

# When Gray Markets Have Silver Linings: All-Unit Discounts, Gray Markets, and Channel Management

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Gray markets are unauthorized channels of distribution for a supplier's authentic products. We study a distribution channel that consists of a supplier who offers *all-unit* quantity discounts for batch orders to enjoy cost savings, and a reseller who may divert some goods to the gray markets. We show that the impact of gray markets depends on the reseller's inventory holding cost. When the reseller's inventory holding cost is high, diversion to the gray markets improves the channel performance by enabling the reseller to make batch orders. Because the reseller's order costs decrease through quantity discounts, diversion to the gray markets reduces the resale price and expands sales to the authorized channel. On the other hand, when the reseller's inventory holding cost is low, the reseller would make the batch orders even without the gray markets. In this case the diversion to the gray markets may improve the reseller's performance by shortening the order cycles and reducing the inventory holding costs. Interestingly, because diversion to the gray markets decreases the reseller's cycle inventory volume, the reseller has the reduced incentive to push its inventory, and, consequently, the resale price rises and sales volume decreases in the authorized channel. Moreover, there exists a range of reseller's inventory holding cost and supplier's cost of scale economy such that it is optimal for the supplier to induce reseller's gray market diversion through an all-unit discount. We show that these results are robust when the gray market overlaps with the authorized channel or when the gray market price is sensitive to reseller's diversion volume.

*Key words:* gray markets; channel management; inventory; quantity discounts; supplier pricing

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## 1. Introduction

The diversion of branded goods to unauthorized channels, also known as gray markets, is of substantial strategic interest to manufacturers. Gray market channels that exist domestically and alongside the authorized channels are called domestic gray markets. Industry reports show that domestic gray market channels account for a significant portion of markets in a broad set of industries ranging from pharmaceuticals to consumer electronics. Even powerful manufacturers like IBM circa 1984 face domestic gray markets: internal IBM studies indicate gray market activity by authorized IBM distributors accounted for at least 5% of total IBM PC sales (Banerji 1990). Cespedes et al. (1988) estimated that the annual U.S. domestic gray market size amounted to \$7–\$10 billion. According to a KPMG (2008) study, the gray market accounted

for as much as \$58 billion in the 2008 information technology (IT) hardware industry. The same report shows many authorized resellers acknowledging sales to gray market agents, often in response to incentive programs offered by suppliers and limited effort on the behalf of suppliers to police gray market diversions. With this in mind, we take the premise that a supplier manages the gray market through tolerance of violation (Dutta et al. 1994, Bergen et al. 1998, Antia et al. 2004). In other words, the supplier chooses not to pursue enforcement through monitoring and legal action. Instead, the supplier anticipates the reseller's access to the gray market and formulates the pricing strategy accordingly.

The focus of this paper is on the operational and marketing issues associated with the domestic gray market diversion induced by all-unit quantity

discounts. An all-unit quantity discount applies the same price discount to every unit purchased once the purchase volume threshold is achieved. Munson and Rosenblatt (1998) reported that all-unit discounts are the most prevalent form of quantity discount offered by suppliers, and are predominantly driven by economies of scale in production and transportation. Our analysis shows that the all-unit discount scheme is unique (as opposed to the incremental discount and two-part tariff scheme) in providing an incentive for the reseller to order up to a threshold and then use the gray market to divert possible excess inventory. Our research goals are to understand how operational factors (e.g., inventory costs for the reseller and scale economies for the supplier) interact with pricing decisions (e.g., reseller pricing and supplier all-unit discounts) in markets where the resellers can divert to the gray market. We also investigate how the presence of the gray market affects the profits of the resellers and sales in the authorized channel. Consistent with the findings of Munson and Rosenblatt (1998) and like those of Lal and Staelin (1984), our model builds on the assumption that the supplier enjoys cost benefits when the resellers order in batches. In addition to the KPMG (2008) survey, a wide variety of industrial literature has indicated reseller's overordering in response to supplier's discount pricing practices as a key driver for a persistent gray market (Ramirez 1985, Lowe and McCrohan 1988, Jorgenson 1999, Gilroy 2004). Even in ostensibly well-regulated industries such as pharmaceuticals, abuse of promotions has proved difficult to counteract. Eban (2005) reported that up to four-fifths of American nursing homes and similar healthcare institutions take advantage of wholesalers' discounting practices to profit from sales of prescription medications to gray market channels.

A widely cited concern over gray markets is that they compete for customers with authorized resellers operating in the same channel and/or location at the same time. Unlike a typical salvage channel where either the primary and secondary markets are independent or the cannibalization happens sequentially (e.g., consumers anticipate future salvage of leftover inventory; see Su and Zhang 2008), the gray market coexists with an authorized channel. The extent of concurrent cannibalization depends on the price sensitivity of customers, the search costs associated with locating and acquiring gray market goods, and the degree of differentiation between the gray market and authorized market good (e.g., gray market products often lack a warranty). This cannibalization aspect of the gray market is closely related to concerns over counterfeit goods. Although gray markets sell authentic products, counterfeit goods do not originate from the trademark holder (Duhan and Sheffet

1988, Grossman and Shapiro 1988). Cho et al. (2011) studied how suppliers may counter the counterfeits using price reduction, quality differentiation, marketing campaigns, and strict enforcement. The problem with counterfeits can be further complicated by their sales through licit or illicit supply chains. However, unlike counterfeit goods, gray market supply originates from the genuine supplier, and thus the gray market can increase the supplier's aggregate sales. Moreover, the supplier has marketing levers that can directly impact the diversion behavior of resellers. These two unique aspects of the gray market largely motivate this paper.

Gray market supply may also result from excess stock due to demand uncertainty (Xia and Bassok 2005, Altug and van Ryzin 2009, Dasu et al. 2012). When realized demand is below expectations, the gray market provides a channel for resellers to dispose of overstock after the regular sales horizon. In practice we expect both drivers of excess stock and quantity discount to contribute to the gray market. The gray market diversion driven by overstock under demand uncertainty can be analyzed in a newsvendor setting, and is particularly appropriate when goods are perishable and demand is periodic. This setup lends itself to seasonal and fashion goods. Both drivers may contribute to domestic gray markets as well as parallel imports. In the latter case, products are diverted internationally, and the drivers may act to reinforce price differences across countries (Ahmadi and Yang 2000, Dasu et al. 2012).

### 1.1. Contributions

Our model and analysis integrate the operational and marketing decisions. First, our model considers a reseller optimizing over lot-sizing and resale price decisions when facing an authorized channel, a gray market and a supplier offering all-unit quantity discounts. The closed-form analysis of the reseller's lot-sizing problem yields a novel solution that links the cost of holding inventory to the supply of goods to the gray market. Specifically, we find gray market diversion occurs only in a middle range of the retailer's inventory holding cost. Within this range, the reseller finds it beneficial to use the gray markets to reduce its inventory holding costs. The gray market may allow the reseller to improve operational efficiency while enjoying the batch quantity discounts. Second, we examine the impact of gray market diversion on the resale price and sales volumes in the authorized channel. Interestingly, the effect depends on the reseller's inventory holding cost. When the inventory holding cost is sufficiently high such that the reseller would not order in batches without the gray market, diversion allows the reseller to enjoy the quantity discount and in turn reduce the resale price. As a result, the

presence of gray markets expands sales in the authorized channel. However, when the inventory holding cost is low enough that the reseller would order in batches even without the gray market, the diversion reduces the reseller's peak cycle inventory and expedites the order frequency. As the reseller faces reduced pressure to push out the inventory, the resale price increases and sales volumes decrease in the authorized channel. Third, we study the effect of gray market diversion on the performance of the distribution channel. When gray market diversion enables the reseller to take advantage of the quantity discount, the supplier enjoys increased operational efficiency and total channel performance improves. Because of the Stackelberg structure of our model, the supplier captures all or most of the efficiency gains.

Our paper contributes to both the marketing and operations management (OM) literature. The marketing literature on gray markets typically neglects inventory costs and limits attention to single-period models (Howell et al. 1986, Wilcox et al. 1987, Banerji 1990). These restrictions on the analysis do not permit operational characteristics, in particular inventory holding costs, to influence the reseller's strategies or the diversion to a gray market. Though Lal and Staelin (1984) indeed considered the effect of the reseller's holding cost on the supplier's quantity discount design in an economic order quantity (EOQ) setting, the authors assume that all products are sold to the authorized channel at an exogenous resale price. In this paper we incorporate the reseller's operational decisions and derive the resale price as a function of the operating cost in the presence of a gray market. As a result, the supplier can take a reseller's operating environment into account when making the channel management decisions. The obtained insights are more applicable to the domestic gray markets, where the diverting authorized channel coexists with the gray market.

The OM literature tends to emphasize the algorithmic problems determining optimal lot sizes (reseller ordering policy) in response to offered discounts due to the lack of tractability of the general multiperiod problem. An overview of the area is covered by a pair of surveys by Benton and Park (1996) and Munson and Rosenblatt (1998). Despite an impressive breadth of work including many extensions of the lot-sizing problem under all-unit discounts, gray markets and, more generally, the ability to salvage surplus inventory have been discussed in only a couple of instances. Sethi (1984) and Arcelus and Rowcroft (1992) have examined the optimal lot-sizing problem with an all-unit discount and a fixed value for salvaged inventory. They both developed algorithms to numerically solve the reseller's lot-sizing problem.

This paper contributes to this literature by developing an explicit solution to a representative lot-sizing problem where the salvage channel is replaced by a gray market which may cannibalize some portion of authorized channel demand. The closed-form solution allows optimal analysis of the reseller's pricing and inventory decisions, and the resulting profit implications to the supplier and welfare consequences to the consumers.

We organize the rest of this paper as follows. In §2 we describe the main model. In §3 we assume an exogenous resale price and focus on the effect of a gray market on the reseller's operational decisions and the subsequent effect on the supplier's profit. In §4 we incorporate the reseller's pricing decision and investigate the interaction between operational and marketing decisions. We then extend the model in §5 to allow for an endogenous gray market price. Finally, we conclude in §6. All proofs can be found in the online appendix (available at <http://dx.doi.org/10.1287/msom.1120.0416>).

## 2. Model and Preliminary Analysis

### 2.1. Model and Assumptions

We consider a market where a monopoly supplier sells its products to end consumers through a single reseller. The reseller can sell the goods through an authorized channel and a domestic gray market. The reseller may be viewed as either a retailer selling directly to consumers or an intermediate distributor selling to authorized retailers.

**2.1.1. Supplier's Cost Structure.** We assume there exists scale economy in production and distribution when the reseller orders in batches. Specifically, we let  $c_o$  represent the per unit supply cost when the reseller orders one unit of the product each time, and we let  $c_\eta (< c_o)$  be the per unit supply cost when the reseller orders in batches of  $\eta$ , which is exogenously determined. The economies of scale are most commonly motivated by operational advantages (Munson and Rosenblatt 1998) and are typically associated with an exogenous batch size. For instance, this batch size may correspond to a pallet or the production lot size. For simplicity, we assume there is only one cost breakpoint because it is sufficient to demonstrate the incentive for induced batch ordering and gray market diversion.

**2.1.2. Reseller's Cost Structure.** The supplier offers an all-unit discount. Following the convention in the OM literature, we represent the supplier's lot-size-based all-unit quantity discount by the reseller's order cost function, denoted by  $C(q)$ , where  $q$  is the order size, as follows:  $C(q) = w_o q$  if  $0 \leq q < \eta$ , and

$C(q) = w_\eta q$  if  $q \geq \eta$ , where  $w_o$  is the list price before the discount, and  $w_\eta$  is the discount price. If  $w_o = w_\eta$ , the discount schedule reduces to the trivial case of no quantity discount. As mentioned, all-unit discounts are the most popular type of quantity discounts and provide a unique incentive for overbuying. In practice, all-unit discounts shift the batch-breaking decision from the supplier to the reseller, who shares the cost savings through the quantity discount. We assume the discount breakpoint is the same as the supply cost breakpoint  $\eta$ , which is consistent with the Robinson–Patman Act, which forbids discriminatory pricing in the United States through a quantity discount unless justified by underlying costs (Coughlan et al. 2001). For simplicity, we assume there is no fixed order costs because the all-unit discount has already incorporated an incentive for batch ordering. The obtained insights from the model on gray market diversion remain when there is a fixed order cost. Moreover, the reseller has the option to divert goods to the gray market at a per unit gray market wholesale price  $s$ , which is assumed to be below the batch supply cost, i.e.,  $s < c_\eta$ , to eliminate the supplier's indirect arbitrage opportunity. This is a common assumption in the OM literature; for example, similar assumptions are made in the newsvendor problem. This assumption also eliminates another possible benefit from diversion for the channel, namely, profitable sales to a new market segment (see, e.g., Ahmadi and Yang 2000). Last, for each unit of goods in inventory, the reseller is charged for holding cost at  $h$  per unit per unit of time.

**2.1.3. Demand Structure.** We assume that demand is deterministic in both markets. Specifically, in the authorized channel, the market demand (or order) arrives continuously with a deterministic rate determined by a modified isoelastic demand structure that takes into account the cannibalization between the authorized channel and gray market:  $\lambda(p, p_s) = m/(p - \gamma p_s)^\alpha$ , where  $p$  is the authorized channel resale price, and  $p_s$  is the gray market resale price. This demand structure is mostly applicable to domestic gray markets. (In parallel imports, gray market diversion to another country may not necessarily cannibalize the diverter's own channel, but will cannibalize the foreign authorized channels.) We do not model the detailed gray market pricing mechanism, because emergence and pricing of the gray market can be attributed to many factors. The gray market prices are treated as exogenously determined in the main model. This is likely the case when there are a large number of resellers spreading across many geographic territories, and the diversion of the focal monopoly reseller in one territory contributes only a tiny proportion of the entire supply to the online gray market. In §5, we examine the robustness of

our results in scenarios where the gray market prices are sensitive to diversion volume. The parameter  $m$  denotes the size of the market,  $\gamma < 1$  parameterizes the sensitivity of the authorized channel's demand to the gray market resale price  $p_s$ , and  $-\alpha$  measures the demand elasticity to the adjusted market price difference  $(p - \gamma p_s)$ . The higher the value of  $\gamma$ , the more sensitive the authorized channel's demand to the gray market resale price.

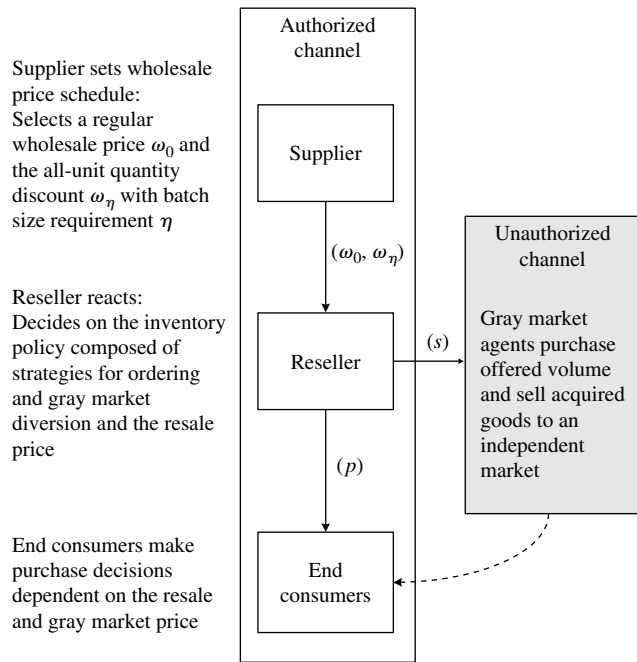
**2.1.4. Sequence of Events.** We consider a Stackelberg game: the supplier first announces the all-unit discount  $w_\eta$ , then the reseller responds. The list price  $w_o$  is assumed to be determined before the supplier optimizes the quantity discount  $w_\eta$ . The list price may be determined exogenously, which is common across studies of quantity discount practices (e.g., Lal and Staelin 1984), or by an optimization on top of the Stackelberg game. After the supplier offers the discount, the reseller makes lot-sizing decisions and, possibly, the resale pricing decision. At any time  $t$ , the reseller's lot-sizing decisions include ordering  $q(t)$  units from the supplier at a cost of  $C(q(t))$  and diverting  $g(t)$  units into the gray market. We allow replenishment to be instantaneous, which is equivalent to assuming a deterministic lead time. The reseller's inventory level at time  $t$  is denoted by  $I(t)$ . Profits for both reseller and supplier will be considered over an infinite horizon under the long-run average cost criterion. This setting fits well with products of low demand seasonality and moderate life cycles, e.g., hard drives and network routers. Such products are sold over very long timelines through periodic incremental specification improvements (e.g., a 1 GB hard drive is replaced by a 1.5 GB hard drive at the same price point). We summarize the sequence of events in Figure 1.

## 2.2. Preliminary Analysis: All-Unit Discount Induced Gray Market Diversion

We first analyze a static model to investigate the possible incentive for overbuying and diverting to the gray market under an all-unit quantity discount, upon which our dynamic problem builds with consideration of holding inventory. Suppose the reseller orders  $q$  units to fulfill demand  $d$  in the authorized channel and diverts the remaining  $g = q - d$  units to the gray market. The reseller's total effective cost associated with such a policy is  $c(q, d) = C(q) - s(q - d)$ .

If the resale price is exogenous, then maximizing the reseller's profit becomes equivalent to minimizing the reseller's cost. We calculate the optimal order quantity as a function of replenishment need  $d$ ,  $q^*(d) \equiv \max\{\arg \min_{q \geq d} c(q, d)\}$ . We let  $c^*(d) \equiv c(q^*(d), d)$  and  $g^*(d) \equiv q^*(d) - d$  denote the reseller's optimal order cost and the optimal size of diversion to the gray market, respectively.

Figure 1 Model Structure



PROPOSITION 1 (OPTIMAL STATIC ORDER AND DIVERSION STRATEGY). Under the all-unit discount  $C(q)$ , the optimal order and diversion strategy of the reseller with a demand size  $d \geq 0$  is

$$q^*(d) = \begin{cases} \eta & \text{if } \hat{q} \leq d < \eta, \\ d & \text{otherwise,} \end{cases}$$

where  $\hat{q} = ((w_\eta - s)/(w_0 - s))\eta \leq \eta$  is the threshold demand size above which the reseller orders up to  $\eta$ . The optimal effective order cost is a continuous function in form of

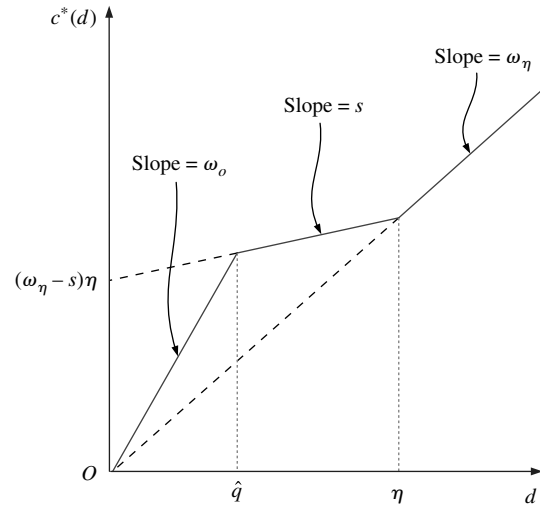
$$c^*(d) = \begin{cases} w_0 d & \text{if } 0 \leq d < \hat{q}, \\ (w_\eta - s)\eta + sd & \text{if } \hat{q} \leq d \leq \eta, \\ w_\eta d & \text{otherwise.} \end{cases}$$

The diversion to the gray market is

$$g^*(d) = \begin{cases} \eta - d & \text{if } \hat{q} \leq d \leq \eta, \\ 0 & \text{otherwise.} \end{cases}$$

Note that incremental discounts and two-part tariffs do not generate incentive for overbuying and diversion to the gray market (see the online appendix). All-unit discounts have a *negative marginal cost* property, i.e., there exists  $q$  for which  $C(q + 1) - C(q) < 0$ . Both incremental discount and two-part tariff policies (which combine a constant marginal cost per unit with a fixed order cost) lack this feature. Without marginal costs being negative, the loss from overbuying and selling a unit to the gray market will never

Figure 2 Optimal Order Costs vs. Demand



be profitable. The introduction of holding costs in the following sections further relies on this property to generate an incentive of overbuying because they are increasing in the size of the order.

The diversion to the gray market occurs only in the middle range of demand  $d \in [\hat{q}, \eta]$ . In this range, the benefit from receiving the discount outweighs the loss from diverting the excess purchases to the gray market. The reseller orders  $\eta$  units regardless of the size of demand in this range; an incremental demand means a unit reduction of the reseller’s supply to the gray market, and hence an increase in the reseller’s total cost by  $s$ . If  $d = \hat{q}$ , the reseller is indifferent between (1) making an order  $\hat{q}$  at the list price  $w_0$  and (2) ordering up to  $\eta$  at the discount price  $w_\eta$  and then diverting  $(\eta - \hat{q})$  units to the gray market; for consistency, we designate the reseller’s behavior when  $d = \hat{q}$  to be the latter case of ordering and diversion. We illustrate this optimal order cost  $c^*(d)$  in Figure 2.

The above static analysis has clearly demonstrated the cause and consequence of an all-unit discount induced gray market at the reseller’s level. Next we analyze the full model where both the reseller and supplier consider their decisions, taking into account inventory holding costs in an infinite-horizon setting.

### 3. Model Analysis: Exogenous Resale Price

In this section we study the case where the resale price  $p$  is exogenous. We will solve the Stackelberg game backward by first analyzing the reseller’s optimal inventory decisions and then examining the supplier’s optimal discount price.

#### 3.1. Reseller’s Inventory Policy

With an exogenous resale price, the reseller’s profit-maximization problem can be solved by minimizing

the total of the reseller’s order cost and inventory holding cost. Because demand is stationary, it is expected that the optimal policy is of a stationary type among all dynamic policies. The stationary inventory policy includes a target inventory level  $I$ , an order size  $q$ , a diversion amount  $g$  to the gray market, and the timing of ordering and diversion.

**LEMMA 1 (RESELLER’S ZERO-INVENTORY POLICY).** *The reseller’s optimal inventory policy is a zero-inventory policy: with a target cycle inventory level  $I$ , the reseller orders  $q^*(I)$  every cycle and immediately diverts  $g^*(I)$  to the gray market at the exact times when the inventory is replenished.*

The zero-inventory property in Lemma 1 is typically associated with EOQ models. In the optimal inventory policy, the inventory level reaches a peak of  $I = \eta - g$  at the beginning of each order cycle. Because the demand arrives at a constant rate  $\lambda$ , the length of each cycle is  $I/\lambda = ((\eta - g)(p - \gamma p_s)^\alpha)/m$ . For simplicity of notation, we define the average unit holding cost per cycle for a full batch order  $I = \eta$  without diversion, when the demand rate is  $\lambda = m$ , as  $H \equiv (h\eta)/(2m)$ . As an aggregate measurement, the holding cost  $H$  contains information not only about  $h$  but also about the cycle length, and is a more convenient measure of inventory costs in this paper.

**PROPOSITION 2 (EXOGENOUS RESALE PRICE: RESELLER PROBLEM).** *The optimal inventory policy of the reseller is a zero-inventory policy with the cycle inventory as follows: if  $s < w_\eta < (w_o + s)/2$ ,*

$$I^* = \begin{cases} \eta & \text{if } H \leq \frac{w_\eta - s}{(p - \gamma p_s)^\alpha}, \\ I^o \equiv \eta \sqrt{\frac{w_\eta - s}{H(p - \gamma p_s)^\alpha}} & \\ \text{if } \frac{w_\eta - s}{(p - \gamma p_s)^\alpha} < H \leq \frac{(w_o - s)^2}{4(w_\eta - s)(p - \gamma p_s)^\alpha}, & \\ 0 & \text{otherwise;} \end{cases}$$

if  $(w_o + s)/2 \leq w_\eta \leq w_o$ ,

$$I^* = \begin{cases} \eta & \text{if } H \leq \frac{w_o - w_\eta}{(p - \gamma p_s)^\alpha}, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 2 shows that there are three distinct zero-inventory policies obtained that may be optimal for the reseller. First, the *order-as-you-go* strategy, or  $I = 0$  for short, is defined by the reseller not carrying any inventory. Second, the *batch* strategy, or  $I = \eta$ , is characterized by the reseller ordering  $\eta$  and not diverting any quantity to the gray market. Finally, the

*diversion* strategy is defined by the reseller ordering  $\eta$  and immediately diverting a positive amount  $\eta - I^o$  into the gray market.

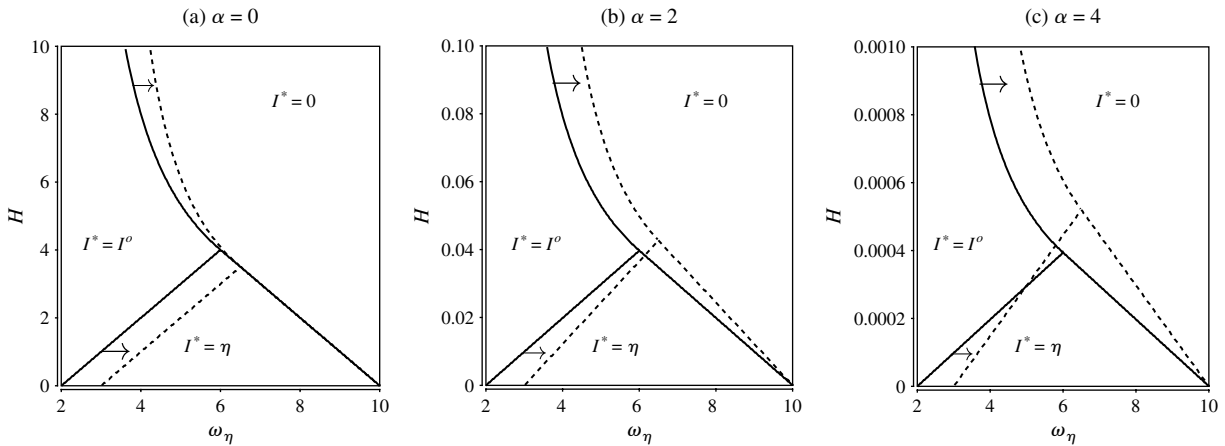
Proposition 2 shows that, irrespective of other parameters, sufficiently high  $H$  leads the reseller to use the order-as-you-go strategy, and sufficiently low  $H$  leads the reseller to use the batch strategy. As the reseller increases the order size to  $\eta$ , additional units will either incur a negative profit margin or additional holding costs depending on whether or not they are diverted to the gray market. A necessary condition for the diversion strategy to be optimal is that  $w_\eta - s < w_o - w_\eta$ , which requires the benefit from the quantity discount to outweigh the margin loss on diverting a unit. On one hand, if  $H$  is below a threshold, the negative profit margin on units sold to the gray market outweighs additional holding costs. On the other hand, for the reseller to be better off diverting goods to the gray market rather than keeping all products in inventory, the immediate diversion loss  $(w_\eta - s)$  needs to be smaller than the average unit holding cost  $H(p - \gamma p_s)^\alpha$  the reseller would have to incur in a cycle without diversion. Then it follows that the diversion strategy is only adopted in an intermediate range of  $H$ .

With the understanding that the gray market wholesale and resale prices are positively correlated, a higher gray market wholesale price will have two implications. First, as the per-unit diversion loss becomes smaller, a *diversion* effect increases the reseller’s incentive to resort to the gray market. Second, a *cannibalization* effect makes the gray market purchase less attractive and increases demand in the authorized channel, which reduces the reseller’s incentive for gray market diversion. These two effects clearly work in opposition. Depending on the relative magnitude of each effect, an increase in the gray market prices may either reinforce or reduce the reseller’s incentive for gray market diversion.

We illustrate the key results of Proposition 2 with Figure 3. The figure shows the parameter spaces under which each of the three inventory policies are optimal. Whereas the scale for  $w_\eta$  is the same in the three panels, the scale for  $H$  is very different, with much smaller units in panels (b) and (c) owing to reduced demand from increased price elasticity.

To illustrate the diversion effect, in Figure 3(a) we eliminate cannibalization by setting  $\alpha = 0$ . When  $s$  increases from 2 to 3, the boundary lines for the diversion strategy versus batch and order-as-you-go strategies both shift toward the right as the diversion strategy becomes more attractive. The boundary between diversion and batch strategies shifts in parallel from  $H = w_\eta - 2$  to  $H = w_\eta - 3$ . In this case, the gray market allows the reseller to reduce cycle inventory in addition to enjoying the discount price. Such a benefit

**Figure 3** Reseller's Optimal Inventory Policy with Exogenous Resale Price



Note. The solid (respectively, dashed) line indicates policy boundaries for  $s = 2$  (respectively,  $s = 3$ );  $m = 10$ ,  $\eta = 20$ ,  $w_o = 10$ ,  $\gamma = 0.7$ ,  $p = 12$ ,  $p_s = 1.4s$ .

is independent of the holding cost. On the other hand, the boundary between diversion and order-as-you-go strategies, present at higher values of  $H$ , shifts disproportionately further to the right as  $H$  increases. In this case, the gray market offers the reseller a chance to order additional units to enjoy the discount that benefits the reseller more at higher holding costs.

The cannibalization effect is illustrated in Figure 3(b), where  $\alpha = 2$ . Clearly the diversion effect still exists: when  $s$  is increased from 2 to 3, the boundary lines shift rightward, and a diversion strategy is more likely to be optimal. However, positive values of parameters  $\alpha$  and  $\gamma$  lead to cannibalization of the authorized channel demand. As a result, to be optimal, the batch strategy requires a much smaller  $H$ , compared to that in Figure 3(a). The interaction between the diversion and cannibalization effects on the reseller's strategies can also be observed in Figure 3(b). In contrast to Figure 3(a), rather than shifting in parallel, the slope of the boundary line between diversion and batch strategies increases when  $s$  is increased from 2 to 3. Although a larger  $s$  makes the diversion strategy more efficient, the effect is smaller with relatively larger values of  $w_\eta$  because the gray market resale price  $p_s$  is also increasing. The increased  $p_s$  reduces the cannibalization effect, increasing market demand in the authorized channel and making the batch strategy more profitable. Figure 3(c) illustrates that the reduction in the cannibalization effect is stronger at a larger value of  $\alpha$ . The cannibalization effect eclipses the diversion effect when  $w_\eta$  is close to 6. However, the boundary between diversion and order-as-you-go strategies is affected differently. As  $s$  is increased, the diversion and reduced cannibalization effects are complementary in making the diversion strategy even more attractive relative to the order-as-you-go policy and pushing this boundary further to the right.

### 3.2. Supplier's Discount Policy

When deciding on the discount price, the supplier needs to anticipate the best response from the reseller. Based on Proposition 2, we can derive the supplier's profit function and optimize its all-unit discount policy. We summarize the results as follows.

**PROPOSITION 3 (EXOGENOUS RESALE PRICE: SUPPLIER PROBLEM).** *The profit-maximizing supplier's optimal discount wholesale price is, if  $c_o - c_\eta > (w_o - s)/2$ ,*

$$w_\eta^* = \begin{cases} w_o - H(p - \gamma p_s)^\alpha & \text{if } H \leq \frac{w_o - s}{2(p - \gamma p_s)^\alpha}, \\ s + \frac{(w_o - s)^2}{4H(p - \gamma p_s)^\alpha} & \text{if } \frac{w_o - s}{2(p - \gamma p_s)^\alpha} < H \leq \frac{(2c_o - w_o - s)(w_o - s)}{4(c_\eta - s)(p - \gamma p_s)^\alpha}, \\ w_o & \text{otherwise;} \end{cases}$$

if  $c_o - c_\eta \leq (w_o - s)/2$ ,

$$w_\eta^* = \begin{cases} w_o - H(p - \gamma p_s)^\alpha & \text{if } H \leq \frac{c_o - c_\eta}{(p - \gamma p_s)^\alpha}, \\ w_o & \text{otherwise.} \end{cases}$$

Proposition 3 shows that the supplier's decision to offer a quantity discount largely depends on the economies of scale ( $c_o - c_\eta$ ) and the reseller's holding cost  $H$ . Small economies of scale allow only a small window of profitable discounts, insufficient to profitably induce a reseller with large values of  $H$  to hold inventory. Significant economies of scale may lead the supplier to provide a discount leading to batch orders. Moreover, the inventory holding cost matters. When the holding costs are low, the supplier offers a discount of  $H(p - \gamma p_s)^\alpha$  from  $w_o$ . This discount exactly accounts for the reseller's incremental

holding cost from ordering in batches without diversion. In this case,  $w_\eta^* = w_o - H(p - \gamma p_s)^\alpha > (w_o + s)/2$ , which is equivalent to  $H(p - \gamma p_s)^\alpha < w_\eta^* - s$ . Thus, the supplier can induce batch orders with a discount price  $w_\eta^*$  sufficiently high in comparison to the gray market wholesale price to discourage any diversion. The supplier enjoys all the net benefits for the entire channel,  $c_o - c_\eta - H(p - \gamma p_s)^\alpha$  per unit, resulting from economies of scale.

Gray market diversion occurs only when the holding cost is in an intermediate range. Proposition 3 clearly indicates the importance of considering the reseller's operational parameters when investigating gray market diversion. Such an intermediate range exists only when the condition  $c_o - c_\eta > (w_o - s)/2$  holds. In this diversion range, the optimal discount size is less than  $H(p - \gamma p_s)^\alpha$ . This discount size perfectly offsets the reseller's holding cost for a less-than-full-cycle inventory as well as the loss incurred in gray market diversion. The supplier enjoys economies of scale by making it just incentive compatible for the reseller to order in batches followed by an immediate diversion.

**COROLLARY 1 (EXOGENOUS RESALE PRICE: BENEFIT ALLOCATION).** *In the case of an exogenous resale price, when it is optimal for the supplier to offer an all-unit discount to the reseller to enjoy economies of scale, the supplier takes all the net benefits.*

In Lal and Staelin (1984), the motivation behind the supplier's quantity discount is increased channel efficiency resulting from economies of scale. The size of the discount is just to offset the reseller's extra inventory holding costs. However, in this paper, the possible channel efficiency from economies of scale is also affected by the gray market. When the gray market prices (both wholesale and resale prices) increase, the diversion and cannibalization effects are complementary in helping the supplier to achieve economies of scale: First, the diversion loss becomes smaller. Second, the higher gray market resale price leads to less cannibalization and hence increases the authorized channel demand.

#### 4. Model Analysis: Endogenous Resale Price

In this section, we extend the previous analysis to allow the resale price to be an endogenous decision variable of the reseller. For tractability, we fix the elasticity by setting  $\alpha = 2$ , which leads to a demand rate function  $\lambda(p) = m/(p - \gamma p_s)^2$ , where the gray market resale price  $p_s$  is exogenously given. Recall that we also assume  $\gamma < 1$ , which means the authorized channel demand is more sensitive to its own channel's price changes. As in the previous section, we

will solve the Stackelberg game backward by first examining the reseller's optimal coordinated pricing and inventory decisions in response to the supplier's quantity discount schedule, and then solving the supplier's optimal quantity discount problem.

##### 4.1. Reseller's Pricing and Inventory Policies

The reseller jointly determines the optimal resale price and inventory policy by balancing revenues with ordering, diversion, and inventory costs. We solve for the optimal resale pricing and inventory policy given the supplier's discount schedule and summarize the results in the following proposition.

**PROPOSITION 4 (ENDOGENOUS RESALE PRICE: RESELLER PROBLEM).** *The optimal pricing and inventory policy for the reseller with the demand function  $\lambda(p) = m/(p - \gamma p_s)^2$  is, if  $s < w_\eta < \sqrt{(w_o - \gamma p_s)(s - \gamma p_s)} + \gamma p_s$ ,*

$$(p^*, I^*) = \begin{cases} (2w_\eta - \gamma p_s, \eta) & \text{if } H \leq \frac{w_\eta - s}{4(w_\eta - \gamma p_s)^2}, \\ \left( \frac{2(s - \gamma p_s)}{1 - 2\sqrt{(w_\eta - s)H}} + \gamma p_s, \left[ \sqrt{\frac{w_\eta - s}{4H}} - (w_\eta - s) \right] \frac{\eta}{(s - \gamma p_s)} \right) & \text{if } \frac{w_\eta - s}{4(w_\eta - \gamma p_s)^2} < H \leq \frac{[1 - \sqrt{(s - \gamma p_s)/(w_o - \gamma p_s)}]^2}{4(w_\eta - s)}, \\ (2w_o - \gamma p_s, 0) & \text{otherwise;} \end{cases}$$

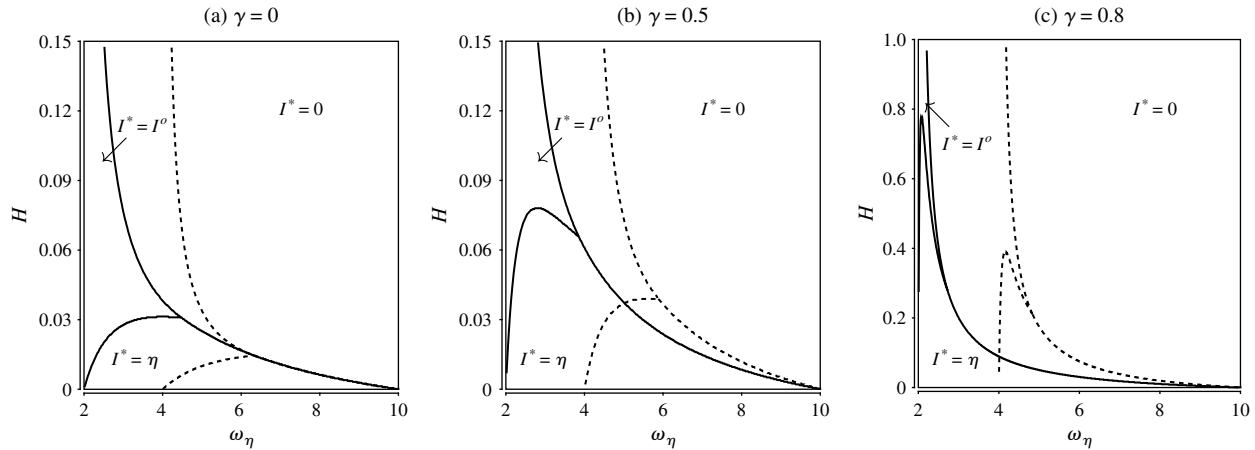
if  $\sqrt{(w_o - \gamma p_s)(s - \gamma p_s)} + \gamma p_s \leq w_\eta \leq w_o$ ,

$$(p^*, I^*) = \begin{cases} (2w_\eta - \gamma p_s, \eta) \\ \text{if } H \leq \frac{1}{4(w_\eta - \gamma p_s)} - \frac{1}{4(w_o - \gamma p_s)}, \\ (2w_o - \gamma p_s, 0) & \text{otherwise.} \end{cases}$$

The above proposition describes the impact of marketing and operational parameters on the resale price in the authorized market. As when the resale price is exogenous, gray market diversion is optimal only when the discount wholesale price is sufficiently low and the inventory holding cost is moderate. This is illustrated in Figure 4, which depicts parameter spaces where each type of strategy is optimal. In these examples, as the gray market wholesale price  $s$  increases, the gray market resale price  $p_s$  also increases according to a fixed markup. As a result, market demand in the authorized channel increases as cannibalization is reduced. Cannibalization increases across panels (a), (b), and (c) through the increase in



**Figure 4 Reseller's Optimal Inventory Policy with Endogenous Resale Price**



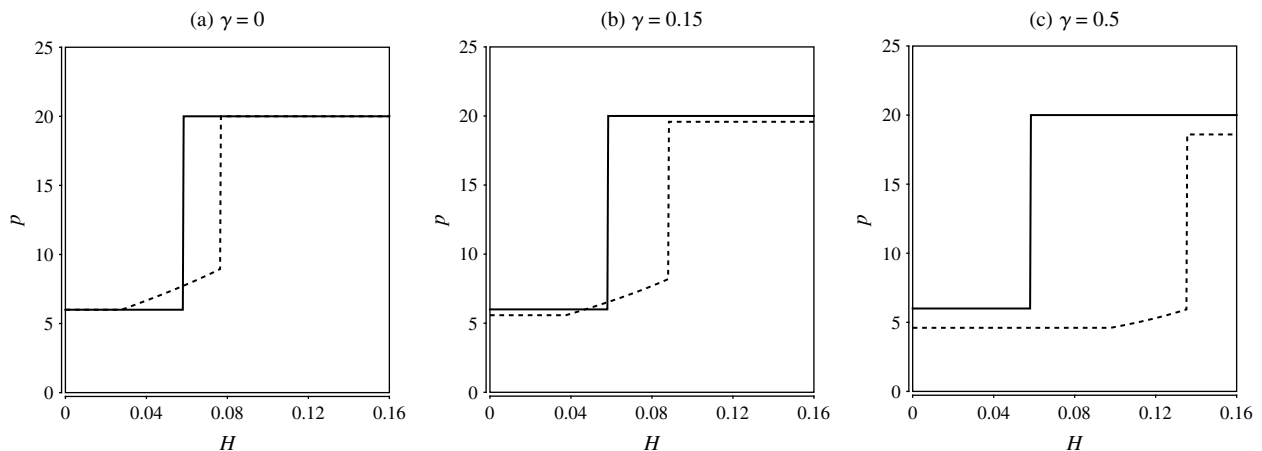
*Note.* The solid (respectively, dashed) line indicates policy boundaries for  $s = 2$  (respectively,  $s = 4$ );  $m = 10$ ,  $\eta = 20$ ,  $w_o = 10$ ,  $\alpha = 2$ ,  $p_s = 1.2s$ .

values of  $\gamma$ . The resulting patterns are very similar to those in Figure 3, where comparative statics are conducted on  $\alpha$ . In particular, in Figure 4(a),  $\gamma = 0$  eliminates cannibalization, and the diversion effect allows a diversion strategy to be optimal over a range of discounts. In Figures 4(b) and 4(c), both cannibalization and diversion effects are present. When  $s$  is increased from 2 to 4, Figure 4 shows that the diversion strategy becomes more prominent.

It should be noted that both lowering the resale price and diverting to the gray market allow the reseller to mitigate increases in holding costs while enjoying the quantity discount. Lowering the resale price generates additional demand in the authorized channel, which shortens the reorder cycle. The optimal resale pricing and inventory policy strikes a balance between lowering the price to increase demand and diverting to the gray market to lower the cycle inventory.

Figure 5 illustrates the effect of a gray market on the optimal resale price. In Figure 5(a) with the cannibalization effect eliminated ( $\gamma = 0$ ), without a gray market ( $s = 0$ ), the optimal resale price (solid line) is low when the firm enjoys the discount wholesale price  $w_\eta$  due to a low holding cost  $H$ , and is high when the firm pays the regular wholesale price  $w_o$  due to a high holding cost  $H$ . With a positive gray market wholesale price ( $s = 2$ ), diversion is used in an intermediate range of holding costs (approximately  $H = 0.02$  to  $0.08$ ). The resale price increases from the diversion-free price at the low end of this range but decreases at the high end of the range. First, when the holding cost is relatively low, the reseller would order in batches anyway regardless of  $s$ . An increase in  $s$  makes diversion more attractive, which reduces the reseller's inventory holding cost. The reseller has a reduced incentive to use a lower price to attract consumers so the resale price increases. As a

**Figure 5 Reseller's Optimal Endogenous Resale Price**



*Note.* The solid (respectively, dashed) line indicates the resale price for  $s = 0$  (respectively,  $s = 2$ );  $m = 10$ ,  $\eta = 20$ ,  $w_o = 10$ ,  $w_\eta = 3$ ,  $\alpha = 2$ ,  $p_s = 1.2s$ .

consequence, sales in the authorized channel would decrease. Second, when the holding cost is relatively high, without the gray market the reseller would follow the order-as-you-go strategy. An increase in  $s$  allows the reseller to benefit from the discount by diverting part of the order. The reduction in order costs decreases the resale price, which expands the market coverage in the authorized channel.

Figures 5(b) and 5(c) show the interaction of diversion and cannibalization effects. Given the assumed demand function, price elasticity increases with  $\gamma$ . Thus, as shown in Proposition 4, for a particular inventory strategy, the optimal resale price has a tendency to decrease as  $\gamma$  and/or  $p_s$  increases. With a large value of  $\gamma$  as shown in Figure 5(c), the optimal resale price in the authorized channel always drops when the gray market wholesale price  $s$  is increased from 0 to 2 with a fixed 20% markup gray market mechanism. The reduced resale price is expected to increase market coverage of the authorized channel.

#### 4.2. Supplier's Discount Policy

In this section we examine the supplier's optimal discount price, taking into consideration the reseller's best response to the discount schedule. The supplier's profit depends on the effective wholesale price with or without discount, the reseller's best-response resale pricing and inventory policies, and the effective supply cost with or without economies of scale. By Proposition 4, it is easy to derive the supplier's profit function given the reseller's best response. Then we can proceed to solve the supplier's profit maximization problem. In addition to the previous assumptions  $s < c_\eta$  and  $\gamma < 1$ , we make an additional assumption  $w_o < 4s - 3\gamma p_s$  for tractability. This is not restrictive when the two markets are relatively differentiated (i.e.,  $\gamma$  is not too close to 1).

**PROPOSITION 5 (ENDOGENOUS RESALE PRICE: SUPPLIER PROBLEM).** *Assume  $w_o < 4s - 3\gamma p_s$  and  $\lambda(p) = m/(p - \gamma p_s)^2$ . The profit-maximizing supplier's optimal discount price is as follows.*

*Case (i). If  $0 \leq 4H < \max\{0, 1/(2(c_\eta - \gamma p_s)) - 1/(w_o - \gamma p_s)\}$ ,*

$$w_\eta^* = \begin{cases} 2c_\eta - \gamma p_s & \text{if } (w_o - \gamma p_s)^2 \geq 4(c_\eta - \gamma p_s)(w_o - c_o), \\ w_o & \text{otherwise.} \end{cases}$$

*Case (ii). If  $\max\{0, 1/(2(c_\eta - \gamma p_s)) - 1/(w_o - \gamma p_s)\} \leq 4H < 1/\sqrt{(w_o - \gamma p_s)(s - \gamma p_s)} - 1/(w_o - \gamma p_s)$ ,*

$$w_\eta^* = \begin{cases} \tilde{w} & \text{if } 4H \leq \frac{1}{2(c_\eta - \gamma p_s)} - \frac{1}{(w_o - \gamma p_s)} \\ & + \sqrt{\frac{1}{4(c_\eta - \gamma p_s)^2} - \frac{w_o - c_o}{(c_\eta - \gamma p_s)(w_o - \gamma p_s)^2}}, \\ w_o & \text{otherwise.} \end{cases}$$

*Case (iii). If  $4H \geq 1/\sqrt{(w_o - \gamma p_s)(s - \gamma p_s)} - 1/(w_o - \gamma p_s)$ ,*

$$w_\eta^* = \begin{cases} \hat{w} & \text{if } 4H \leq \frac{1}{c_\eta - s} \left( \sqrt{\frac{(s - \gamma p_s)}{w_o - \gamma p_s}} - \frac{(s - \gamma p_s)}{w_o - \gamma p_s} \right) \\ & \cdot \left( \sqrt{\frac{w_o - \gamma p_s}{(s - \gamma p_s)}} - \frac{w_o - c_o}{w_o - \gamma p_s} - 1 \right), \\ w_o & \text{otherwise.} \end{cases}$$

*For all cases,  $\tilde{w} \equiv 1/[4H + 1/(w_o - \gamma p_s)] + \gamma p_s$ , and  $\hat{w} \equiv s + (1 - \sqrt{(s - \gamma p_s)/(w_o - \gamma p_s)})^2/(4H)$ .*

If the value of  $H$  is relatively low, the supplier sets the optimal all-unit discount such that, in response, the reseller sets a low resale price to drive up demand and achieve the minimum batch threshold without diversion (see the first subcases in Cases (i) and (ii)). However, if the value of  $H$  is high enough, the high inventory cost may lead to an optimal supplier's discount policy under which the reseller simultaneously resorts to gray market diversion and a low resale price (see the first subcase in Case (iii)). Consistent with its exogenous resale price counterpart, Proposition 5 shows that gray market diversion occurs if and only if the economies of scale are sufficiently large and the reseller's inventory holding cost falls into an intermediate range of values.

**COROLLARY 2 (ENDOGENOUS RESALE PRICE: BENEFIT ALLOCATION).** *In the case of an endogenous resale price, benefits in the supplier's scale economy may be shared with the reseller. Consumers in the authorized channel enjoy a lower resale price with scale economy than without.*

Recall that when the resale price is exogenous, the supplier enjoys all benefits from economies of scale. In the case of an endogenous resale price, the supplier can benefit from additional authorized market demand, when the reseller orders in batches and lowers the resale price. In a standard sequential monopoly game with the supplier as the leader and the reseller as the follower who faces the downward-sloping demand function  $\lambda(p) = m/(p - \gamma p_s)^2$ , the supplier's optimal wholesale price with an effective supply cost  $c$  is  $(2c - \gamma p_s)$ . When the  $H$  is sufficiently low as in Case (i), charging the lower discount wholesale price at  $(2c_\eta - \gamma p_s)$  may provide greater net benefits to the supplier than charging a wholesale price that just offsets the reseller's increased inventory holding cost. This allows the reseller and consumers to enjoy part of the supplier's economies of scale. In Case (ii),  $(2c_\eta - \gamma p_s)$  is no longer a feasible discount price. In this case, when an optimal quantity discount is offered, it precisely compensates for the reseller's additional inventory holding costs. In Case (iii), when an optimal discount wholesale price is offered, it

precisely compensates for increased costs associated with a diversion strategy and an endogenously optimized resale price. In both Cases (ii) and (iii), the reseller shares no economic benefits resulting from batch orders.

## 5. Model Extension: Endogenous Gray Market Prices

So far we have assumed that gray market prices are insensitive to the reseller's diversion. In this section, we extend the main model and allow the gray market wholesale price (the price the reseller receives from diversion) and resale price (the price consumers pay for gray market products) to decrease with the amount of diversion from the reseller. Such a scenario becomes possible when the reseller's diversion accounts for a noticeable share of total gray market supply. We next show that our key results in the main model still hold with diversion-sensitive gray market prices. Specifically, in a general setting we identify the conditions under which (1) the diversion strategy is optimal for the reseller in the second stage and (2) in the first stage it is optimal for the supplier to induce a diversion strategy from the reseller.

We assume that gray market prices are general functions of the reseller's diversion rate, namely, the average diversion volume per unit of time. We assume the following regularity assumption.

**ASSUMPTION (R).** *The gray market wholesale price  $s(g)$  and gray market resale price  $p_s(g)$  are continuous and decreasing functions of the diversion rate  $g$ . To eliminate arbitrage, we assume  $s(0) \leq c_\eta$ .*

The diversion rate is determined by the volume of each diversion and the frequency of diversion. Following the same argument as Lemma 1, it is still optimal for the reseller to adopt a zero-inventory strategy; that is, if the reseller diverts, it is optimal to divert in the beginning of an order cycle. Hence, the reseller's strategy can be characterized by a pricing and zero-inventory policy pair  $(p, I)$ . For any strategy  $(p, I)$ , the gray market diversion rate  $g$  in equilibrium is implicitly given by

$$g = \frac{\eta - I}{I/\lambda(p, p_s(g))},$$

where  $\eta - I$  is the diversion amount in the beginning of every order cycle, and  $I/\lambda(p, p_s(g))$  is the cycle length.

**ASSUMPTION (G).** *For any reseller's strategy  $(p, I)$ , there exists a unique solution, denoted as  $g(p, I)$ , to the above equilibrium equation.*

Note that the reseller's diversion decision affects the gray market prices which will, in turn, affect the gray market demand and hence the authorized channel demand. The reseller needs to anticipate such chain reactions when making the diversion decision.

Like in the main model, the reseller has three possible strategies: the order-as-you-go, diversion, and batch strategies. If the reseller adopts a diversion strategy  $(p, I)$ , then the associated long-run average profit rate for the reseller can be written as

$$\pi_d(p, I; h, w_\eta) \equiv p\lambda(p, p_s(g(p, I))) - \left[ h\frac{I}{2} + \frac{w_\eta\eta - (\eta - I)s(g(p, I))}{I/\lambda(p, p_s(g(p, I)))} \right].$$

If the cycle inventory  $I$  is equal to the batch size  $\eta$  (i.e.,  $g = 0$ ), the profit rate  $\pi_d(p, I)$  boils down to that under the batch strategy with zero diversion. Hence, the definition of the profit rate  $\pi_d(p, I; h)$  for a diversion strategy includes the batch strategy as a degenerate case. Because the derivative of  $\pi_d(p, I; h)$  with respect to  $p$  and  $I$  decreases with  $h$ , the cycle inventory level  $I^*(h)$  in the optimal diversion strategy is a decreasing function of  $h$ . Intuitively, in one extreme where  $h = 0$ , because diversion is costly and inventory holding is free, we have  $I^*(h = 0) = \eta$ . In another extreme where  $h = \infty$ , we have  $I^*(h = \infty) = 0$  because inventory holding is extremely costly. We can thus conclude there must exist a *range* of reseller's inventory holding cost  $h$  within which the optimal inventory policy to maximize the profit rate  $\pi_d(p, I)$  is a diversion strategy with a *positive* amount of diversion. Under this range of inventory holding costs, the optimal diversion strategy (with  $g > 0$ ) is more profitable than the batch strategy (with  $g = 0$ ). Moreover, by comparing the profit rate under the optimal diversion strategy and that under the pay-as-you-go strategy, we can identify a *threshold* on the reseller's inventory holding cost  $h$  below which the optimal diversion strategy is more profitable. If the inventory holding cost  $h$  is in the nonempty intersection of these two ranges, the diversion strategy should be the reseller's optimal strategy.

**PROPOSITION 6 (ENDOGENOUS GRAY MARKET PRICES: RESELLER PROBLEM).** *Under Assumptions (R) and (G), given any supplier's discount wholesale price, if it is optimal for the reseller to adopt a diversion strategy, then the reseller's inventory holding cost must be in an intermediate range.*

Now we consider the supplier's problem in the first stage. To facilitate the analysis, we make two more assumptions.

**ASSUMPTION (CL).** *The cycle length  $I/\lambda(p, p_s(g(p, I)))$  is strictly increasing in  $I$ .*

**ASSUMPTION (P).** *The optimal resale price  $p^*(I) = \arg \max_p \pi_d(p, I)$  in a diversion strategy with a cycle inventory  $I$  is increasing in  $I$ .*

Assumptions (CL) and (P) includes exogenous gray market prices in the main model as a special case. Assumption (CL) requires that the common

operational relationship that cycle lengths are increasing in cycle inventory is not affected by pricing effects in the gray market. This assumption holds when the authorized channel demand is not too sensitive to the gray market resale price, or the gray market resale price is not too sensitive to the diversions. Under this assumption, the reseller's optimal diversion strategy pair  $(p^*(w_\eta), I^*(w_\eta))$  is increasing in  $w_\eta$ . Furthermore, under this assumption, we can show that the optimal supplier's discount price  $w_\eta^*(c_\eta)$ , given the reseller's best response with a diversion strategy, is increasing in the batch supply cost  $c_\eta$ . These monotone properties allow us to identify a *range* on the supplier's batch supply cost  $c_\eta$  where if the supplier offers an optimal discount wholesale price, the best response of the reseller is a diversion strategy.

Assumption (P) requires nothing but the following simple argument to hold for the gray market: More cycle inventory implies less diversion and thus higher gray market prices. Then, because of substitutability between the authorized and gray market channels, the reseller can charge a higher optimal resale price. Under Assumption (P), we can further identify a *threshold* on the supplier's batch supply cost  $c_\eta$  below which offering an optimal discount wholesale price  $w_\eta^*(c_\eta)$  is more profitable for the supplier than not offering any discount. If the batch supply cost  $c_\eta$  is in the nonempty intersection of those two ranges, it is optimal for the supplier to induce a diversion strategy from the reseller.

**PROPOSITION 7 (ENDOGENOUS GRAY MARKET PRICES: SUPPLIER PROBLEM).** *Under Assumptions (R), (G), (CL), and (P), for any holding cost in the range identified in Proposition 6 for some discount wholesale price, if it is optimal for the supplier to induce the reseller to adopt a diversion strategy, then the supplier's batch supply cost must be in an intermediate range.*

The above proposition indicates that for endogenous gray market prices, gray market diversion due to the incentive of batch ordering under an all-unit quantity discount is expected for a range of the supplier's batch supply cost and reseller's inventory holding cost. These ranges are more likely to be valid in industries where the authorized channel demand is not very sensitive to the gray market resale price because of high differentiation between the two channels (e.g., inadequate customer service and lack of warranty coverage for gray market products), or in industries where high search costs impede access to the gray market. These regions can also be more relevant when gray market prices are not very sensitive to diversions induced by all-unit discounts. This scenario happens when the gray market prices are primarily determined by other hard-to-prevent diversion behavior, induced by alternative incentive programs such as one-time promotions and marketing

development funds, and exacerbated by the lack of monitoring systems for incentive claims and serial number tracking.

## 6. Conclusions

This paper examines the impact of a domestic gray market on authorized channel members' decision making and the welfare of consumers served by the authorized channel. The gray market, unlike a separate salvage channel, acts as an *autonomous* channel that cannibalizes demand from the authorized reseller network. The multiple levels of interaction—diversion-induced emergence and the entailed cannibalization—between the authorized and secondary channels differentiate a gray market from a typically independent salvage channel. Compared to an independent salvage channel for moving excess inventories, the gray market is less favorable to the reseller because the reseller's diversion may lead to sizable cannibalization of its regular sales in authorized channel.

Our results yield several useful implications on how firms may manage their distribution channels with potential gray market leakage. First, in industries with sufficiently attractive gray market prices, only those resellers in the intermediate range of inventory holding costs may divert goods to the gray market. This may help managers to identify the resellers prone to gray market diversion. Managers can easily monitor the gray market wholesale price and examine its gap from the discounted price to assess the attractiveness of the gray market. Although our results cannot prescribe specific parameter ranges to predict a reseller's gray market activities, the managers could use the model parameters to estimate the gray market activities. One may expect a reseller's holding cost and market demand to depend on the reseller's geographical location. For instance, a downtown location may have a higher unit holding cost than a suburban location, and a young and educated city may have a higher consumption rate for IT products than a rural and less tech-savvy market.

Second, managers should be aware of the complex effect of gray market diversion on the authorized channel. Among resellers engaging in gray market activities, those with relatively smaller holding costs use gray market diversion to reduce inventory costs and increase the prices in the authorized channel. This would reduce sales in the authorized channel, which typically serves more valuable customers. The managers may consider treating them differently from those who benefit from diversion to alleviate high inventory holding costs and pass a part of savings to consumers in the authorized channel. Because the interests of resellers and their customers in the authorized channel may be at odds, the supplier should

take an integrated perspective in deciding the tolerance for gray markets.

This paper shifts from the usual focus on legal issues in gray markets and studies the strategic impact to the channel members. Gray markets have existed for a very long time and have gained momentum, growing in recent years through the online channel. The emergence of online markets makes it easier to establish independent resale operations and to reach a much larger geographic market. Because the gray market will remain for the foreseeable future, firms should fully consider its operational and marketing impacts on their authorized resellers. This paper demonstrates that simply looking at one aspect (e.g., the pricing decisions) but ignoring another (e.g., the inventory decisions) could lead to erroneous conclusions on gray markets.

Although our results apply quite generally, one should be aware that our core investigation is limited to the context of a chain of sequential monopolies with a domestic gray market, deterministic demand, and full information. Relaxing these restrictions can be important avenues for future research. First, adding uncertainty and asymmetric information between the supplier and its reseller may result in interactions with the bullwhip effect. Under full information for resellers with low holding costs, our model predicts increased order frequency due to reduced effective ordering costs. We thus surmise that in this case the bullwhip effect may be moderated. However, for resellers with high holding costs who shift from the pay-as-you-go strategy to the diversion strategy, the bullwhip effect can be amplified. Second, our model needs to be modified for international gray markets. Parallel imports to another country may cannibalize a foreign reseller without hurting the reseller responsible for the diversion. In this case, the diversion behavior may have a different impact on the supplier who sells to both countries. Finally, future research should incorporate the effects of competition. Competition can exist between the authorized resellers within a supplier's distribution network, and can also result from substitutable suppliers.

### Electronic Companion

An electronic companion to this paper is available as part of the online version at <http://dx.doi.org/10.1287/msom.1120.0416>.

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