Online Appendix to the Paper "Newsvendor Selling to Loss Averse Consumers with Stochastic Reference Points"

Appendix. A. Extensions

In this section, we relax some assumptions of the base model and show that our main findings are robust for these extensions.

A.1. Inventory Rationing During Sales

We consider the extension where the newsvendor firm rations inventory during sales. When the product is priced at the sale price, both the loss-averse consumers and bargain hunters are present. Let $\delta \in [0,1]$ denote the chance of consumers being served during sales. Let $\overline{p}^*(\delta)$, $\tau^*(\delta)$ and $\Pi^*(\delta)$ be the firm's optimal full price, optimal sales threshold and optimal expected profit given a rationing parameter δ. We have the following proposition:

Proposition 10 *Given a fixed order quantity q, $\overline{p}^*(\delta)$ is increasing, $\tau^*(\delta)$ is decreasing, and $\Pi^*(\delta)$ is increasing in the rationing parameter δ.*

Proposition 10 suggests that when consumers have greater chances of being served, the fill rate will increase, which drives up the full price and demands less frequent sales, so the firm is better off, with a higher profit margin and less sales. On the other hand, if the bargain hunters have greater chances of being served, then the firm needs to run more frequent sales and lower the full price to compensate for the loss-averse consumers, because those consumers see relatively lower fill rates compared to when they are prioritized over bargain hunters. Proposition 10 implies that if viable, the firm would prefer to prioritize consumers during sales, as assumed in the base model.

A.2. Consumer Heterogeneity

A.2.1. Heterogeneity in Loss Aversion

We consider a market with heterogeneity in loss aversion. Consumers have the same consumption valuation v, but differ in the degree of loss aversion. One segment is loss neutral, i.e., $\lambda = 1$, and the other is loss averse, i.e., $\lambda > 1$. The fraction of loss neutral (averse) consumers is α_l (α_h), with $\alpha_l + \alpha_h = 1$. The loss-neutral consumers buy at a price p if and only if $p \leq v$. Conditional on the relations between p and v, there are three cases of personal equilibria for the loss-averse consumers. For each case, we analyze the firm's optimal decisions on the order quantity and contingent pricing, and show that most managerial insights obtained from Propositions 3 – 4 sustain under mild conditions.

Equilibrium 1. In the first (possible) equilibrium, the loss neutral consumers buy at both the sales price s and the regular price p_l, while the loss averse consumers *only* buy at the sales price s. For this equilibrium to sustain, the following three constraints must be satisfied simultaneously:

$$\phi_l = \int_0^{\tau_l} \frac{\min\{x,q\}}{x} dF(x), \quad p_l \geq v + \frac{\phi_l(s+v)-v}{1+\eta\lambda}(\lambda-1)\eta, \quad p_l \leq v. \tag{A-1}$$

In this case, the reference points for the loss averse consumers are $(v,-s)$ with probability ϕ_l and $(0,0)$ with probability $1-\phi_l$. The second constraint of (A-1) is derived from $U((v,-p_l)|\Gamma) \leq U((0,0)|\Gamma)$ where $\Gamma = \{(v,-s),(0,0)\}$, ensuring that the full price is sufficiently high such that the loss averse consumers would not deviate from their purchase plans. The third constraint of (A-1) ensures that the loss neutral consumers buy at the full price p_l. *Equilibrium 1* corresponds to the Case (iv) of Proposition 16 in the Supplemental Note, where consumers with loss aversion λ less than a threshold choose to buy at both prices. In this equilibrium, the firm's expected profit is $\Pi_l(p_l,\tau_l) = sqF(\tau_l) + p_l \int_{\tau_l}^{+\infty} \min\{\alpha_l x, q\} dF(x) - cq$, The optimal solution to $\Pi_l(p_l,\tau_l)$ subject to constraints in (A-1) is $p_l^* = v,\ \tau_l^* = \min\left\{\frac{sq}{\alpha_l v},\ F^{-1}\left(\frac{v}{v+s}\right)\right\}$. Note that the optimal solution only depends on order quantity q and is independent of loss aversion parameter λ, so the firm's expected profit is a function of q. Based on these properties, we have comparative statics results as (i) of Proposition 11.

Equilibrium 2. In the second (possible) equilibrium, the loss averse consumers buy at both the sales price s and the regular price p_h, while the loss neutral consumers only buy at the sales price s. For this equilibrium to sustain, the following constraints must be satisfied simultaneously:

$$\begin{aligned} \phi_h &= \int_0^{\tau_h} \frac{\min\{x,q\}}{x} dF(x) + \int_{\tau_h}^{+\infty} \frac{\min\{\alpha_h x, q\}}{\alpha_h x} dF(x), \\ p_h &= v + \frac{\phi_h s F(\tau_h) - [1-\phi_h(2-F(\tau_h))]v}{1+\eta[\lambda-\phi_h(\lambda-1)(1-F(\tau_h))]}(\lambda-1)\eta, \quad p_h > v. \end{aligned} \tag{A-2}$$

The third constraint in (A-2) ensures that the loss neutral consumers will not buy at the full price. *Equilibrium 2* corresponds to Case (iii) of Proposition 16 in the Supplemental Note, where consumers with loss aversion λ greater than a threshold choose to buy at both prices. In this equilibrium, the firm's expected profit is $\Pi_h(p_h,\tau_h) = sqF(\tau_h) + p_h \int_{\tau_h}^{+\infty} \min\{\alpha_h x, q\} dF(x) - cq$, Note that the fill rate in the homogenous consumer case, i.e., ϕ defined as (10), is independent of the sales threshold τ; while the fill rate ϕ_h, defined as the first constraint in (A-2), is dependent of τ_h and is non-increasing in τ_h. Therefore, the sales threshold τ_h affects the regular price, both as a direct factor in the functional form of the regular price, and as an indirect factor through the fill rate ϕ_h. This dependence can be explicitly shown in the first order condition as follows.

Given a fixed order quantity q, the optimal sales threshold τ_h^* must satisfy the following first order condition:

$$\frac{\partial \Pi_h}{\partial \tau_h} = f(\tau_h)\left[sq - p_h \alpha_h \tau_h + \left(\frac{\partial p_h}{\partial F} + \frac{\partial p_h}{\partial \phi_h}\frac{\partial \phi_h}{\partial \tau_h}\right)\int_{\tau_h}^{\infty} \min\{\alpha_h x, q\} dF(x)\right] = 0. \tag{A-3}$$

where $\partial\phi_h/\partial\tau_h = \kappa(\tau_h) = \begin{cases} 0, & \text{if } \tau_h \le q; \\ q/\tau_h - 1, & \text{if } \tau_h > q. \end{cases}$ Comparing (11) with the terms in the bracket at the right hand side of (A-3), we note that the existence of $\kappa(\tau_h)$ captures the dependence of the fill rate ϕ_h on the sales threshold τ_h. The comparative statics of *Equilibrium 2* are summarized as Proposition 12.

Equilibrium 3. In the third (possible) equilibrium, both the loss neutral consumers and the loss averse consumes buy at the full price p. For this equilibrium to sustain, the following constraints must be satisfied simultaneously:

$$\phi_o = \int_0^{+\infty} \frac{\min\{x,q\}}{x} dF(x), \quad p_o \le v + \frac{\phi_o sF(\tau) - [1-\phi_o(2-F(\tau))]v}{1+\eta[\lambda - \phi_o(\lambda-1)(1-F(\tau))]}(\lambda-1)\eta, \quad p_o \le v. \quad \text{(A-4)}$$

The third constraint in (A-4) ensures that the loss neutral consumers also buy at the regular price p_o. In this equilibrium, the firm's expected profit is $\Pi_o(p_o, \tau_o) = sqF(\tau_o) + p_o \int_{\tau_o}^{+\infty} \min\{x,q\}dF(x) - cq$. The firm's objective is to maximize $\Pi_o(p_o, \tau_o)$ subject to constraints in (A-4). In optimum, either the second constraint or the third constraint in (A-4) must be binding. Conditional on the binding constraint, we have the firm's pricing strategy and the comparative statics results as points (ii) and (iii) of Proposition 11.

Proposition 11 *In the case of two consumer types, we have*

- *If* **Equilibrium 1** *prevails, then*

(i) both the optimal order quantity and the optimal sales threshold (when order quantity is optimized) are decreasing in the procurement cost c;

- *If* **Equilibrium 3** *prevails, then*

(ii) in the regime where the second constraint in (A-4) is binding in optimum, the results of Proposition 3 to Proposition 4 hold;

(iii) in the regime where the third constraint in (A-4) is binding in optimum, the regular price is v and the optimal sales threshold is $\tau_o^* = sq/v$, and both the optimal order quantity and the optimal sales threshold (when order quantity is optimized) are decreasing in the procurement cost c.

Proposition 12 *If* **Equilibrium 2** *prevails, then*

(i) In the neighborhood where the optimal sales threshold τ_h^* is decreasing in the order quantity q,

(a) the optimal full price p_h^* and the expected price $\widehat{p}_h^* = sF(\tau_h^*) + p_h^*(1-F(\tau_h^*))$ is increasing in the order quantity q;

(b) the results of Propositions 7 and 4 continue to hold.

(ii) If $F(\tau_h^*(\lambda)) < \widehat{F}$, then the optimal sales frequency $F(\tau_h^*)$ is decreasing, the optimal full price p_h^* is increasing, and the firm's expected profit Π_h^* is strictly increasing in the loss aversion parameter λ.

A.2.2. Heterogeneity in Consumption Valuation

We consider a market with consumers who are heterogenous in their consumption valuation. Consumers have the same degree of loss aversion $\lambda > 1$, but differ in their consumption valuation: One segment of consumers has a consumption valuation of v_h and the other segment has a consumption valuation of v_l, with $0 < v_l < v_h$. It can be shown that two cases of personal equilibria would prevail: One is that only the high-type consumers would buy at the full price, and the other is that both types of consumers would buy at the full price. As we have done for the extension of consumer heterogeneity in loss aversion, we show that the results of Propositions 3 – 4 continue to hold with mild conditions.

Proposition 13 *When consumers differ in consumption valuation,*

(i) for the personal equilibrium in which only the high-type consumers buy at both the full and sale price, while the low-type consumers buy only at the sale price, the results of Proposition 12 hold;

(ii) for the personal equilibrium in which both types of consumers buy at both the full and sale price, the results of Propositions 3 – 4 continue to hold.

A.3. Competition

Lastly, we consider a duopoly of competing newsvendors with an identical procurement cost. Both firms simultaneously choose their initial inventory levels and commit to pricing strategies that are contingent on demand realizations (see, e.g., Wang and Hu 2014). By observing the price distributions and product availabilities, consumers decide which firm to frequent by forming expectations about their consumption outcomes. Consumers are allowed to shop around: if the full prices of both firms are acceptable, consumers will first go to the firm with a lower full price; if that firm has insufficient inventory, those unsatisfied consumers will then go to the other firm. Heidhues and Koszegi (2014) show that under perfect ex ante competition, firms would *not* implement a randomized price strategy in equilibrium. In other words, in equilibrium, both firms would set a deterministic price equal to the marginal cost c and thus both firms would earn zero profits. This finding no longer holds when firms face demand uncertainty.

Proposition 14 *When newsvendors in a duopoly compete, setting a deterministic price would not be an equilibrium strategy.*

The reason for the difference is as follows. In Heidhues and Koszegi (2014), demand is deterministic, so a firm can only benefit from price randomization by manipulating consumers' reference price points. When two firms compete in the market, a firm that randomizes its prices would lower consumers' expected total utility and lead consumers to prefer the other firm when forming purchase plans. In contrast, when there is demand uncertainty, contingent pricing not only benefits a firm by manipulating consumers' reference distribution, but also benefits the firm by better matching demand with supply. When demand is low, both firms are willing to set the price to be s to clear their inventories. Therefore, setting a deterministic price would never be an equilibrium strategy.

Note that our assumption about the consumer choice is different from that of Heidhues and Koszegi (2014). They consider perfect ex ante competition for consumers, i.e., the consumers observe the price distributions of the two competing retailers, form the expected utilities of buying from each of the retailers ex ante, and then commit to buying from one of them. While in Proposition 14, we allow consumers to switch from one retailer to another ex post. However, even under the setting of perfect ex ante competition, the retailers might still adopt the randomized pricing strategy when there is demand uncertainty. The rationale is as follows. Suppose that retailer i sets a deterministic price. For retailer $-i$, the best response problem is to adopt a pricing strategy so as to maximize her profit, subject to the constraint that the expected utility of buying from retailer $-i$ is no less than that of buying from retailer i. Mathematically, the best response problem is equivalent to adding more constraints to the monopoly firm's problem formulated in Section 3.2. As a result, the optimal pricing strategy might still be a randomized one (as long as the optimal contingent pricing policy does not degenerate to a deterministic price). In summary, the driving force behind contingent pricing is that the retailers face demand uncertainty even if consumers commit to purchase plans ex ante. While in Heidhues and Koszegi (2014), if the consumers commit to buying from one of the retailers, there is no demand uncertainty. It is the demand uncertainty that incentivizes retailers to set prices contingently.

A.4. Price-Dependent Fill Rate

We consider the extension where regular consumers are strategic to an extent that they can distinguish the product availabilities by observing which price is charged. That is, the perceived product availability conditional on seeing the sale price is $\phi(s) = 1$ because consumers are assumed to be prioritized over bargain hunters during sales, and the perceived product availability conditional on seeing the full price is $\phi(\overline{p}) = \int_{\tau}^{\infty} \frac{\min(x,q)}{x} dF(x) / \int_{\tau}^{\infty} dF(x)$. The following proposition shows results similar to Propositions 1 and 2, when perceived fill rates are price dependent.

Proposition 15 *When fill rates are price dependent, the pricing scheme is of the threshold form and the optimal full price is*

$$\overline{p}^* = v + \frac{(s+v)F(\tau) + v(2\phi(\overline{p}^*)(1-F(\tau)) - 1)}{(1+\eta\lambda) + \eta(\lambda-1)\phi(\overline{p}^*)(1-F(\tau))}(\lambda-1)\eta.$$

Let τ^* still denote the optimal sales threshold when fill rates are price dependent, then the optimal full price $\overline{p}^*$ is higher than loss-averse consumers' valuation v, if and only if one of the following two cases holds: (i) $\phi(\overline{p}^*) \geq 1/2$; (ii) $\phi(\overline{p}^*) < 1/2$ and $F(\tau^*) > \frac{v(1-2\phi(\overline{p}^*))}{s+v(1-2\phi(\overline{p}^*))}$.

Proposition 15 shows that a higher fill rate for regular consumers (case (i)) or higher sales frequency when the fill rate for regular consumers is low (case (ii)) can help strengthen the positive attachment effect.

Unlike the base model where the optimal fill rate can be shown to depend only on the initial order quantity (see Equation (10)), the product availability conditional on seeing the full price, when fill rates are price dependent, depends on the optimal full price as well. As a result, the optimal full price, $\overline{p}^*$, the optimal sales frequency, τ^*, and the product availability conditional on seeing the full price, $\phi(\overline{p}^*)$, are characterized by a set of three complicated nonlinear equations. This complication makes the comparative statics problems much less tractable, if not impossible. However, we expect similar insights to Propositions 3 and 9 to continue to hold. This is because that the driving force behind these comparative statics results in the base model is that the positive attachment effect is reinforced by higher product availability. The same driving force is still expected to hold for price-dependent fill rates.

Appendix. B. Proofs.

Proof of Lemma 1

According to the outcome distribution (6), the consumer's total utility from outcome $(v,-\overline{p})$ is $U((v,-\overline{p})|\mathbf{\Gamma}(g,\phi,\overline{p})) = (v-\overline{p}) - \eta\lambda(\overline{p}-s)\cdot\phi g(s) + (\eta v - \eta\lambda\overline{p})\cdot(1-\phi)$, where $(v-\overline{p})$ is the consumption utility, $-\eta\lambda(\overline{p}-s)\cdot\phi g(s)$ is the gain-loss utility comparing outcome $(v,-\overline{p})$ to outcome $(v,-s)$, and $(\eta v - \eta\lambda\overline{p})\cdot(1-\phi)$ is the gain-loss utility comparing outcome $(v,-\overline{p})$ to outcome $(0,0)$.

The consumer's total utility from outcome $(0,0)$ is $U((0,0)|\mathbf{\Gamma}(g,\phi,\overline{p})) = \sum_{p=\overline{p},s}(\eta p - \eta\lambda v)\cdot\phi g(p)$. where $(\eta p - \eta\lambda v)\cdot\phi g(p)$ is the gain-loss utility comparing outcome $(0,0)$ to outcome $(v,-p)$, $p=\overline{p},s$.

According to (3), by equating $U((v,-\overline{p})|\mathbf{\Gamma}(g,\phi,\overline{p}))$ and $U((0,0)|\mathbf{\Gamma}(g,\phi,\overline{p}))$, and after some algebra, we have the desired result.

Proof of Proposition 1

First, we characterize the full price and optimal fill rate when q is given. If the firm can successfully induce consumers to buy at both prices, then according to (6) in Lemma 1, the full price $\overline{p}^*$ must satisfy

$$\overline{p}^* = v + \frac{[\phi\psi s - (1-\phi(2-\psi))v]}{1+\eta[\lambda - \phi(\lambda-1)(1-\psi)]}(\lambda-1)\eta, \tag{A-5}$$

where ψ is the probability of s being charged. Since $\overline{p}^*$ is increasing in s, which justifies the modeling assumption that the sale price should not be lower than s. By Assumption (V), it follows that $\overline{p}^* > s$. In this case, the pricing scheme is $p = \begin{cases} s & \text{if } x \in \Omega, \\ \overline{p}^* & \text{if } x \in \Omega^c, \end{cases}$ where Ω satisfies $\int_\Omega dF(x) = \psi$. Note that the fill rate ϕ and sales frequency ψ are two independent decision variables for now.

Note that $\overline{p}^*$ defined in (A-5) is strictly increasing in fill rate ϕ, i.e.,

$$\frac{\partial \overline{p}^*}{\partial \phi} = \frac{[\psi s + (2-\psi)v] + \eta\lambda\psi s + \eta v(\lambda + 1 - \psi)}{\{1+\eta[\lambda - \phi(\lambda-1)(1-\psi)]\}^2}(\lambda - 1)\eta > 0. \tag{A-6}$$

Therefore, regardless how ψ and Ω are chosen, the strategy to maximize fill rate is to set the product availability for each demand realization to be $\xi(\overline{p}^*, q, x) = \frac{\min\{x,q\}}{x}$. This strategy increases both the full price and the sales to consumers. As a result, the optimal fill rate should be $\phi^* = \int_0^\infty \xi(\overline{p}^*, q, x)dF(x) = \int_0^\infty \frac{\min\{x,q\}}{x}dF(x)$.

Next, the remaining task is to determine the sales frequency and the set of demand realizations chosen for sales. For a fixed ψ, we show that it is optimal for the firm to set $\Omega = [0, \tau]$ such that $\psi = \int_0^\tau dF(x)$. First, note that given any Ω that satisfies $\int_\Omega dF(x) = \psi$, the firm's expected profit is $\Pi = sq\int_\Omega dF(x) + \overline{p}^* \int_{\Omega^c} \xi(\overline{p}^*, q, x)\cdot x \cdot dF(x) - cq = sq\psi + \overline{p}^*\int_{\Omega^c}\min\{x,q\}dF(x) - cq$. Hence, to maximize profit is equal to maximize $\int_{\Omega^c}\min\{x,q\}dF(x)$. Subject to $\int_\Omega dF(x) = \psi$, the best choice of Ω to maximize $\int_{\Omega^c}\min\{x,q\}dF(x)$ is $\Omega = [0,\tau]$ then $\psi = \int_0^\tau dF(x) = F(\tau)$, and $\int_{\Omega^c}\min\{x,q\}dF(x) = \int_\tau^\infty \min\{x,q\}dF(x)$. Consequently, the sales frequency ψ and the threshold for running sales τ are interchangeable decision variables to the firm. We will focus on determining τ.

Because $\Omega = [0,\tau]$, the firm's expected profit function can be rewritten as $\Pi = sq\int_0^\tau dF(x) + \overline{p}^*\int_\tau^\infty \min\{x,q\}dF(x) - cq$. Invoking the chain rule to compute the derivative of $\overline{p}^*$ with respect to τ yields

$$\frac{\partial \overline{p}^*}{\partial \tau} = \frac{\partial \overline{p}^*}{\partial F}\frac{\partial F(\tau)}{\partial \tau} = f(\tau)\frac{\partial \overline{p}^*}{\partial F}. \tag{A-7}$$

Taking derivative of Π with respect to τ and using (A-7), we have

$$\frac{\partial \Pi}{\partial \tau} = f(\tau)\left[sq - \overline{p}^*\cdot\min\{\tau,q\} + \frac{\partial \overline{p}^*}{\partial F}\int_\tau^\infty \min\{x,q\}dF(x)\right], \tag{A-8}$$

Note that

$$\frac{\partial \overline{p}^*}{\partial F} = \frac{\eta\phi(\lambda-1)[s(1+\eta\lambda) - v(1+\eta) - \eta\phi(\lambda-1)(v+s)]}{[1+\eta\lambda - \eta\phi(\lambda-1) + \eta\phi F(\tau)(\lambda-1)]^2} < 0, \tag{A-9}$$

Equation (A-9) follows because according to Assumption (V), we must have $s(1+\eta\lambda) - v(1+\eta) \le 0$.

For $\tau > q$, we have $sq - \overline{p}^*\cdot\min\{\tau,q\} < 0$, so according to (A-8), we have $\partial\Pi/\partial\tau < 0$, i.e., it is not optimal to set the sales threshold τ above q. Consequently, it is either $\partial\Pi/\partial\tau < 0$ for all $\tau \in [0,q]$, or there exists $\tau^* \in [0,q]$ such that the first order condition is satisfied, i.e., $\partial\Pi/\partial\tau|_{\tau^*} = 0$. Because we assume $f(\tau) > 0, \forall\tau > 0$, (A-8) is equivalent to (11).

Proof of Corollary 1

By (A-9), we have $\frac{\partial \overline{p}^*}{\partial F} < 0$. By (11), we have $sq - \overline{p}^*\tau^* \ge 0$, which is $\tau^* \le sq/\overline{p}^*$. Under Conditions (12) and (13), we must have $\overline{p}^* \ge v$ by Proposition 2 and thus $\tau^* \le sq/v$. The result follows by realizing that sq/v is the "optimal" contingent discount threshold in the absence of consumers' loss aversion, i.e., the optimal contingent pricing strategy is to set $p = s$ when demand is lower than sq/v and to set $p = v$ otherwise.

Proof of Proposition 3

By (10), there is a one-to-one increasing correspondence between q and ϕ^*. Hence comparative statics with respect to q are equivalent to those with respect to ϕ^*. Note that ϕ^* affects $\overline{p}^*$ through two channels: the first is the direct effect of ϕ^* on $\overline{p}^*$ and the second is the indirect effect through the optimal threshold for sales τ^*. Under the assumption that τ^* is determined by (11), we can write τ^* as a function of ϕ^*, i.e., $\tau^*(\phi^*)$, and the full price is a function of ϕ^* as $\overline{p}^*(\phi^*,\tau^*)$.

To show this result, by the chain rule, we can write

$$\frac{\mathrm{d}\overline{p}^*}{\mathrm{d}\phi^*} = \frac{\partial\overline{p}^*}{\partial\phi^*} + \frac{\partial\overline{p}^*}{\partial\tau^*}\frac{\partial\tau^*}{\partial\phi^*}, \tag{A-10}$$

where we have $\partial\overline{p}^*/\partial\phi^* > 0$ and $\partial\overline{p}^*/\partial\tau^* < 0$ by (A-7) and (A-9).

Applying the Implicit Function Theorem [1] to the first order condition in (11) yields

$$\frac{\partial\tau^*}{\partial\phi^*} = -\frac{\partial^2\Pi(\tau^*,\phi^*)}{\partial\tau^*\partial\phi^*}\bigg/\frac{\partial^2\Pi(\tau^*,\phi^*)}{\partial\tau^{*2}}, \tag{A-11}$$

in which $\partial^2\Pi(\tau^*,\phi^*)/\partial\tau^{*2} < 0$ as long as τ^* is a local interior maximizer. For $\partial^2\Pi(\tau^*,\phi^*)/\partial\tau^*\partial\phi^*$, we have

$$\frac{\partial^2\Pi(\tau^*,\phi^*)}{\partial\tau^*\partial\phi^*} = f(\tau^*)\left\{-\tau^*\frac{\partial\overline{p}^*}{\partial\phi^*} + \frac{\partial^2\overline{p}^*}{\partial F\partial\phi^*}\int_{\tau^*}^{\infty}\min\{x,q\}dF(x)\right\}. \tag{A-12}$$

Note that the denominator of (A-9) is positive and decreasing in ϕ, and the numerator of (A-9) is negative and decreasing in ϕ, so $\partial\overline{p}^*/\partial F$ is decreasing in ϕ^*, and thus $\partial^2\overline{p}^*/\partial F\partial\phi^* < 0$. Applying this result to (A-12), we must have $\partial^2\Pi(\tau^*,\phi^*)/\partial\tau^*\partial\phi^* < 0$, and by (A-11), we have $\partial\tau^*/\partial\phi^* < 0$.

Based on the above results, and by (A-10), we have $\mathrm{d}\overline{p}^*/\mathrm{d}\phi^* > 0$. The average price is $\widehat{p}^* = F(\tau^*)s + (1-F(\tau^*))\overline{p}^*$, then we have $\frac{\partial\widehat{p}^*}{\partial\phi^*} = \underbrace{(s-\overline{p}^*)}_{<0} f(\tau^*)\underbrace{\frac{\partial\tau^*}{\partial\phi^*}}_{<0} + (1-F(\tau^*))\underbrace{\frac{\mathrm{d}\overline{p}^*}{\mathrm{d}\phi^*}}_{>0} > 0$.

Proof of Proposition 4

Using the chain rule, we can write out the net effect $\mathrm{d}\overline{p}^*/\mathrm{d} = \underbrace{\frac{\partial\overline{p}^*}{\partial\phi^*}}_{>0}\underbrace{\frac{\partial\phi^*}{\partial q^*}}_{>0}\frac{\partial q^*}{\partial c} + \underbrace{\frac{\partial\overline{p}^*}{\partial F}}_{<0}\underbrace{\frac{\partial F(\tau^*)}{\partial\tau^*}}_{>0}\underbrace{\frac{\partial\tau^*}{\partial q^*}}_{<0}\frac{\partial q^*}{\partial c}$.

These signs are based on the results of Proposition 3. To determine the sign of $\mathrm{d}q^*/\mathrm{d}c$, we note that $\partial\Pi^*/\partial q^* = 0$, and by the Implicit Function Theorem, we have $\frac{\mathrm{d}q^*}{\mathrm{d}c} = -\frac{\partial^2\Pi^*}{\partial q^*\partial c}\Big/\frac{\partial^2\Pi^*}{\partial q^{*2}}$. It can be verified that $\partial^2\Pi^*/\partial q^*\partial c = -1$, and we know $\partial^2\Pi^*/\partial q^{*2} \leq 0$ because q^* is a maximizer of the profit function. Therefore, we have $\mathrm{d}q^*/\mathrm{d}c \leq 0$. Furthermore, we have $\mathrm{d}F(\tau^*)/\mathrm{d}c = \frac{\partial F(\tau^*)}{\partial\tau^*}\frac{\partial\tau^*}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}c} > 0$.

[1] Here and below we note that the conditions required for the implicit function theorem to hold are all satisfied whenever the optimal sales threshold is determined by (11).

Proof of Proposition 5

By the Implicit Function Theorem, we have

$$\frac{\partial \tau^*}{\partial \lambda} = -\frac{\partial^2 \Pi^*(\tau^*, \lambda)}{\partial \tau^* \partial \lambda} \Bigg/ \frac{\partial^2 \Pi^*(\tau^*, \lambda)}{\partial \tau^{*2}}. \tag{A-13}$$

Note that $\partial^2 \Pi^*(\tau^*, \lambda)/\partial \tau^{*2} \le 0$ because τ^* is the local maximization. Taking derivative of (A-8) with respect to λ gives rise to

$$\frac{\partial^2 \Pi^*(\tau^*, \lambda)}{\partial \tau^* \partial \lambda} = f(\tau^*) \left(\frac{\partial^2 \overline{p}^*}{\partial F \partial \lambda} \int_{\tau^*}^{\infty} \min\{x, q\} dF(x) - \frac{\partial \overline{p}^*}{\partial \lambda} \tau^* \right). \tag{A-14}$$

Define $B \equiv 1 + \eta[\lambda - \phi^*(\lambda - 1)(1 - F(\tau^*))]$, $C \equiv s\phi^* F(\tau^*) - (1 - \phi^*(2 - F(\tau^*)))v$ and $\widehat{\tau} = F^{-1}\left(\min\left\{1, \widehat{F}\right\}\right)$, then we have

$$\frac{\partial^2 \overline{p}^*}{\partial F \partial \lambda} = \frac{\eta(1+\eta)}{B^2} \phi^*(s - v) + \frac{1}{B^3} \left[2\eta(\lambda - 1) \frac{\partial B}{\partial F} \frac{\partial B}{\partial \lambda} - \eta(\lambda - 1) B \frac{\partial^2 B}{\partial F \partial \lambda} - B \frac{\partial B}{\partial F} \right] C. \tag{A-15}$$

In the derivation of (A-15), we used the results $\partial^2 C/\partial F \partial \lambda = 0, \partial C/\partial \lambda = 0$. When $\tau^*(\lambda) < \widehat{\tau}$, one can verify that $(s - v) < 0$, $\eta(\lambda - 1) \left[2 \frac{\partial B}{\partial F} \frac{\partial B}{\partial \lambda} - B \frac{\partial^2 B}{\partial F \partial \lambda} \right] - B \frac{\partial B}{\partial F} < 0$, and $C > 0$ so we must have $\frac{\partial^2 \overline{p}^*}{\partial F \partial \lambda} < 0$.

Taking derivative from (9) with respect to λ gives

$$\frac{\partial \overline{p}^*}{\partial \lambda} = \frac{s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v}{\left[1 + \eta[\lambda - \phi^*(\lambda - 1)(1 - F(\tau^*))]\right]^2} (1 + \eta). \tag{A-16}$$

Therefore, the sign of $\partial \overline{p}^*/\partial \lambda$ is determined by the sign of $s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v$. When $\tau^*(\lambda) < \widehat{\tau}$, we have $s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v > 0$, so $\partial \overline{p}^*/\partial \lambda > 0$ holds. In this case, by (A-14), we know $\partial^2 \Pi^*(\tau^*, \lambda)/\partial \tau^* \partial \lambda < 0$, and by (A-13), we know that $\partial \tau^*/\partial \lambda < 0$.

The net effect of consumers' loss aversion on the firm's regular price is

$$\frac{\mathrm{d}\overline{p}^*}{\mathrm{d}\lambda} = \frac{\partial \overline{p}^*}{\partial \lambda} + \underbrace{\frac{\partial \overline{p}^*}{\partial F}}_{<0} \underbrace{\frac{\partial F(\tau^*)}{\partial \tau^*}}_{>0} \frac{\partial \tau^*}{\partial \lambda}. \tag{A-17}$$

According to the above results, when $\tau^*(\lambda) < \widehat{\tau}$, $\tau^*(\lambda)$ is decreasing in λ (i.e., $\partial \tau^*/\partial \lambda < 0$), and so is $F(\tau^*)$. By Proposition 2, we must have $s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v > 0$, so $\partial \overline{p}^*/\partial \lambda > 0$. Plugging these results into (A-17), we must have $\mathrm{d}\overline{p}^*/\mathrm{d}\lambda > 0$.

Proof of Proposition 6

First, we show that for any local extreme point λ^* of the profit function $\Pi^*(\tau^*, \lambda)$, if such λ^* exists, the optimal sales threshold τ^*, as a function of λ, has the property $\partial \tau^*/\partial \lambda|_{\lambda^*} < 0$. Invoking the Envelope Theorem at τ^*, we have $\frac{\mathrm{d}\Pi^*(\tau^*, \lambda)}{\mathrm{d}\lambda} = \frac{\partial \Pi^*(\tau^*, \lambda)}{\partial \lambda}$. Taking derivative from (15) with respect to λ, we have

$$\frac{\mathrm{d}\Pi^*(\tau^*, \lambda)}{\mathrm{d}\lambda} = \frac{\partial \Pi^*(\tau^*, \lambda)}{\partial \lambda} = \frac{\partial \overline{p}^*}{\partial \lambda} \int_{\tau^*}^{\infty} \min\{x, q\} dF(x), \tag{A-18}$$

where $\partial\overline{p}^*/\partial\lambda$ is given by (A-16). Since λ^* is a local extreme point, we must have $\partial\Pi^*(\tau^*,\lambda)/\partial\lambda|_{\lambda^*}=0$, and because $\tau^*<q$, according to (A-18), that is equivalent to $\partial\overline{p}^*/\partial\lambda|_{\lambda^*}=0$. By (A-16), this holds, if and only if, λ^* satisfies $F(\tau^*(\lambda^*))=\frac{2\phi^*-1}{\phi^*}\frac{v}{v-s}=\widehat{F}$, or equivalently, $\tau^*(\lambda^*)=\widehat{\tau}$. By (A-18) and (A-16), for any other value $\lambda\neq\lambda^*$, $\Pi^*(\tau^*,\lambda)$ is increasing in λ if $\tau^*(\lambda)<\widehat{\tau}$ and decreasing if $\tau^*(\lambda)>\widehat{\tau}$.

According to the proof of Proposition 5, $\partial\tau^*/\partial\lambda$ is expressed as (A-13), and its sign is determined by the sign of $\partial^2\Pi^*(\tau^*,\lambda)/\partial\tau^*\partial\lambda$, as given in (A-14). Then at λ^*, we have $C(F(\tau^*),\lambda^*)=0$, and according to (A-15), we further have $\left.\frac{\partial^2\overline{p}^*}{\partial F\partial\lambda}\right|_{\lambda^*}=\frac{\eta(1+\eta)}{B^2}\phi^*(s-v)<0$. Since $\partial\overline{p}^*/\partial\lambda|_{\lambda^*}=0$, the second term in (A-14) is zero at $\lambda=\lambda^*$, and $\partial^2\Pi^*(\tau^*,\lambda)/\partial\tau^*\partial\lambda|_{\lambda^*}<0$, which implies that $\left.\frac{\mathrm{d}\tau^*}{\mathrm{d}\lambda}\right|_{\lambda^*}<0$.

The result $\left.\frac{\mathrm{d}\tau^*}{\mathrm{d}\lambda}\right|_{\lambda^*}<0$ indicates that the sales threshold τ^*, as a function of λ, can only cross the horizontal line $\widehat{\tau}$ at most once from above. Therefore, three possibilities can emerge:

Case 1: $\tau^*(\lambda)>\widehat{\tau}$ for all $\lambda>1$, in which case $\Pi^*(\tau^*,\lambda)$ is monotonically decreasing in λ;

Case 2: $\tau^*(\lambda)<\widehat{\tau}$ for all $\lambda>1$, in which case $\Pi^*(\tau^*,\lambda)$ is monotonically increasing in λ;

Case 3: $\tau^*(1)>\widehat{\tau}$ and $\tau^*(\infty)<\widehat{\tau}$, and $\tau^*(\lambda)$ crosses $\widehat{\tau}$ at a single point λ^*, so that $\Pi^*(\tau^*,\lambda)$ is monotonically decreasing in $\lambda\in[1,\lambda^*]$ and monotonically increasing in $\lambda\in[\lambda^*,\infty]$.

Proof of Proposition 7

By the Implicit Function Theorem, we have

$$\frac{\mathrm{d}q^*}{\mathrm{d}\lambda}=-\frac{\partial^2\Pi^*}{\partial q^*\partial\lambda}\bigg/\frac{\partial^2\Pi^*}{\partial q^{*2}}\,. \tag{A-19}$$

Since q^* is a maximizer of profit function, we must have $\partial^2\Pi^*/\partial q^{*2}<0$. By (14), we can explicitly write out $\partial^2\Pi^*/\partial q^*\partial\lambda$ as

$$\frac{\partial^2\Pi^*}{\partial q^*\partial\lambda}=\frac{\partial\overline{p}^*}{\partial\lambda}(1-F(q^*))+\frac{\partial^2\overline{p}^*}{\partial\phi^*\partial\lambda}\frac{\partial\phi^*}{\partial q^*}\int_\tau^\infty\min\{x,q^*\}dF(x). \tag{A-20}$$

where the expression of $\partial\overline{p}^*/\partial\lambda$ is given in (A-16). Taking derivative from (A-16) with respect to ϕ^* gives

$$\frac{\partial^2\overline{p}^*}{\partial\phi^*\partial\lambda}=\frac{sF(\tau^*)+(2-F(\tau^*))v+\eta\lambda sF(\tau^*)+\eta\lambda v(2-F(\tau^*))-\eta v(\lambda-1)(1-F(\tau^*))}{\{1+\eta[\lambda-\phi^*(\lambda-1)(1-F(\tau^*))]\}^3}\eta(1+\eta)>0. \tag{A-21}$$

By (A-16), when $s\phi^*F(\tau^*)-[1-\phi^*(2-F(\tau^*))]v>0$, i.e., the condition under which $\overline{p}^*>v$, we have $\partial\overline{p}^*/\partial\lambda>0$, so $\mathrm{d}q^*/\mathrm{d}\lambda>0$, i.e., the optimal order quantity is increasing in consumers' loss-averseness. As the order quantity increases, according to Proposition 3, the full price $\overline{p}^*$ increases in the order quantity and is greater than v, so the inequality $s\phi^*F(\tau^*)-[1-\phi^*(2-F(\tau^*))]v>0$ still holds, by which we must have $\partial\overline{p}^*/\partial\lambda>0$. Hence, both terms in the right hand side of (A-20) are positive. Together with (A-19), we can further conclude $\mathrm{d}q^*/\mathrm{d}\lambda>0$.

In summary, once λ is such that $\mathrm{d}q^*/\mathrm{d}\lambda>0$, the sign of $\mathrm{d}q^*/\mathrm{d}\lambda$ will not become negative as λ increases. A possible cut-off point $\widehat{\lambda}$ immediately follows.

Proof of Proposition 8

(i) Since the firm optimizes the sales threshold and the order quantity simultaneously, we treat (τ, q) as a vector of decision variables. Using the Implicit Function Theorem for vector variable, we have:

$$\begin{bmatrix} \frac{\partial \tau^*}{\partial \lambda} \\ \frac{\partial q^*}{\partial \lambda} \end{bmatrix} = \frac{-1}{|H|} \begin{bmatrix} \frac{\partial^2 \Pi^*}{\partial q^2}\frac{\partial^2 \Pi^*}{\partial \tau \partial \lambda} - \frac{\partial^2 \Pi^*}{\partial q \partial \tau}\frac{\partial^2 \Pi^*}{\partial q \partial \lambda} \\ \frac{\partial^2 \Pi^*}{\partial \tau^2}\frac{\partial^2 \Pi^*}{\partial q \partial \lambda} - \frac{\partial^2 \Pi^*}{\partial q \partial \tau}\frac{\partial^2 \Pi^*}{\partial \tau \partial \lambda} \end{bmatrix},$$

where $H = \begin{pmatrix} \frac{\partial^2 \Pi^*}{\partial q^2} & \frac{\partial^2 \Pi^*}{\partial q \partial \tau} \\ \frac{\partial^2 \Pi^*}{\partial q \partial \tau} & \frac{\partial^2 \Pi^*}{\partial \tau^2} \end{pmatrix}$ is the profit function's Hessian matrix. Note that $|H| > 0$, i.e., negative semidefinite, because the optimal decision vector $\mathbf{z}^* = (\tau^*, q^*)$ is a maximizer.

First, we note that since τ^* and q^* are maximizers, we must have $\partial^2 \Pi^* / \partial \tau^{*2} \leq 0$ and $\partial^2 \Pi^* / \partial q^{*2} \leq 0$. Second, we show that when $F(\tau^*) \leq \widehat{F}(q^*)$, then $\frac{\partial^2 \Pi^*}{\partial \tau^* \partial \lambda^*} < 0$ holds. To show this result, we have

$$\frac{\partial^2 \Pi^*}{\partial \tau^* \partial \lambda} = \frac{\partial^2 \overline{p}^*}{\partial \tau^* \partial \lambda} \int_{\tau^*}^{\infty} \min(x, q) dF(x) - \frac{\partial \overline{p}^*}{\partial \lambda} \tau^* f(\tau^*). \tag{A-22}$$

By (A-16), we observe $\partial^2 \overline{p}^* / \partial \tau^* \partial \lambda = \frac{\partial^2 \overline{p}^*}{\partial F \partial \lambda} f(\tau^*) < 0$ when $F(\tau^*) \leq \widehat{F}(q^*)$. In addition, as shown in the proof of Proposition 5, when $F(\tau^*) \leq \widehat{F}(q^*)$, we have $\partial \overline{p}^* / \partial \lambda \geq 0$. With all these results, we conclude that when $F(\tau^*) \leq \widehat{F}(q^*)$, $\partial^2 \Pi^* / \partial \tau^* \partial \lambda < 0$ holds. Third, by the analysis of (A-20), we note that when $F(\tau^*) \leq \widehat{F}(q^*)$, $\partial^2 \Pi^* / \partial q^* \partial \lambda > 0$ holds. Lastly, by (A-12), we have $\partial^2 \Pi^* / \partial \tau^* \partial q^* = \underbrace{\frac{\partial^2 \Pi^*}{\partial \tau^* \partial \phi^*}}_{<0} \underbrace{\frac{\partial \phi^*}{\partial q^*}}_{>0} < 0$.

Consequently, based on the above results, we can conclude that when $F(\tau^*) \leq \widehat{F}(q^*)$, $\partial \tau^* / \partial \lambda = \underbrace{\frac{-1}{|H|}}_{<0} [\underbrace{\frac{\partial^2 \Pi^*}{\partial q^2}}_{<0} \underbrace{\frac{\partial^2 \Pi^*}{\partial \tau \partial \lambda}}_{<0} - \underbrace{\frac{\partial^2 \Pi^*}{\partial q \partial \tau}}_{<0} \underbrace{\frac{\partial^2 \Pi^*}{\partial q \partial \lambda}}_{>0}] < 0$ holds, and as a result $\partial F(\tau^*) / \partial \lambda = f(\tau^*) \frac{\partial \tau^*}{\partial \lambda} < 0$ also holds. Similarly, we have $\partial q^* / \partial \lambda = \underbrace{\frac{-1}{|H|}}_{<0} [\underbrace{\frac{\partial^2 \Pi^*}{\partial \tau^2}}_{<0} \underbrace{\frac{\partial^2 \Pi^*}{\partial q \partial \lambda}}_{>0} - \underbrace{\frac{\partial^2 \Pi^*}{\partial q \partial \tau}}_{<0} \underbrace{\frac{\partial^2 \Pi^*}{\partial \tau \partial \lambda}}_{<0}] > 0$.

(ii) Since the firm optimizes both the sales threshold τ and the order quantity q simultaneously under loss aversion parameter λ, we can write τ^* and q^* as functions of λ, i.e., $\tau^*(\lambda)$ and $q^*(\lambda)$. Using the chain rule, we have

$$\begin{aligned} \frac{\mathrm{d}\Pi(\tau^*(\lambda))}{\mathrm{d}\lambda} &= \frac{\partial \Pi^*}{\partial \lambda} + \frac{\partial \Pi^*}{\partial \tau^*} \frac{\tau^*(\lambda)}{\partial \lambda} + \frac{\partial \Pi^*}{\partial q^*} \frac{q^*(\lambda)}{\partial \lambda} = \frac{\partial \Pi^*}{\partial \lambda} = \frac{\partial \overline{p}^*}{\partial \lambda} \int_{\tau^*}^{\infty} \min(x, q) dF(x) \\ &= \frac{s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v}{[1 + \eta\lambda - \phi^*(\lambda - 1)(1 - F(\tau^*))]^2} (1 + \eta) \int_{\tau^*}^{\infty} \min(x, q) dF(x). \end{aligned}$$

because $\partial \Pi^* / \partial \tau^* = 0$ and $\partial \Pi^* / \partial q^* = 0$ as τ^* and q^* are maximizes. Therefore, the sign of $\mathrm{d}\Pi(\tau^*(\lambda))/\mathrm{d}\lambda$ are determined by the sign of $s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v$. Reshuffling the inequality $s\phi^* F(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v \geq 0$ yields $F(\tau^*) \leq \widehat{F}(q^*)$.

Proof of Proposition 9

Suppose $q < \overline{d}$. The fill rate is a function of the distance d: $\widetilde{\phi}(\sigma) = \theta \frac{\min\{q, d_l\}}{d_l} + (1-\theta)\frac{q}{d_h}$. Conditional on $\sigma \in [0, \overline{d} - q]$ and $\sigma \in (\overline{d} - q, \overline{d}]$, we can explicitly write the fill rate and the marginal fill rate as:

$$\widetilde{\phi}(\sigma) = \begin{cases} \frac{\theta q}{\overline{d} - d} + \frac{(1-\theta)q}{\overline{d} + \frac{\theta}{1-\theta} d}, & \text{if } \sigma \leq \overline{d} - q; \\ \theta + \frac{(1-\theta)q}{\overline{d} + \frac{\theta}{1-\theta} d}, & \text{if } \sigma > \overline{d} - q. \end{cases} \quad \frac{\partial \widetilde{\phi}(\sigma)}{\partial \sigma} = \begin{cases} \theta q \left[\frac{1}{(\overline{d} - \sigma)^2} - \frac{1}{\left(\overline{d} + \frac{\theta}{1-\theta}\sigma\right)^2} \right] > 0, & \text{if } \sigma \leq \overline{d} - q; \\ -\frac{\theta q}{\left(\overline{d} + \frac{\theta}{1-\theta}\sigma\right)^2} < 0, & \text{if } \sigma > \overline{d} - q. \end{cases}$$

Let $\widetilde{p}_0$ and $\widetilde{p}_1$ denote the full prices when the sales threshold is set at 0 and d_l, respectively. The expression of $\widetilde{p}_0$ ($\widetilde{p}_1$) is (9) with ϕ^* being replaced by $\widetilde{\phi}$ and $F(\tau^*)$ being replaced by 0 (θ). By the proof of Proposition 3, both $\widetilde{p}_0$ and $\widetilde{p}_1$ are increasing in the fill rate. The firm's expected profit and marginal profit when the sales threshold is $\tau = 0$ is written as

$$\widetilde{\Pi}_0 = \begin{cases} \widetilde{p}_0 q - cq, & \text{if } \sigma \leq \overline{d} - q; \\ \widetilde{p}_0[\theta d_l + (1-\theta)q] - cq, & \text{if } \sigma > \overline{d} - q. \end{cases} \quad \frac{\partial \widetilde{\Pi}_0}{\partial \sigma} = \begin{cases} \frac{\partial \widetilde{p}_0}{\partial \widetilde{\phi}} \frac{\partial \widetilde{\phi}}{\partial \sigma} q > 0, & \text{if } \sigma \leq \overline{d} - q; \\ \frac{\partial \widetilde{p}_0}{\partial \widetilde{\phi}} \frac{\partial \widetilde{\phi}}{\partial \sigma} [\theta d_l + (1-\theta)q] - \theta \widetilde{p}_0 < 0, & \text{if } \sigma > \overline{d} - q. \end{cases}$$

Similarly, we can write the firm's expected profit and marginal profit when sales threshold is $\tau = d_l$ as

$$\widetilde{\Pi}_1 = \theta s q + (1-\theta)\widetilde{p}_1 q - cq, \quad \frac{\partial \widetilde{\Pi}_1}{\partial \sigma} = \begin{cases} \frac{\partial \widetilde{p}_1}{\partial \widetilde{\phi}} \frac{\partial \widetilde{\phi}}{\partial \sigma} (1-\theta) q > 0, & \text{if } \sigma \leq \overline{d} - q; \\ \frac{\partial \widetilde{p}_1}{\partial \phi} \frac{\partial \widetilde{\phi}}{\partial \sigma} (1-\theta) q < 0, & \text{if } \sigma > \overline{d} - q. \end{cases}$$

The firm's optimal profit is $\widetilde{\Pi}^*(\sigma) = \max\left\{\widetilde{\Pi}_0(\sigma), \widetilde{\Pi}_1(\sigma)\right\}$. From the above analysis, $\widetilde{\Pi}^*$ is strictly increasing in $\sigma \in \left[0, \overline{d} - q\right]$ and strictly decreasing in $d \in \left(\overline{d} - q, \overline{d}\right]$, i.e., the optimal profit is unimodular in d. The proof for the case $q \geq \overline{d}$ is analogous and details are omitted for conciseness.

Proof of Proposition 10

When $x > \tau$, the firm sells the product at the full price and only loss averse consumers are present, so the product availability is $\min\{x, q\}/x$. When $x \leq \tau$, the firm sells the product at the sales price and both the loss averse consumers and the bargain hunters are present. Since the number of bargain hunters is abundant by assumption, how to allocate the limited inventory q to this mix of customers becomes the firm's *inventory rationing* problem.

Let δ be defined as in Proposition 10. Specifically, $\delta = 1$ indicates that the consumers are fully prioritized over bargain hunters (which is assumed in the rest of this paper), $\delta = 0$ indicates that bargain hunters are fully prioritized over the consumers. Under inventory rationing, the fill rate for loss averse consumers is

$$\phi(\delta) = \delta \int_0^{\tau} \frac{\min(x, q)}{q} dF(x) + \int_{\tau}^{+\infty} \frac{\min(x, q)}{q} dF(x).$$

It is clear that the fill rate under rationing is increasing in δ.

Utilizing the results of Proposition 1, given a rationing parameter δ, the optimal full price can be written as

$$\overline{p}^*(\delta) = v + \frac{\phi(\delta) s F(\tau^*(\delta)) - [1 - \phi(\delta)(2 - F(\tau^*(\delta)))]v}{1 + \eta\lambda - \phi(\delta)(\lambda - 1)(1 - F(\tau^*(\delta)))]}(\lambda - 1)\eta,$$

and the optimal sales threshold $\tau^*(\delta)$ must satisfy the following first-order condition:

$$sq - \overline{p}^*(\delta)\tau^*(\delta) + \frac{\partial \overline{p}^*(\delta)}{\partial F}\int_{\tau^*(\delta)}^{\infty} \min(x,q)dF(x) = 0,$$

By the proof of Proposition 3, the optimal full price is increasing in, and the optimal sales threshold is decreasing in the fill rate. Since the fill rate $\phi(\delta)$ is increasing in δ, the optimal full price $\overline{p}^*(\delta)$ is increasing in, and the optimal sales threshold $\tau^*(\delta)$ is decreasing in δ.

For the last part, suppose $0 \le \delta_1 < \delta_2 \le 1$. Under δ_2, the firm can always mimic any sales threshold strategy under δ_1 but have a higher full price, so the firm's expected profit under δ_2 is always higher than that under δ_1.

Proof of Proposition 11

- If ***Equilibrium 1*** prevails:

(i) The firm's expected profit becomes $\Pi_l(v,\tau_l^*)$. Since τ_l^* is a function of order quantity q, profit $\Pi_l(v,\tau_l^*)$ is a function of order quantity q, and the optimal order quantity q_l^* must satisfy the firm order condition $\partial\Pi_l(v,\tau_l^*)/\partial q = 0$. Analogous to the proof of Proposition 4, we have $\mathrm{d}q_l^*/\mathrm{d}c = -\frac{\partial^2\Pi_l^*}{\partial q_l^*\partial c}\Big/\frac{\partial^2\Pi_l^*}{\partial {q_l^*}^2}$. Since $\partial^2\Pi_l^*/\partial q_l^*\partial c = -1 < 0$ and $\partial^2\Pi_l^*/\partial {q_l^*}^2 \le 0$, we must have $\mathrm{d}q_l^*/\mathrm{d}c \le 0$. Furthermore, we have $\mathrm{d}\tau_l^*/\mathrm{d}c = \frac{s}{\alpha_l v}\frac{\mathrm{d}q_l^*}{\mathrm{d}c} \le 0$ or $\mathrm{d}\tau_l^*/\mathrm{d}c = 0$.

- If ***Equilibrium 3*** prevails:

(ii) In the regime where the second constraint in (A-4) is binding in optimum, the firm's optimization problem degenerates to the homogenous loss averse consumer case as in Section 4, and the analysis of the comparative statics is analogous to the proofs of Proposition 3 to Proposition 4, so the results of these propositions still hold.

(iii) In the regime where the third constraint in (A-4) is binding, the regular price is $p_o^* = v$. Solving for the first order condition $\partial\Pi_o(v,\tau_o)/\partial\tau_o = 0$ yields $\tau_o^* = sq/v$, so the optimal sales threshold is only dependent on the order quantity and is increasing in the order quantity. The optimal order quantity q_o^* satisfies the first order condition $\partial\Pi_o\left(v,\frac{sq}{v}\right)/\partial q = 0$. Analogous to the proof above, we can show $\mathrm{d}q_o^*/\mathrm{d}c \le 0,\ \mathrm{d}\tau_o^*/\mathrm{d}c = \frac{s}{v}\frac{\mathrm{d}q_o^*}{\mathrm{d}c} \le 0$.

Proof of Proposition 12

If the loss averse consumers buy at both the full price p_h and the sales price s, while the loss neutral consumers only buy at the sales price s, then the fill rate ϕ_h, sales threshold τ_h and full price p_h must satisfy the three constraints in (A-2) simultaneously.

(i) Since the order quantity q affects the regular price both through the fill rate ϕ_h^* and the optimal sales threshold τ_h^*, the net effect of q on p_h^* can be written as:

$$\frac{\mathrm{d}p_h^*}{\mathrm{d}q} = \underbrace{\frac{\partial p_h^*}{\partial\phi_h^*}}_{\ge 0}(\underbrace{\frac{\partial\phi_h^*}{\partial q}}_{>0} + \underbrace{\frac{\partial\phi_h^*}{\partial\tau_h^*}}_{\le 0}\frac{\mathrm{d}\tau_h^*}{\mathrm{d}q}) + \underbrace{\frac{\partial p_h^*}{\partial\tau_h^*}}_{\le 0}\frac{\mathrm{d}\tau_h^*}{\mathrm{d}q}.$$

Therefore, if we have $\mathrm{d}\tau_h^*/\mathrm{d}q \le 0$, then $\mathrm{d}p_h^*/\mathrm{d}q > 0$ must follow.

(ii) The proof follows exactly those of Proposition 7 and Proposition 4.

(iii) By the Implicit Function Theorem, we have $\frac{\partial \tau_h^*}{\partial \lambda} = -\frac{\partial^2 \Pi_h^*(\tau_h^*,\lambda)}{\partial \tau_h^* \partial \lambda} \Big/ \frac{\partial^2 \Pi_h^*(\tau_h^*,\lambda)}{\partial \tau_h^{*2}}$. Note that $\partial^2 \Pi_h^*(\tau_h^*,\lambda)/\partial \tau_h^{*2} \le 0$ because τ_h^* is the local maximization. Hence, the sign of $\partial \tau_h^*/\partial \lambda$ is the same as $\partial^2 \Pi_h^*(\tau_h^*,\lambda)/\partial \tau_h^* \partial \lambda$. Taking derivative of (A-3) with respect to λ gives

$$\frac{\partial^2 \Pi_h^*(\tau_h^*,\lambda)}{\partial \tau_h^* \partial \lambda} = f(\tau_h^*)\left[\left(\frac{\partial^2 p_h^*}{\partial F \partial \lambda} + \frac{\partial^2 p_h^*}{\partial \phi_h^* \partial \lambda}\kappa(\tau_h^*)\right)\int_{\tau_h^*}^{\infty} \min\{\alpha_h x, q\} dF(x) - \frac{\partial p_h^*}{\partial \lambda}\alpha_h \tau_h^*\right].$$

Analog to the proof of Proposition 5, we can show $\partial^2 p_h^*/\partial F \partial \lambda \le 0$, and $\partial p_h^*/\partial \lambda > 0$. Furthermore, given that $F(\tau_h^*) \le \widehat{F}(\phi_h^*(\tau_h^*))$, we take derivative of (A-16) with respect to ϕ_h, then it is straightforward to check that $\partial^2 p_h^*/\partial \phi_h^* \partial \lambda > 0$. Based on the above results and note that $\kappa(\tau_h^*) \le 0$, we have $\partial^2 \Pi_h^*(\tau_h^*,\lambda)/\partial \tau_h^* \partial \lambda < 0$. Therefore, we have $\partial \tau_h^*/\partial \lambda < 0$.

The net effect of consumers' loss aversion on the firm's regular price is

$$\frac{\mathrm{d}p_h^*}{\mathrm{d}\lambda} = \frac{\partial p_h^*}{\partial \lambda} + \underbrace{\frac{\partial p_h^*}{\partial F}}_{<0}\underbrace{\frac{\partial F(\tau^*)}{\partial \tau^*}}_{>0}\underbrace{\frac{\partial \tau^*}{\partial \lambda}}_{<0} + \underbrace{\frac{\partial p_h^*}{\partial \phi_h^*}}_{>0}\underbrace{\frac{\partial \phi_h^*}{\partial \tau^*}}_{\le 0}\underbrace{\frac{\partial \tau^*}{\partial \lambda}}_{<0},$$

where $\partial p_h^*/\partial \lambda$ is given by (A-16) with τ_h^* replacing τ^* and ϕ_h^* replacing ϕ^*. Since $F(\tau_h^*) \le \widehat{F}(\phi_h^*(\tau_h^*))$, the numerator of (A-16) must be greater than zero and thus $\partial p_h^*/\partial \lambda > 0$. Therefore, we have $\mathrm{d}p_h^*/\mathrm{d}\lambda > 0$.

Invoking the Envelope Theorem at τ_h^*, we have $\frac{\mathrm{d}\Pi_h^*(\tau_h^*,\lambda)}{\mathrm{d}\lambda} = \frac{\partial \Pi_h^*(\tau_h^*,\lambda)}{\partial \lambda}$. Taking derivative of $\Pi_h^*(\tau_h^*,\lambda)$ with respect to λ, we have $\frac{\mathrm{d}\Pi_h^*(\tau_h^*,\lambda)}{\mathrm{d}\lambda} = \frac{\partial \Pi_h^*(\tau_h^*,\lambda)}{\partial \lambda} = \frac{\partial p_h^*}{\partial \lambda}\int_{\tau_h^*}^{\infty} \min\{\alpha_h x, q\} dF(x)$, since $\partial p_h^*/\partial \lambda > 0$ when $F(\tau_h^*) \le \widehat{F}(\phi_h^*(\tau_h^*))$, we must have $\mathrm{d}\Pi_h^*(\tau_h^*,\lambda)/\mathrm{d}\lambda > 0$.

Proof of Proposition 13

For conciseness, we call consumers with consumption valuation v_l (v_h) the *low* (*high*) type consumers. Analogous to the case of consumer heterogeneity in loss aversion, we assume that the fraction of low type consumers is α_l and the fraction of high type consumers is α_h, with $\alpha_l + \alpha_h = 1$.

Consider a personal equilibrium in which low type consumers buy at the full price. According to Lemma 1, the sufficient and necessary conditions for this purchase plan to be a personal equilibrium is

$$\begin{aligned}&(v_l - \overline{p}) + (\eta v_l - \eta\lambda\overline{p}) + \eta(\lambda-1)\sum_{p=\overline{p},s}(v_l + p)\cdot\phi g(p)\\ &= v_l[1+\eta+\eta(\lambda-1)\sum_{p=\overline{p},s}\phi g(p)] - \overline{p}[1+\eta\lambda-\eta(\lambda-1)\phi g(\overline{p})] + \eta(\lambda-1)s\phi g(s) \ge 0. \quad \text{(A-23)}\end{aligned}$$

Note that since $v_l < v_h$, the inequality (A-23) must still hold with v_l being replaced by v_h. Hence, buying at the full price is also a personal equilibrium for any individual high type consumer. As a result, whenever low type consumers find that buying at the full price is a personal equilibrium, the high type consumers also find that buying at the full price is a personal equilibrium. However, the reverse is not true. In conclusion, there are two cases of personal equilibria when consumers differ in consumption valuation: (a) only the high type consumers buy at the full price, (b) both types of consumers buy at the full price.

For personal equilibrium (a) to sustain, the following constraints must be satisfied simultaneously: $\phi_a = \int_0^{\tau_a} \frac{\min\{x,q\}}{x} dF(x) + \int_{\tau_a}^{+\infty} \frac{\min\{\alpha_h x,q\}}{\alpha_h x} dF(x)$ and $\overline{p}_a = v_h + \frac{\phi_a sF(\tau_a) - [1-\phi_a(2-F(\tau_a))]v_h}{1+\eta[\lambda-\phi_a(\lambda-1)(1-F(\tau_a))]}(\lambda-1)\eta$. These constraints are identical to those constraints in (A-2) in the analysis of consumer heterogeneity in loss aversion. Therefore, the analysis of **Equilibrium 2** in the case of consumer heterogeneity in loss aversion applies and we have the same comparative statics results as Proposition 12.

For personal equilibrium (b) to sustain, the following constraints must be satisfied simultaneously: $\phi_b = \int_0^{+\infty} \frac{\min\{x,q\}}{x} dF(x)$ and $\overline{p}_b = v_l + \frac{\phi_b sF(\tau_b) - [1-\phi_b(2-F(\tau_b))]v_l}{1+\eta[\lambda-\phi_b(\lambda-1)(1-F(\tau_b))]}(\lambda-1)\eta$. These constraints are identical to (9) and (10) of Proposition 1 in the homogenous consumer case. Therefore, the comparative statics analysis in the homogenous consumer case applies and Proposition 3 to Proposition 4 continue to hold.

Proof of Proposition 14

We prove by contradiction. Suppose both firms set a deterministic price in equilibrium. Without loss of generality, we assume firm 1(2)'s price as p_1 (p_2) with $s < p_1 < p_2$, and firm 1(2)'s order quantity as q_1 (q_2). In equilibrium, buying at both prices must be consumers' **PE**; Otherwise, it will degenerate to a monopoly setting.

First, we show that firm 1 will deviate. According to our assumptions on consumer behavior, after demand realizes, consumers will first go to firm 1 to purchase the product. If a consumer cannot obtain the product from firm 1 due to stockout, she will then go to firm 2. Thus, the consumer's reference distribution resulting from consumer's strategy of buying at both prices is

$$\mathbf{\Gamma}(\mathbf{r}; p_1, p_2) = \begin{cases} g_1 & \text{if } \mathbf{r} = (v, -p_1), \\ g_2 & \text{if } \mathbf{r} = (v, -p_2), \\ 1 - g_1 - g_2 & \text{if } \mathbf{r} = (0,0). \end{cases} \tag{A-24}$$

where $g_1 = \int_0^{q_1} dF(x) + \int_{q_1}^{+\infty} \frac{q_1}{x} dF(x)$ and $g_2 = \int_{q_1}^{q_1+q_2} \frac{x-q_1}{x} dF(x) + \int_{q_1+q_2}^{+\infty} \frac{q_2}{x} dF(x)$.

According to (4), if buying at both price p_1 and p_2 is consumers' **PE**, we must have

$$U((v,-p_2)|\mathbf{\Gamma}) \geq U((0,0)|\mathbf{\Gamma}).$$

We discuss two cases as follows:

Case 1: $U((v,-p_2)|\mathbf{\Gamma}) > U((0,0)|\mathbf{\Gamma})$.

In this case, we show that firm 1 can randomize pricing among s and p_1 contingent on demand realization and obtains a higher profit. Let $\tau = \min\left\{\frac{sq_1}{p_1}, F^{-1}\left(\frac{U((v,-p_2)|\mathbf{\Gamma}) - U((0,0)|\mathbf{\Gamma})}{\eta(p_1-s)(\lambda-1)}\right)\right\}$ and $\xi =$

$\int_0^\tau dF(x)$, suppose firm 1 sets its price at s when $x \in [0,\tau]$ and at p_1 when $x \in (\tau,+\infty)$. Under this new price, the reference distribution becomes

$$\mathbf{\Gamma}'(\mathbf{r};s,p_1,p_2) = \begin{cases} \xi & \text{if } \ \mathbf{r} = (v,-s), \\ g_1 - \xi & \text{if } \ \mathbf{r} = (v,-p_1), \\ g_2 & \text{if } \ \mathbf{r} = (v,-p_2), \\ 1 - g_1 - g_2 & \text{if } \ \mathbf{r} = (0,0). \end{cases}$$

It is straightforward to show that $U((v,-p_2)|\mathbf{\Gamma}') = U((v,-p_2)|\mathbf{\Gamma}) + \xi\eta\lambda(s-p_1)$ and $U((0,0)|\mathbf{\Gamma}') = U((0,0)|\mathbf{\Gamma}) + \xi\eta(s-p_1)$. Therefore, we must have

$$U((v,-p_2)|\mathbf{\Gamma}') - U((0,0)|\mathbf{\Gamma}') = U((v,-p_2)|\mathbf{\Gamma}) - U((0,0)|\mathbf{\Gamma}) - \xi\eta\lambda(p_1-s)(\lambda-1) \geq 0,$$

which follows by the definition of ξ. Hence, buying at s, p_1 and p_2 is still consumers' **PE**. Let Π_1 be firm 1's profit when it sets a deterministic price p_1 and Π_1' be firm 1's profit when it randomizes among s and p_1 according to sales threshold τ, then we have

$$\begin{aligned} \Pi_1' &= s\int_0^\tau q_1 dF(x) + p_1 \int_\tau^{+\infty} \min\{x,q_1\} dF(x) - c \cdot q_1 \\ &> p_1 \int_0^\tau x dF(x) + p_1 \int_\tau^{+\infty} \min\{x,q_1\} dF(x) - c \cdot q_1 = \Pi_1. \end{aligned}$$

Therefore, setting a deterministic price p_1 is not firm 1's optimal strategy.

Case 2: $U((v,-p_2)|\mathbf{\Gamma}) = U((0,0)|\mathbf{\Gamma})$.

Suppose now firm 1 increase its price from p_1 to $p_1+\epsilon$, where ϵ is positive and sufficiently small, then the reference distribution becomes

$$\mathbf{\Gamma}''(\mathbf{r};p_1,p_2) = \begin{cases} g_1 & \text{if } \ \mathbf{r} = (v,-p_1-\epsilon), \\ g_2 & \text{if } \ \mathbf{r} = (v,-p_2), \\ 1 - g_1 - g_2 & \text{if } \ \mathbf{r} = (0,0). \end{cases}$$

It is straightforward to show that

$$U((v,-p_2)|\mathbf{\Gamma}'') - U((0,0)|\mathbf{\Gamma}'') = U((v,-p_2)|\mathbf{\Gamma}) - U((0,0)|\mathbf{\Gamma}) + g_1\eta(\lambda-1)\epsilon > 0.$$

Hence, buying at both $p_1+\epsilon$ and p_2 is still consumers' **PE**, but firm 1 is strictly better off, and the discussion goes back to **Case 1**.

Next, given price randomization by firm 1, we argue that firm 2 will also be better off by randomize its price. At the very least, firm 2 can follow firm 1's sales strategy by setting its price at s when demand realization is lower than or equal to τ. By doing so, firm 2 can earn an additional amount of $sq_2F(\tau)$ in expected profit without changing consumers' **PE**.

Proof of Proposition 15

Suppose the firm implements a contingent pricing policy $p(x)$ and results in a price distribution $g(p)$ as follows

$$p(x) = \begin{cases} s, \text{ if } x \in [0,\tau]; \\ \overline{p}, \text{ if } x \in (\tau,+\infty), \end{cases} \quad g(p) = \begin{cases} F(\tau), & \text{if } p = s; \\ 1-F(\tau), & \text{if } p = \overline{p}. \end{cases}$$

By Proposition 1, given a fixed order quantity q, the sales threshold τ is smaller than the order quantity in optimum. When the firm charges the sale price, i.e., $p=s$, the loss averse consumers know that the realized demand is lower than the sales threshold, i.e., $x \le \tau$, so the product availability conditional on $p=s$ is

$$\phi(s)=\frac{\int_0^{\tau}\frac{\min(x,q)}{x}dF(x)}{\int_0^{\tau}dF(x)}=\frac{\int_0^{\tau}dF(x)}{\int_0^{\tau}dF(x)}=1.$$

When the firm charges the full price, i.e., $p=\overline{p}$, the loss averse consumers know that the realized demand is higher than the sales threshold, i.e., $x>\tau$, so the product availability conditional on $p=\overline{p}$ is

$$\phi(\overline{p})=\frac{\int_{\tau}^{\infty}\frac{\min(x,q)}{x}dF(x)}{\int_{\tau}^{\infty}dF(x)}=\frac{\int_{\tau}^{q}dF(x)+\int_{q}^{\infty}\frac{q}{x}dF(x)}{\int_{\tau}^{\infty}dF(x)}=\frac{F(q)-F(\tau)+\int_{q}^{\infty}\frac{q}{x}dF(x)}{1-F(\tau)}.$$

Therefore, $\phi(p), p=s,\overline{p}$, is the price-dependent fill-rate. Consumers' reference distribution is:

$$\mathbf{\Gamma}(\mathbf{r};g,\phi,\overline{p})=\begin{cases}\phi(s)g(s), & \text{if } \mathbf{r}=(v,-s);\\ \phi(\overline{p})g(\overline{p}), & \text{if } \mathbf{r}=(v,-\overline{p});\\ 1-\sum_{p=s,\overline{p}}\phi(p)g(p), & \text{if } \mathbf{r}=(0,0).\end{cases}$$

The utilities of consumption outcomes $(v,-\overline{p})$ and $(0,0)$ when a consumer decides to buy at the full price $\overline{p}$ are:

$$U(\mathbf{k}|\mathbf{\Gamma})=\begin{cases}(v-\overline{p})-\eta\lambda\phi(s)g(s)(\overline{p}-s)+(1-\phi(s)g(s)-\phi(\overline{p})g(\overline{p}))\eta(v-\lambda\overline{p}), & \text{if } \mathbf{k}=(v,-\overline{p});\\ \eta\phi(s)g(s)(-\lambda v+s)+\phi(\overline{p})g(\overline{p})(-\lambda v+\overline{p})\eta, & \text{if } \mathbf{k}=(0,0).\end{cases}$$

According to Lemma 1, the credible full price $\overline{p}^*$ satisfies

$$U((v,-\overline{p})|\mathbf{\Gamma})=U((0,0)|\mathbf{\Gamma}).$$

Note that $\phi(s)=1$ and let ϕ^* denote $\phi(\overline{p}^*)$, then the credible full price can be expressed as

$$\overline{p}^*=v+\frac{(s+v)F(\tau)+v(2\phi^*(1-F(\tau))-1)}{(1+\eta\lambda)+\eta(\lambda-1)\phi^*(1-F(\tau))}(\lambda-1)\eta,$$

$\overline{p}^*\ge v$ holds, if and only if, the inequality $(s+v)F(\tau^*)+v(2\phi^*(1-F(\tau^*))-1)\ge 0$ holds, i.e.,

$$F(\tau^*)(s+v(1-2\phi^*))\ge v(1-2\phi^*). \tag{A-25}$$

If $1-2\phi^*\le 0$, then we have either (i) $v(1-2\phi^*)<s+v(1-2\phi^*)\le 0$, or (ii) $v(1-2\phi^*)<0<s+v(1-2\phi^*)$. Since $0<F(\tau^*)<1$, $v(1-2\phi^*)<F(\tau^*)[s+v(1-2\phi^*)]\le 0$ must hold for Case (i). If $1-2\phi^*>0$, dividing both sides of (A-25) by $s+v(1-2\phi^*)$ gives rise to $F(\tau^*)>\frac{v(1-2\phi^*)}{s+v(1-2\phi^*)}$.

Supplemental Note to the Paper "Newsvendor Selling to Loss Averse Consumers with Stochastic Reference Points"

Analysis of Personal Equilibrium of Continuous Heterogenous Loss Averse Consumers

We consider a market with a continuum of heterogeneous consumers in loss aversion. Consumers have the same consumption valuation v but differ in the degree of loss aversion, parameterized by a cumulative distribution function $H(\cdot)$ of λ. Let $\Re$ be the support of $H(\cdot)$. For tractability, we assume that functions $H(\cdot)$ and $F(\cdot)$ are independent. Hence, for a demand realization x, $xH(y)$ is the number of consumers with a loss aversion parameter λ no more than y.

We discuss how the heterogeneity of consumer loss aversion would affect the existence and uniqueness of the equilibrium when the firm uses the contingent two-price strategy as in Proposition 1. Since consumers do not reveal their private loss aversion levels to the firm, the firm cannot discriminate consumers and ration inventory among them. As a result, consumers with different degrees of loss aversion should have the same product availability if they follow the same purchase plan.

Consumers with $\lambda \in \Delta$ (where $\Delta \subseteq \Re$) choose to purchase at both prices, and consumers with $\lambda \in \Delta^c$ (where $\Delta^c = \Re \backslash \Delta$) choose to purchase only at the sales price. Let $\xi_\Delta = \int_{\lambda \in \Delta \xi} dH(\lambda)$, then the product availability (fill rate) for every type of consumer in set Δ is

$$\phi_\Delta = \int_0^\tau \frac{\min\{x, q\}}{x} dF(x) + \int_\tau^{+\infty} \frac{\min\{\xi_\Delta x, q\}}{\xi_\Delta x} dF(x), \tag{A-1}$$

and the product availability for every type of consumer in set Δ^c is

$$\phi_c = \int_0^\tau \frac{\min\{x, q\}}{x} dF(x). \tag{A-2}$$

We further define the following notations:

- $A_\Delta = \phi_\Delta(v + sg(s) + g(\overline{p}_\Delta)\overline{p}_\Delta) - \overline{p}_\Delta$;
- $A_{\Delta^c} = \phi_c g(s)(s+v) - \overline{p}_\Delta$;
- λ_Δ is the solution to equation $(v - \overline{p}_\Delta) + \eta[v - (v+s)\phi_\Delta g(s) - (v + \overline{p}_\Delta)\phi_\Delta g(\overline{p}_\Delta))] + \eta\lambda A_\Delta = 0$;
- λ_{Δ^c} is the solution to equation $(v - \overline{p}_\Delta) + \eta[v - \phi_c g(s)(v+s)] + \eta\lambda A_{\Delta^c} = 0$;
- $\overline{p}_\Delta$ is defined as (9) with λ replaced by λ_Δ and ϕ^* replaced by ϕ_Δ.

then the following proposition characterizes the existence of equilibrium and the cut-off point between *buying* and *not-buying* consumers.[1]

[1] *Buying* consumers refer to those who buy the product at both prices and *not-buying* consumers refer to those who buy the product only at the sales price.

Proposition 16 (Existence of Personal Equilibrium) *For consumers with a continuous distribution $H(\cdot)$ of loss aversion parameter λ, there exists an equilibrium, if and only if, the following two conditions are satisfied simultaneously:*

(i) $\lambda_\Delta = \lambda_{\Delta^c}$*;*

(ii) $A_\Delta = A_{\Delta^c}$*.*

Furthermore, if an equilibrium exists, then the partition of consumers must have a cut-off structure as follows:

(iii) if $\phi_c g(s)(s+v) - \overline{p} > 0$*, then* $\Delta = \{\lambda | \lambda \geq \lambda_\Delta\}$*, i.e., consumers with* $\lambda < \lambda_\Delta$ *choose to buy only at the sales price, while consumers with* $\lambda \geq \lambda_\Delta$ *choose to buy at both prices;*

(iv) if $\phi_c g(s)(s+v) - \overline{p} \leq 0$*, then* $\Delta = \{\lambda | \lambda \leq \lambda_\Delta\}$*, i.e., consumers with* $\lambda \leq \lambda_\Delta$ *choose to buy at both price, while consumers with* $\lambda > \lambda_\Delta$ *choose to buy only at the sales price.*

Proposition 16 also suggests that there could exist multiple equilibria. When there are multiple equilibria, the firm would choose the equilibrium that yields the highest profit and adopt the pricing strategy accordingly. A equilibrium *prevails* if it is chosen by the firm.

Proof of Proposition 16

Now suppose there is an equilibrium, in which consumers with loss aversion $\lambda \in \Delta$ choose to buy at both prices and their reference points are

$$\mathbf{\Gamma}(\mathbf{r}; g(\cdot), \phi_\Delta, \overline{p}) = \begin{cases} \phi_\Delta g(s) \ , & \text{if } \ \mathbf{r} = (v, -s); \\ \phi_\Delta g(\overline{p}) \ , & \text{if } \ \mathbf{r} = (v, -\overline{p}); \\ 1 - \phi_\Delta \ , & \text{if } \ \mathbf{r} = (0, 0). \end{cases}$$

where ϕ_Δ is defined in (A-1). Consumers with loss aversion $\lambda \in \Delta^c$ choose to buy only at the sales price s, so their references points are

$$\mathbf{\Gamma}(\mathbf{r}; g(\cdot), \phi_c, \overline{p}) = \begin{cases} \phi_c g(s) & , \text{ if } \ \mathbf{r} = (v, -s); \\ 0 & , \text{ if } \ \mathbf{r} = (v, -\overline{p}); \\ 1 - \phi_c g(s) & , \text{ if } \ \mathbf{r} = (0, 0). \end{cases}$$

In equilibrium, the consumers with $\lambda \in \Delta$ will be worse off by deviating from *buying* at $\overline{p}$, i.e.,

$$\begin{aligned} & U((v, -\overline{p}) | \mathbf{\Gamma}(\mathbf{r}; g(\cdot), \phi_\Delta, \overline{p})) \geq U((0,0) | \mathbf{\Gamma}(\mathbf{r}; g(\cdot), \phi_\Delta, \overline{p})) \\ \Rightarrow & (v - \overline{p}) + (\eta v - \eta \lambda \overline{p}) + \eta(\lambda - 1) \sum_{p = \overline{p}, s} (v + p) \cdot \phi_\Delta g(p) \geq 0, \quad \text{(see the first equation in Lemma 1)} \\ \Rightarrow & (v - \overline{p}) + \eta[v - (v+s)\phi_\Delta g(s) - (v + \overline{p})\phi_\Delta g(\overline{p}))] + \eta\lambda \underbrace{[\phi_\Delta(v + sg(s) + g(\overline{p})\overline{p}) - \overline{p}]}_{A_\Delta} \geq 0, \qquad \text{(A-3)} \end{aligned}$$

Clearly, the partition of consumers into *buying* and *not buying* groups depends on the value of A_Δ. Suppose λ_Δ is the solution to equality (A-3). If $A_\Delta > 0$, then consumers with $\lambda < \lambda_\Delta$ will choose not to buy at $\overline{p}$, and consumers with $\lambda \geq \lambda_\Delta$ will choose to buy at $\overline{p}$. Otherwise, if $A_\Delta \leq 0$, then consumers with $\lambda < \lambda_\Delta$ will choose to buy at $\overline{p}$, and consumers with $\lambda \geq \lambda_\Delta$ will choose not to buy at $\overline{p}$.

Analogously, for consumers with $\lambda \in \Delta^c$, they will be worse off by deviating from *not buying* at $\overline{p}$. Note that since every type of consumer is atomic, should λ type of consumers choose to deviate from *not buying* at $\overline{p}$ to *buying* at $\overline{p}$, the product availability is still ϕ_Δ. For these consumers, we have

$$\begin{aligned}
&\phi_\Delta U((v,-\overline{p})|\mathbf{\Gamma}(\mathbf{r};g(\cdot),\phi_c,\overline{p})) + (1-\phi_\Delta)U((0,0)|\mathbf{\Gamma}(\mathbf{r};g(\cdot),\phi_c,\overline{p})) \leq U((0,0)|\mathbf{\Gamma}(\mathbf{r};g(\cdot),\phi_c,\overline{p})) \\
\Rightarrow\ & U((v,-\overline{p})|\mathbf{\Gamma}(\mathbf{r};g(\cdot),\phi_c,\overline{p})) \leq U((0,0)|\mathbf{\Gamma}(\mathbf{r};g(\cdot),\phi_c,\overline{p})) \\
\Rightarrow\ & (v-\overline{p}) + \eta[v-\phi_c g(s)(v+s)] + \eta\lambda \underbrace{[\phi_c g(s)(s+v)-\overline{p}]}_{A_{\Delta^c}} \leq 0, \qquad \text{(A-4)}
\end{aligned}$$

Let λ_{Δ^c} be the solution to equality (A-4). If $A_{\Delta^c} > 0$, then consumers with $\lambda < \lambda_{\Delta^c}$ will choose not to buy at $\overline{p}$, and consumers with $\lambda \geq \lambda_{\Delta^c}$ will choose to buy at $\overline{p}$. Otherwise, if $A_{\Delta^c} \leq 0$, then consumers with $\lambda < \lambda_{\Delta^c}$ will choose to buy at $\overline{p}$, and consumers with $\lambda \geq \lambda_{\Delta^c}$ will choose not to buy at $\overline{p}$.

For the partition of consumers to be consistent, we should have $\lambda_\Delta = \lambda_{\Delta^c}$ and both A_Δ and A_{Δ^c} should have the same sign, and thus $A_\Delta = A_{\Delta^c}$ follows.

Lastly, the characterization of the regular price $\overline{p}_\Delta$ follows by taking equality in (A-3) and solves for $\overline{p}$.