frac{1}{2}(x + \gamma p_2 - \beta w_2). \end{aligned}$$

Note that the above constraint is derived by $p_1^* \geq w_1^*$. Solving the problem yields $z_1^* = \frac{x+\gamma p_2-\beta w_2}{4}$. Plugging back into (D.1) gives the best response of platform 1 as follows,

$$p_1^*(p_2, w_2) = \frac{3}{4}(x+\gamma p_2) + \frac{1}{4}\beta w_2, \quad w_1^*(p_2, w_2) = \frac{1}{4}(x+\gamma p_2) + \frac{3}{4}\beta w_2.$$

By the same token, we obtain

$$p_2^*(p_1, w_1) = \frac{3}{4}(x+\gamma p_1) + \frac{1}{4}\beta w_1, \quad w_2^*(p_1, w_1) = \frac{1}{4}(x+\gamma p_1) + \frac{3}{4}\beta w_1.$$

Solving the set of the above equations yields the resulting equilibrium price p_P^*, wage w_P^*, matching quantity z_P^*, and profit π_P^*. Taking expectation with respect to Ω yields the expected equilibrium profit $E[\pi_P^*]$. □

Proof of Lemma 2. Online Supplement B.1 derives the equilibrium of wage precommitment competition under demand uncertainty. The general idea is as follows. We first analyze the subgame equilibrium conditional on a fixed wage in the first stage and the realization of market size (shown in Lemma B.1), and show that for any realized market size x, the equilibrium price in the second stage takes either the supply-depletion price or profit-maximizing price. Based on the optimal decision in the second stage, we then come back to the first stage and discuss different cases regarding to the magnitude of the wage set in the first stage. There are three cases:

1. The wage is too low such that for any realized market size x, both firms adopt the supply-depletion price in the second stage;
2. The wage is in a medium range such that there exists a threshold of the market size, beyond which both firms adopt the supply-depletion price and adopt the profit-maximizing price otherwise;
3. The wage is too high such that for any realized market size x, both firms adopt the profit-maximizing price in the second stage.

When there is no demand uncertainty, we show that the resulting equilibrium wages and prices are such that the supply is equal to the demand (shown in Lemma B.2), which means the equilibrium price in the second stage must be the supply-depletion price. When the demand variance is sufficiently small, it is still optimal to set the wage ex ante such that both firms adopt a supply-depletion price for any market size. That is, Case 1 occurs. According to Case 1 in Online Supplement B.2, since $\text{Var}(\Omega)$ is sufficiently small, one can check

$$\frac{1+\gamma}{4-3\beta+\gamma-2\gamma^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2} < \frac{1}{3-2\beta-2\gamma+\beta\gamma},$$

which implies $w_{i,c1}^* < w_{ub}^1$. Therefore,

$$w_{wp}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\gamma^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2} E[\Omega].$$

Putting w^*_{wp} back to (B.18) and (B.19) gives the equilibrium price p^*_{wp} and the equilibrium expected profit $E[\pi^*_{wp}]$. □

Proof of Lemma 3. Online Supplement C derives the equilibrium of price precommitment competition under demand uncertainty. The general idea is as follows. We first analyze the subgame equilibrium conditional on a fixed price in the first stage and the realization of market size (shown in Lemma C.1), and show that for any realized market size x, the equilibrium wage in the second stage takes either the demand-depletion wage or profit-maximizing wage. Based on the optimal decision in the second stage, we then come back to the first stage and discuss different cases regarding to the magnitude of the price decided in the first stage. There are three cases:

1. The price is too high such that for any realized market size x, both firms adopt the demand-depletion wage in the second stage;

2. The price is in a medium range such that there exists a threshold of the market size, beyond which both firms adopt the profit-maximizing wage and adopt the demand-depletion wage otherwise;

3. The price is too low such that for any realized market size x, both firms adopt the profit-maximizing wage in the second stage.

When there is no demand uncertainty, we show that the resulting equilibrium wages and prices are such that the supply is equal to the demand (shown in Lemma C.2), which means the equilibrium wage in the second stage must be the demand-depletion wage. When the demand variance is sufficiently small, it is still optimal to set the price ex ante such that both firms adopt a demand-depletion wage for any market size. That is, Case 1 occurs. According to Case 1 in Online Supplement C.2, since Var(Ω) is sufficiently small, one can check

$$\frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2} > \frac{2+\beta-\beta^2}{3-2\gamma+\beta-\beta\gamma-2\beta^2+\beta^2\gamma},$$

which implies $p^*_{i,c1} > p^1_{lb}$. Therefore,

$$p^*_{pw} = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2} E[\Omega].$$

Putting p^*_{pw} back to (C.8) and (C.9) gives the equilibrium wage w^*_{pw} and the equilibrium expected profit $E[\pi^*_{pw}]$. □

Due to the space limit of the Online Appendix, the remaining proofs of lemmas and propositions in the main body are relegated to part D of an Online Supplement, which can be found on the authors' websites.