Online Appendix to

# "Food Delivery Service and Restaurant: Friend or Foe?"

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## A. Tech-Savvy Customers Only

In this section, we temporarily lift the assumption of abundant traditional customers and investigate the case with no traditional customers, i.e., $\Lambda_0 = 0$. This scenario applies to new restaurants without established customer bases or when it becomes too risky to dine in restaurants during a pandemic. Everything else remains the same as our base model. Let $p^*_T$ and $\Pi^*_T$ denote the restaurant's revenue maximizing food price and maximum revenue, respectively, under demand rate $\Lambda_1$ and no food delivery service.

Proposition A.1 **(Equilibrium–Tech-Savvy Only)**. *Under the condition of no traditional customers, i.e., $\Lambda_0 = 0$, if the food delivery service is sufficiently inconvenient and the amount of tech-savvy customers is relatively high, the restaurant operates in a delivery-irrelevant regime. Otherwise, if the food delivery service is sufficiently convenient or if the food delivery service is not so convenient and the amount of tech-savvy customers is low, the restaurant operates in a delivery-only regime–it charges a higher food price compared to that under no food delivery service and all joining tech-savvy customers use food delivery service. Formally, there exist $\phi_1$, $\phi_2$, and $\lambda_3$, such that: If (i) $\phi \le \phi_1$; or (ii) $\phi_1 < \phi \le \phi_2$ and $\Lambda_1 \le \lambda_3$; or (iii) $\phi > \phi_2$ and $\Lambda_1 \le (c-\phi)\,\mu/c$, we have $p^* \ge p^*_T$ and $\Pi^* \ge \Pi^*_T$. Otherwise, if (i) $\phi_1 < \phi \le \phi_2$ and $\Lambda_1 > \lambda_3$; or (ii) $\phi > \phi_2$ and $\Lambda_1 > (c-\phi)\,\mu/c$, we have $p^* = p^*_T$ and $\Pi^* = \Pi^*_T$. Moreover, in all cases, the restaurant's profit $\Pi^*$, platform's profit $\pi^*$, social welfare $S^*$, and the restaurant's demand $\lambda^*_D + \lambda^*_W$ are weakly increasing in $\Lambda_1$ for $\forall \phi$.*

To illustrate the results of Proposition A.1, similar to Figures 1, 2, and 3, we plot the related performance measures as a function of tech-savvy customers' arrival rate $\Lambda_1$, for $\phi = 0.5$, 0.36 and 0.3 in Figures A.1, A.2, and A.3, respectively. Same as Figures 1, 2, and 3, we have $\phi_1 = 0.3554$ in Figures A.1, A.2, and A.3.

When the arrival rate of tech-savvy customers is low (see, e.g., $\Lambda_1 \le 0.4965$ in Figures A.1, A.2, and A.3), the restaurant chooses to set a high food price to grab more residual surplus from food delivery customers, because a low food price cannot attract more customers, due to the absence of traditional customers. In other words, the restaurant choose the delivery-only regime over the delivery-irrelevant regime. On the other hand, when the arrival rate of tech-savvy customers is high, the restaurant will stay in the delivery-only regime only if the food-delivery service is sufficiently convenient (see, e.g., $\phi \le \phi_1$ and $\Lambda_1 > 0.6583$ in Figure A.3); and it will switch to the delivery-irrelevant regime if the food-delivery service is not so convenient (see, e.g., $\phi > \phi_1$ and $\Lambda_1 > 0.6583$

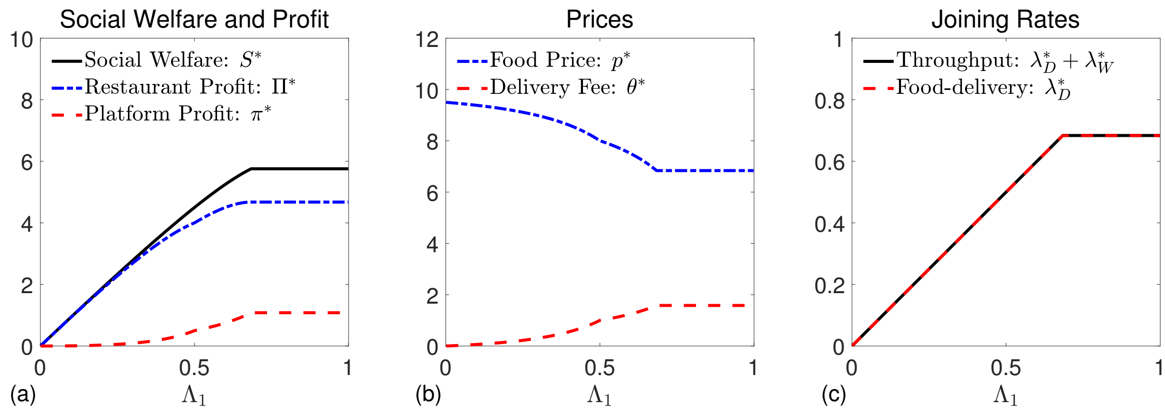


**Figure A.1** **Equilibrium system behavior as a function of $\Lambda_1$ for $\Lambda_0 = 0$, $R = 10$, $\mu = c = 1$, and $\phi = 0.5$.**

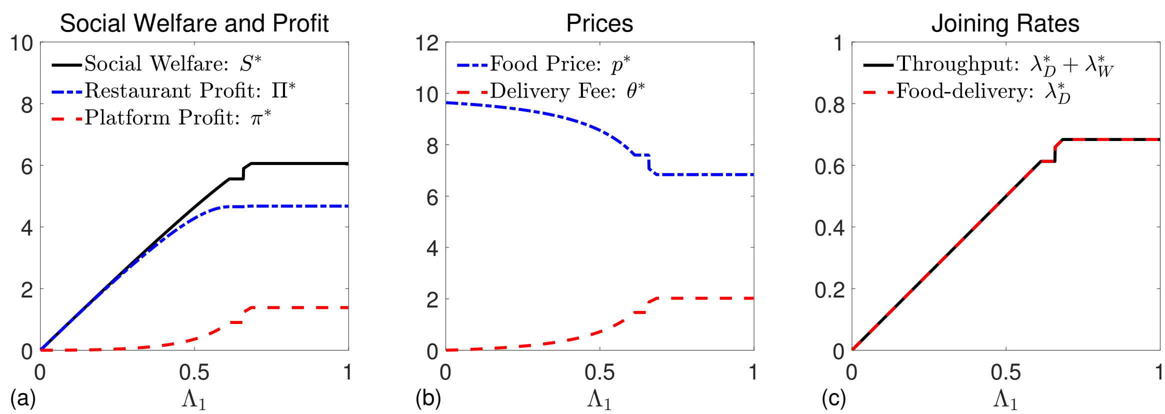


**Figure A.2** **Equilibrium system behavior as a function of $\Lambda_1$ for $\Lambda_0 = 0$, $R = 10$, $\mu = c = 1$, and $\phi = 0.36$.**

in Figure A.2). The same intuition applies. When the food-delivery service is sufficiently convenient, the restaurant can extract more surplus from customers by charging a high food price, which discourages some customers from joining and results in lower throughput; but when the food-delivery service is not convenient enough, the restaurant can gain more profit from a low food price which leads to a higher throughput.

In all cases, the throughput is increasing in the amount of tech-savvy customers; see, e.g., Figures A.1(c), A.2(c), and A.3(c). Naturally, the food delivery platform that connects the restaurant to the growing pool of tech-savvy customers will bring in more demand for the restaurant.

We also consider the profit maximization problem of a centralized owner of the food service chain with control of the food price and delivery fee. Under the condition of no traditional customers, i.e., $\Lambda_0 = 0$, we obtain similar results as those under the condition of abundant traditional customers, i.e., $\Lambda_0 \geq \mu$.

Lemma A.1 **(Optimal Monopoly Prices—Tech-Savvy Only)**. *Suppose there are no traditional customers, i.e., $\Lambda_0 = 0$. If the restaurant sets the optimal monopoly food price $p^o$, the platform's best response will be to charge the optimal monopoly delivery fee $\theta^o$. (The expressions*

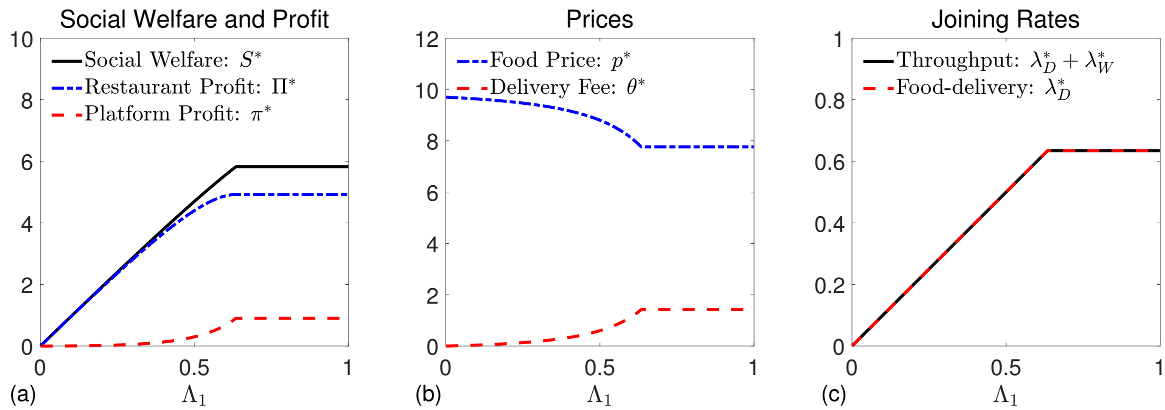


**Figure A.3** **Equilibrium system behavior as a function of $\Lambda_1$ for $\Lambda_0 = 0$, $R = 10$, $\mu = c = 1$, and $\phi = 0.3$.**

*of $p^o$ and $\theta^o$ are given by (D.13) and (D.14).) Moreover, the optimal monopoly food price $p^o$, the total price $p^o + \theta^o$, and the corresponding restaurant profit $\Pi^o$ are weakly decreasing in $\Lambda_1$. The optimal monopoly delivery fee $\theta^o$, the corresponding throughput, delivery platform's profit $\pi^o$, and social welfare $S^o$ are weakly increasing in $\Lambda_1$.*

Proposition A.2 **(Revenue-Sharing Contract—Tech-Savvy Only)**. *Suppose there are no traditional customers, i.e., $\Lambda_0 = 0$. The following RS contracts can coordinate the system and achieve the maximum aggregated profit.*

1. *One-way RS contract with a price ceiling—The platform allocates a fraction $\gamma_1$ of its revenue to the restaurant while the restaurant cannot set a food price higher than $p^o$.*
2. *Two-way RS contract—Both the restaurant and the platform agree that a fraction $\gamma_2$ of their aggregated revenue be allocated to the restaurant.*

*There always exist a range of sharing fractions that make both parties weakly better off than they would be without any contract. In particular, the sharing fractions in (D.15) (resp., in (D.16)) of the Online Appendix B achieve a win-win for both parties under the one-way RS contract with a price ceiling (resp., two-way RS contract).*

## B. Numerical Results - Finite Delivery-Worker Pool

Similar to what we did in our base model, we can apply backward induction to derive the platform's and the restaurant's equilibrium behavior in this Stackelberg game. First, the food delivery platform's optimal delivery fee $\theta^*$ and delivery wage $w^*$ can be derived as the best response to the restaurant's food price $p$. Second, the restaurant's optimal food price $p^*$ can be found by numerical methods. Then, we can derive the desired performance measures including the joining rates of food-delivery and walk-in customers, the profit levels of the restaurant and the platform, the delivery workers' total utility, and social welfare.

### B.1. Uniformly Distributed Opportunity Cost

Next, we study the impact of the delivery-worker pool size $N$ on social welfare when there is sufficient demand from tech-savvy customers. One might expect that when the delivery-worker pool gets larger, it becomes socially cheaper to offer the food delivery service, so social welfare would increase with $N$. However, Figure B.4, where we display social welfare of the decentralized system as a function of $N$ under an uniformly distributed opportunity cost, tells a different story. For $\phi_1 > \phi = 0.2$ or $0.3$, social welfare of the decentralized system may decrease sharply when $N$ increases to a threshold value $\bar{N}$. When the platform's pool of delivery workers is not large, i.e., $N \leq \bar{N}$, as confirmed by Proposition 3, the restaurant overlooks the introduction of the food delivery service and uses the same food price as in an unobservable queue with only traditional customers. Indeed, in this case, social welfare increases if the pool of delivery workers gets larger. However, when the pool is sufficiently large, i.e., $N > \bar{N}$, the restaurant will become a delivery-only kitchen and cater entirely to food-delivery customers out of its own desire for more profit. As a result, social welfare drops dramatically.

Furthermore, combining observations from Proposition 3 and Figure B.4, we note that it may be socially beneficial to limit the delivery-worker pool size. The social planner can prevent the dramatic decline of social welfare by capping the number of delivery workers at $\bar{N}$. Depending on the value of $\bar{N}$, the social planner may set a *temporary* or a *permanent* cap on the pool size. For example, in Figure B.4(b), the value of $\bar{N} = 32.41$ is relatively large; social welfare increases to above that of the decentralized system with an infinite number of delivery workers, and then declines. After the decline at $\bar{N}$, social welfare of the decentralized system stays below that of the decentralized system with an infinite number of delivery workers when the pool size grows. In this case, the social planner prefers a permanent cap on the delivery-worker pool size at $\bar{N}$ so that social welfare stays at the maximum. In Figure B.4(a), the value of $\bar{N} = 11.11$ is relatively small; social welfare declines before increasing to almost that of the decentralized system with an infinite number of delivery workers. After the drop at $\bar{N}$, social welfare of the decentralized system may return to the same level when the delivery-worker pool size grows. In this case, the social planner can use a temporary cap, which can be lifted when the potential number of delivery workers becomes sufficiently large, i.e., when $N$ increases to 27.60.

### B.2. Beta Distributed Opportunity Cost

In this subsection, we relax the assumption of the delivery workers' uniformly distributed opportunity cost (per unit of time) made in Subsection 6 and show that our insights hold for other opportunity cost distributions. Specifically, we consider the situation where the delivery workers' opportunity cost per unit of time $\sigma$ follows a beta distribution with cumulative distribution function $F^{Beta}(\cdot)$ and probability density function $f^{Beta}(\cdot)$. The beta distribution $Beta\,(a, b)$ is a family

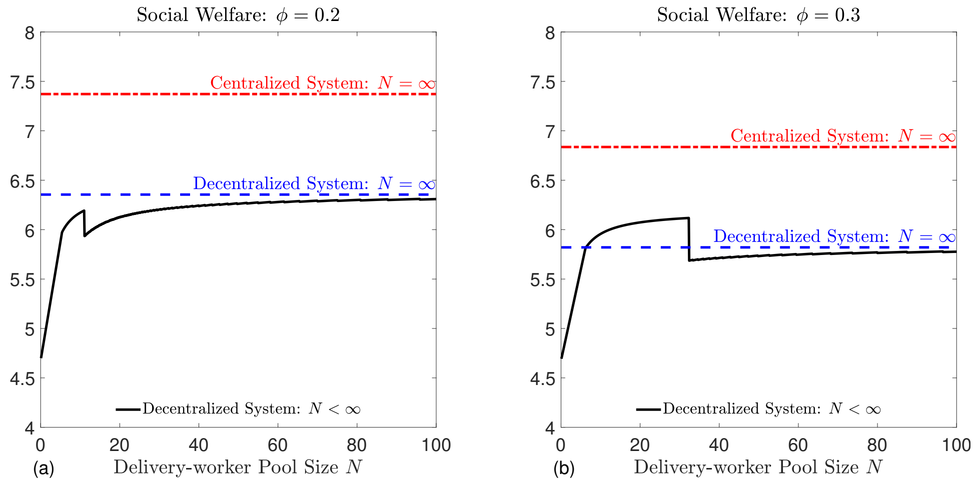


**Figure B.4** **Social welfare of the decentralized system with $N < \infty$, as function of $N$ for the parameter setting $R = 10$, $\Lambda_1 = \Lambda_0 = \mu = c = 1$, and $\phi \in \{0.2, 0.3\}$, where $\phi_1 = 0.3554$.**

of continuous probability distributions defined on the interval $[0,1]$, and can approximate various kinds of opportunity cost distributions. For example, its probability density function $f^{Beta}(\cdot)$ appears as a decreasing function for $(a,b) = (1,3)$, a bell-shaped function for $(a,b) = (2,2)$, and an increasing function for $(a,b) = (5,1)$. We only need to scale the delivery wage $w$ to the interval $[0,1]$ to obtain the supply of delivery workers under wage $w \in [0,\beta]$:

$$\nu(w) = N \cdot F^{Beta}\left(\frac{w}{\beta}\right).$$

Then the platform's profit is

$$\begin{aligned} \pi(p,\theta,w) &= \theta \cdot \min(\lambda_D, \nu(w)) - w \cdot \nu(w) \\ &= \theta \cdot \min\left(\lambda_D, NF^{Beta}\left(\frac{w}{\beta}\right)\right) - wNF^{Beta}\left(\frac{w}{\beta}\right), \end{aligned}$$

where $\lambda_D$ is the tech-savvy customers' unconstrained demand for the food delivery service given by Lemma C.2, and the delivery workers' total utility is

$$\begin{aligned} u_D(\theta,w) &= w \cdot \nu(w) - N\int_0^{\frac{w}{\beta}} x f^{Beta}\left(\frac{x}{\beta}\right) \mathrm{d}x \\ &= wNF^{Beta}\left(\frac{w}{\beta}\right) - N\int_0^{\frac{w}{\beta}} x f^{Beta}\left(\frac{x}{\beta}\right) \mathrm{d}x. \end{aligned}$$

As in Figures 2 and 1, we display these measures as a function of the arrival rate of tech-savvy customers $\Lambda_1$, for $Beta(1,3)$ in Figures B.5 and B.6, for $Beta(2,2)$ in Figures B.7 and B.8, and for $Beta(5,1)$ in Figures B.9 and B.10.

By pairwise comparisons of these figures, we confirm our insight that a limited number of delivery workers can curb the restaurant's self-interested desire to raise the food price in order to extract more surplus from the food delivery service, which hurts the platform and social welfare. Capping the delivery-worker pool size can be an effective means of bringing the platform a higher profit and sustaining higher social welfare for the society.

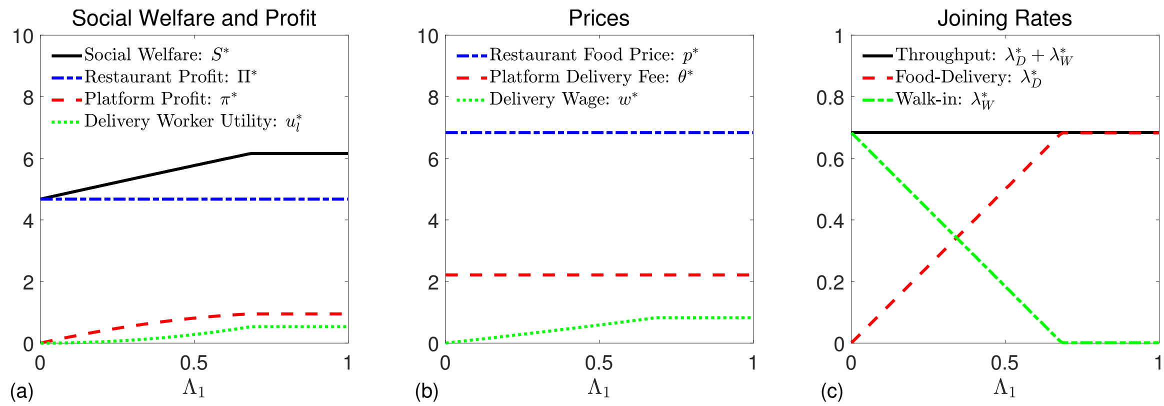


**Figure B.5** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(1,3)$ opportunity cost distribution, for the parameter setting $R = \beta = 10$, $\Lambda = \mu = c = 1$, $\phi = 0.3$, and $N = 3$.**

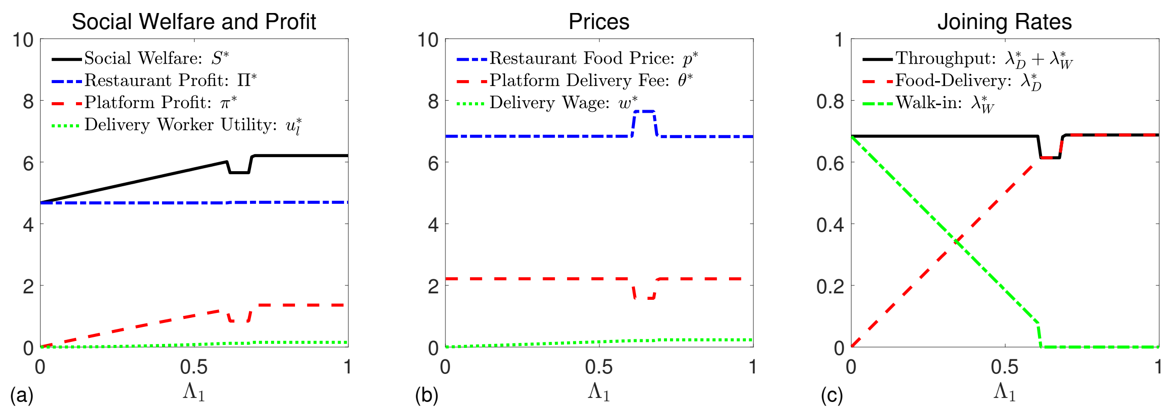


**Figure B.6** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(1,3)$ opportunity cost distribution, for the parameter setting $R = \beta = 10$, $\Lambda = \mu = c = 1$, $\phi = 0.3$, and $N = 10$.**

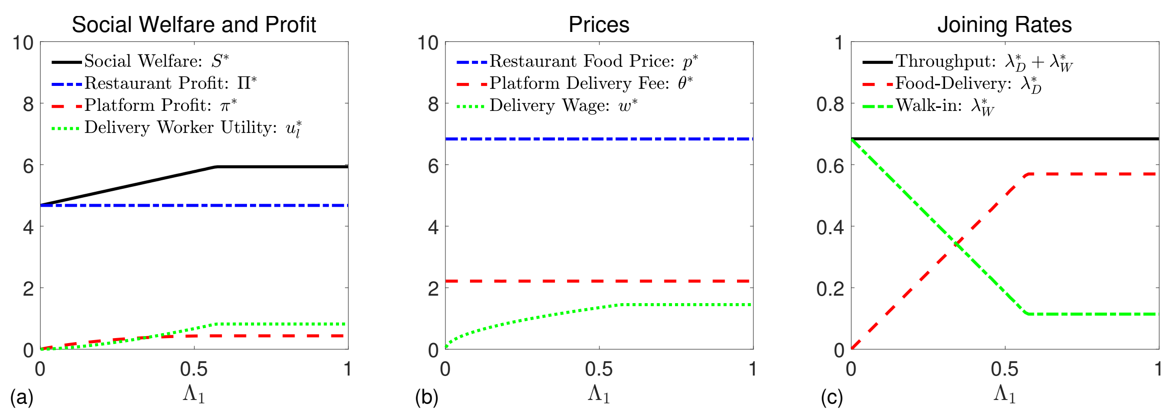


**Figure B.7** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(2,2)$ opportunity cost distribution, for the parameter setting $R = \beta = 10$, $\Lambda = \mu = c = 1$, $\phi = 0.3$, and $N = 10$.**

## C. Proofs of the Main Results

We first label, under the food price $p$ and the delivery fee $\theta$, the expected utility of (tech-savvy or traditional) walk-in customers,

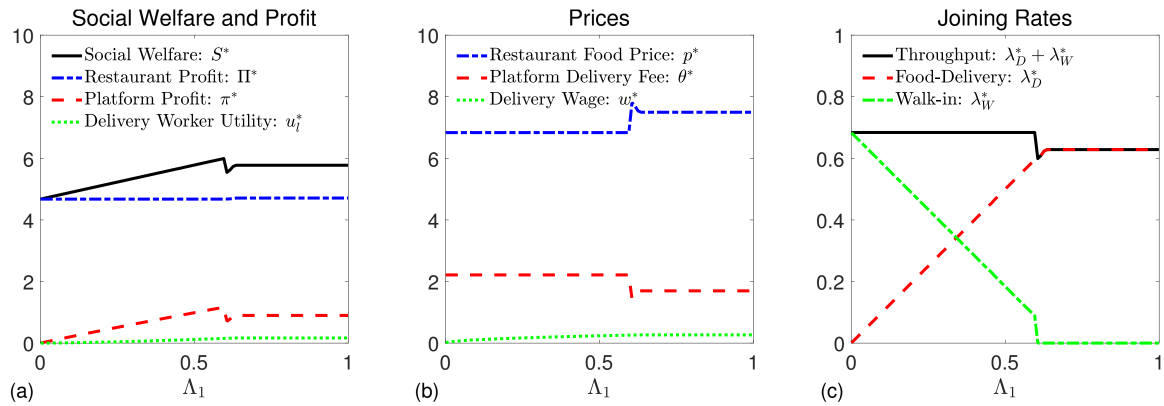


**Figure B.8** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(2,2)$ opportunity cost distribution, for the parameter setting $R=\beta=10$, $\Lambda=\mu=c=1$, $\phi=0.3$, and $N=300$.**

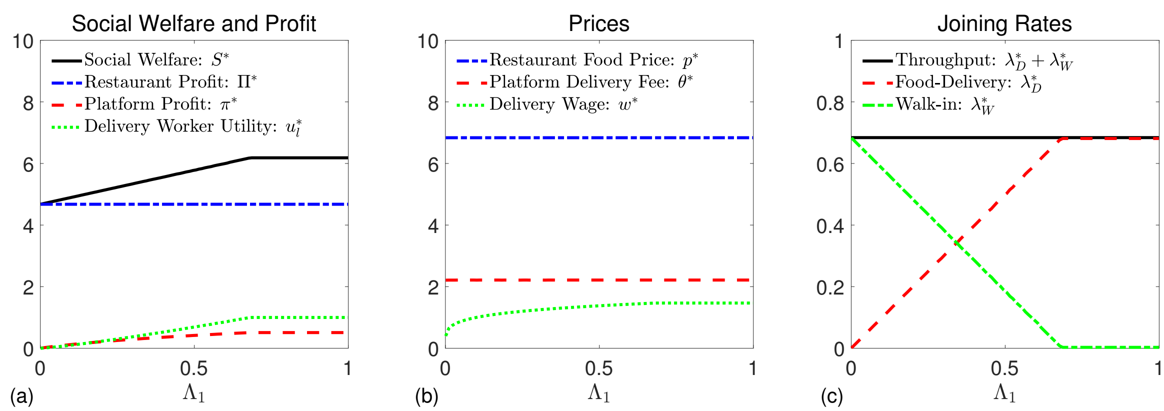


**Figure B.9** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(5,1)$ opportunity cost distribution, for the parameter setting $R=\beta=10$, $\Lambda=\mu=c=1$, $\phi=0.3$, and $N=10^4$.**

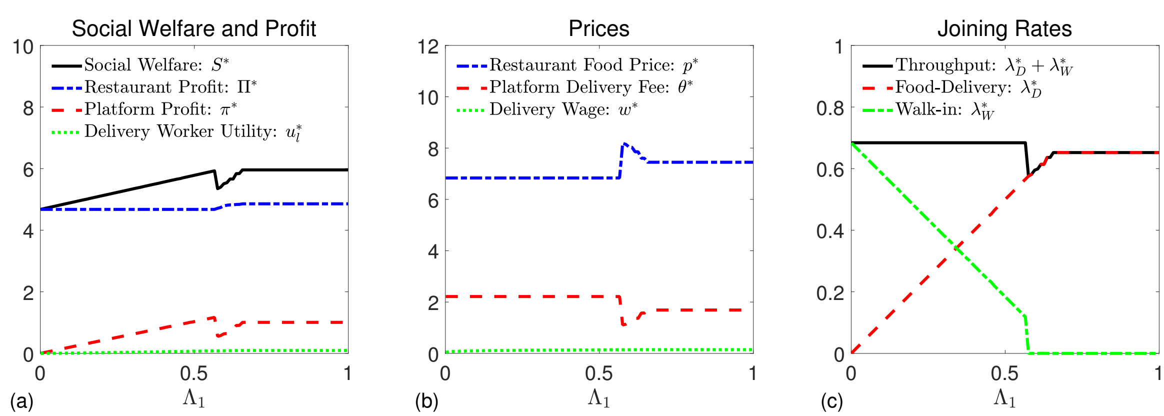


**Figure B.10** **Equilibrium system behavior as a function of $\Lambda_1$ under $Beta\,(5,1)$ opportunity cost distribution, for the parameter setting $R=\beta=10$, $\Lambda=\mu=c=1$, $\phi=0.3$, and $N=10^9$.**

$$U_W(\lambda) = R - p - cW(\lambda) = R - p - c/(\mu-\lambda), \tag{C.1}$$

and the expected utility of food-delivery customers,

$$U_D(\lambda) = R - p - \theta - \phi W(\lambda) = R - p - \theta - \phi/(\mu-\lambda), \tag{C.2}$$

for easy reference in the Online Appendix.

We make the following observations regarding the range of the service reward $R$, the food price $p$, the delivery fee $\theta$, and the arrival rate of tech-savvy customers $\Lambda_1$.

- If $R - \frac{c}{\mu} \geq 0$, the service reward can cover the waiting cost of walk-in customers when there is no line on arrival and the food is free.
- If the food price $p$ is upper bounded by $R - \frac{\phi}{\mu}$, i.e., $p \in \left[0, R - \frac{\phi}{\mu}\right]$, customers are willing to use a food delivery service if the queue is empty and the food delivery service is free.
- If the delivery fee $\theta$ is upper bounded by $R - p - \frac{\phi}{\mu}$, i.e., $\theta \in \left[0, R - p - \frac{\phi}{\mu}\right]$, the service reward is sufficient to offset the sum of the restaurant price $p$, the delivery fee $\theta$, and the waiting cost for a customer's own food preparation $\frac{\phi}{\mu}$; otherwise, no customers will consider using the food delivery service.
- Because the restaurant cannot serve more customers than its capacity $\mu$, the equilibrium behavior in the $\Lambda_1 > \mu$ case is identical to that in the $\Lambda_1 \nearrow \mu$ case. Thus, we focus on the condition of $\Lambda_1 < \mu$ in our analysis.

## C.1. Proof of Proposition 1

To prove Proposition 1, we prove the following Lemmas and Corollaries.

### C.1.1. Customer Strategy

Lemma C.2 **(Customer Strategy)**. *Under the food price $p$ and delivery fee $\theta$, the joining rates of food-delivery and walk-in customers, $\lambda_D$ and $\lambda_W$, are*

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| *When* $p \leq R - \frac{c}{\mu-\Lambda_1}$ | | |
| *if* $0 < \theta \leq \frac{(c-\phi)(R-p)}{c}$ | $\Lambda_1$ | $\mu - \frac{c}{R-p} - \Lambda_1$ |
| *if* $\frac{(c-\phi)(R-p)}{c} < \theta \leq R - p - \frac{\phi}{\mu}$ | $0$ | $\mu - \frac{c}{R-p}$ |
| *When* $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{\phi}{\mu}$ | | |
| *if* $0 < \theta \leq \max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right)$ | $\Lambda_1$ | $0$ |
| *if* $\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \leq \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$ | $\mu - \frac{\phi}{R-p-\theta}$ | $0$ |
| *if* $\theta > \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$ | $0$ | $\mu - \frac{c}{R-p}$ |

To prove Lemma C.2, we first summarize some properties of the two utility functions $U_W(\lambda)$ and $U_D(\lambda)$.

Lemma C.3. *Regarding utility functions $U_W(\lambda) = R - p - \frac{c}{\mu-\lambda}$ and $U_D(\lambda) = R - p - \theta - \frac{\phi}{\mu-\lambda}$, we have*

1. *$U_W(\lambda)$ is a concave decreasing function of $\lambda$ intersecting the $\lambda$-axis at $\lambda_W^X = \mu - \frac{c}{R-p}$, which decreases in $p$.*

   *1.1 When $p = R - \frac{c}{\mu-\Lambda_1}$, $U_W(\lambda)$ intersects the $\lambda$-axis at $\lambda = \Lambda_1$, i.e., $\lambda_W^X = \Lambda_1$.*

2. *$U_D(\lambda)$ is a concave decreasing function of $\lambda$ intersecting the $\lambda$-axis at $\lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$, which decreases in $\theta$.*

   2.1 *When $\theta = 0$, $U_D(\lambda)$ intersects the $\lambda$-axis at $\lambda_D^X(0) = \mu - \frac{\phi}{R-p}$. When $\theta = R - p - \frac{\phi}{\mu}$, $U_D(\lambda)$ intersects the $\lambda$-axis at $\lambda_D^X\left(R - p - \frac{\phi}{\mu}\right) = 0$.*

   2.2 *When $\theta = R - p - \frac{\phi}{\mu-\Lambda_1}$, $U_D(\lambda)$ intersects the $\lambda$-axis at $\lambda = \Lambda_1$, i.e., $\lambda_D^X\left(R - p - \frac{\phi}{\mu-\Lambda_1}\right) = \Lambda_1$. We have $\Lambda_1 \le \lambda_D^X(\theta)$ if $0 < \theta \le R - p - \frac{\phi}{\mu-\Lambda_1}$, and $\Lambda_1 > \lambda_D^X(\theta)$ if $\theta > R - p - \frac{\phi}{\mu-\Lambda_1}$.*

   2.3 *Furthermore, when $p = R - \frac{\phi}{\mu-\Lambda_1}$ and $\theta = 0$, we have $U_D(\lambda)$ intersecting the $\lambda$-axis at $\lambda_D^X(0) = \Lambda_1$.*

3. *$U_W(\lambda)$ and $U_D(\lambda)$ only intersect once at $\lambda^X(\theta) = \mu - \frac{c-\phi}{\theta}$, which increases in $\theta$. Moreover, we have $U_W(\lambda) \ge U_D(\lambda)$ if $\lambda \le \lambda^X(\theta)$, and $U_W(\lambda) < U_D(\lambda)$ if $\lambda > \lambda^X(\theta)$.*

   3.1 *When $\theta = \frac{c-\phi}{\mu-\Lambda}$, $U_W(\lambda)$ and $U_D(\lambda)$ intersect at $\lambda = \Lambda$, i.e., $\lambda^X\left(\frac{c-\phi}{\mu-\Lambda}\right) = \Lambda$. Moreover, we have $U_D(\Lambda) \ge U_W(\Lambda)$ if $\theta \le \frac{c-\phi}{\mu-\Lambda}$; and $U_D(\Lambda) < U_W(\Lambda)$ if $\theta > \frac{c-\phi}{\mu-\Lambda}$.*

   3.2 *When $\theta = \frac{(c-\phi)(R-p)}{c}$, we have $\lambda_W^X = \lambda_D^X(\theta) = \lambda^X(\theta)$. Moreover, we have $\frac{(c-\phi)(R-p)}{c} \le R - p - \frac{\phi}{\mu}$.*

   3.3 *Furthermore, we have*

$$\begin{cases} U_W(\lambda) \text{ and } U_D(\lambda) \text{ have no intersection point, and } \lambda_W^X < \lambda_D^X(\theta) & \text{if } \theta = 0 \\ \lambda^X(\theta) < \lambda_W^X < \lambda_D^X(\theta) & \text{if } 0 < \theta < \frac{(c-\phi)(R-p)}{c} \\ \lambda^X(\theta) = \lambda_W^X = \lambda_D^X(\theta) & \text{if } \theta = \frac{(c-\phi)(R-p)}{c} \\ \lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta) & \text{if } \frac{(c-\phi)(R-p)}{c} < \theta \le R - p - \frac{\phi}{\mu} \end{cases}$$

*Proof of Lemma C.3.* 1. From $\frac{\partial U_W(\lambda)}{\partial \lambda} = -\frac{c}{(\mu-\lambda)^2} < 0$ and $\frac{\partial^2 U_W(\lambda)}{\partial \lambda^2} = -\frac{2c}{(\mu-\lambda)^3} < 0$, we have that $U_W(\lambda)$ is a concave decreasing function of $\lambda$. By solving $U_W(\lambda) = 0$, we have that $U_W(\lambda)$ intersects the $\lambda$-axis at $\lambda_W^X = \mu - \frac{c}{R-p}$, which decreases in $p$.

1.1 When $p = R - \frac{c}{\mu-\Lambda_1}$, we have $U_W(\lambda) = \frac{c(\lambda-\Lambda_1)}{(\lambda-\mu)(\mu-\Lambda_1)}$, which intersects the $\lambda$-axis at $\lambda = \Lambda_1$.

2. From $\frac{\partial U_D(\lambda)}{\partial \lambda} = -\frac{\phi}{(\mu-\lambda)^2} < 0$ and $\frac{\partial^2 U_D(\lambda)}{\partial \lambda^2} = -2\frac{\phi}{(\mu-\lambda)^3} < 0$, we have that $U_D(\lambda)$ is a concave decreasing function of $\lambda$. By solving $U_D(\lambda) = 0$, we have that $U_D(\lambda)$ intersects the $\lambda$-axis at $\lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$, which decreases in $\theta$.

   2.1 When $\theta = 0$, we have $U_D(\lambda) = R - p - \frac{\phi}{\mu-\lambda}$, which intersects the $\lambda$-axis at $\lambda_D^X(0) = \mu - \frac{\phi}{R-p}$. When $\theta = R - p - \frac{\phi}{\mu}$, we have $U_D(\lambda) = \frac{\phi\lambda}{\mu(\lambda-\mu)}$, which intersects the $\lambda$-axis at $\lambda_D^X\left(R - p - \frac{\phi}{\mu}\right) = 0$.

   2.2 When $\theta = R - p - \frac{\phi}{\mu-\Lambda_1}$, we have $U_D(\lambda) = \frac{\phi(\lambda-\Lambda_1)}{(\lambda-\mu)(\mu-\Lambda_1)}$, which intersects the $\lambda$-axis at $\lambda = \Lambda_1$. Simple algebra gives $\Lambda_1 \le \lambda_D^X(\theta) \Leftrightarrow 0 < \theta \le R - p - \frac{\phi}{\mu-\Lambda_1}$.

   2.3 When $p = R - \frac{\phi}{\mu-\Lambda_1}$ and $\theta = 0$, we have $U_D(\lambda) = \frac{\phi(\lambda-\Lambda_1)}{(\lambda-\mu)(\mu-\Lambda_1)}$, which intersects the $\lambda$-axis at $\lambda_D^X(0) = \Lambda_1$.

3. By solving $U_W(\lambda) = U_D(\lambda)$, we have $\lambda^X(\theta) = \mu - \frac{c-\phi}{\theta}$, which increases in $\theta$. We can also verify that $U_W(\lambda) \ge U_D(\lambda) \Leftrightarrow \lambda \le \lambda^X(\theta)$.

3.1 When $\theta = \frac{c-\phi}{\mu-\Lambda}$, we have $U_D(\lambda) = R - p - \frac{c-\phi}{\mu-\Lambda} - \frac{\phi}{\mu-\lambda}$. Solving $U_D(\lambda) = U_W(\lambda) = R - p - \frac{c}{\mu-\lambda}$ gives $\lambda^X\left(\frac{c-\phi}{\mu-\Lambda}\right) = \Lambda$. We can also verify that $U_D(\Lambda) \geq U_W(\Lambda) \Leftrightarrow \theta \leq \frac{c-\phi}{\mu-\Lambda}$.

3.2 When $\theta = \frac{(c-\phi)(R-p)}{c}$, we have $U_D(\lambda) = \frac{\phi(R-p)}{c} - \frac{\phi}{\mu-\lambda}$ and $\lambda_D^X\left(\frac{(c-\phi)(R-p)}{c}\right) = \mu - \frac{c}{R-p}$. Solving $U_D(\lambda) = U_W(\lambda) = R - p - \frac{c}{\mu-\lambda}$ gives $\lambda^X(\theta) = \mu - \frac{c}{R-p}$. Clearly, we have $\lambda_W^X = \lambda_D^X(\theta) = \lambda^X(\theta)$. Furthermore, we have $\frac{(c-\phi)(R-p)}{c} \leq R - p - \frac{\phi}{\mu} \Leftrightarrow p \leq R - \frac{c}{\mu}$, which is clearly true.

3.3 When $\theta = 0$, we can also verify that $U_W(\lambda)$ and $U_D(\lambda)$ have no intersection point and $\lambda_W^X = \mu - \frac{c}{R-p} < \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$. When $0 < \theta < \frac{(c-\phi)(R-p)}{c}$, we have $\lambda^X(\theta) < \lambda_W^X \Leftrightarrow \mu - \frac{c-\phi}{\theta} < \mu - \frac{c}{R-p} \Leftrightarrow \theta < \frac{(c-\phi)(R-p)}{c}$ and $\lambda_W^X < \lambda_D^X(\theta) \Leftrightarrow \mu - \frac{c}{R-p} < \mu - \frac{\phi}{R-p-\theta} \Leftrightarrow \theta < \frac{(c-\phi)(R-p)}{c}$. Thus, we have $\lambda^X(\theta) < \lambda_W^X < \lambda_D^X(\theta)$. Similarly, we can show that when $\frac{(c-\phi)(R-p)}{c} < \theta \leq R - p - \frac{\phi}{\mu}$, we have $\lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta)$. □

The relationship between $R - \frac{c}{\mu-\Lambda_1}$, $R - \frac{\phi}{\mu-\Lambda_1}$, $R - \frac{c}{\mu}$, and $R - \frac{\phi}{\mu}$ plays an important role in characterizing the customers' equilibrium behavior.

Lemma C.4. *If $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, we have $\left(R - \frac{c^2}{\mu\phi} \leq\right) R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu} \leq R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{\phi}{\mu}$. If $\Lambda_1 > \frac{c-\phi}{c}\mu$, we have $R - \frac{c}{\mu-\Lambda_1} < \left(R - \frac{c^2}{\mu\phi} <\right) R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{c}{\mu} < R - \frac{\phi}{\mu}$.*

*Proof of Lemma C.4.* First, because $\phi < c$, we have $R - \frac{c}{\mu} < R - \frac{\phi}{\mu}$, $R - \frac{c}{\mu-\Lambda_1} < R - \frac{\phi}{\mu-\Lambda_1}$ and $R - \frac{c^2}{\mu\phi} < R - \frac{c}{\mu}$. Moreover, from $\mu - \Lambda_1 < \mu$, we have $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu}$ and $R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{\phi}{\mu}$. Thus, we only need to determine the relationship between (i) $R - \frac{\phi}{\mu-\Lambda_1}$ and $R - \frac{c}{\mu}$, and (ii) $R - \frac{c}{\mu-\Lambda_1}$ and $R - \frac{c^2}{\mu\phi}$.

Next we can derive

$$R - \frac{c}{\mu-\Lambda_1} - \left(R - \frac{c^2}{\mu\phi}\right) = \frac{c}{\phi}\frac{c}{\mu(\mu-\Lambda_1)}\left(\frac{c-\phi}{c}\mu - \Lambda_1\right),$$

and

$$R - \frac{\phi}{\mu-\Lambda_1} - \left(R - \frac{c}{\mu}\right) = \frac{c}{\mu(\mu-\Lambda_1)}\left(\frac{c-\phi}{c}\mu - \Lambda_1\right).$$

Clearly, if $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, we have $R - \frac{c^2}{\mu\phi} \leq R - \frac{c}{\mu-\Lambda_1}$ and $R - \frac{c}{\mu} \leq R - \frac{\phi}{\mu-\Lambda_1}$. If $\Lambda_1 > \frac{c-\phi}{c}\mu$, we have $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c^2}{\mu\phi}$ and $R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{c}{\mu}$. When we combine the above results, the proof is complete. □

Next, we prove Lemma C.2. The relative position of the total arrival rate $\Lambda$ compared to $\lambda_W^X$, $\lambda_D^X(\theta)$, and $\lambda^X(\theta)$ decides customers' equilibrium behavior.

Case 1: When $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, we have $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu} \leq R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{\phi}{\mu}$ by Lemma C.4.

1. If $p \leq R - \frac{c}{\mu-\Lambda_1}$, we have $\Lambda_1 \leq \lambda_W^X = \mu - \frac{c}{R-p}$. From Lemma C.3(1), when all customers join with rate $\Lambda \geq \mu > \lambda_W^X = \mu - \frac{c}{R-p}$, the utility of walk-in is negative, i.e., $U_W(\Lambda) < 0$.

1.1 If $\theta \leq \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(3.3), we have $\lambda_W^X \leq \lambda_D^X(\theta)$. Since $\Lambda_1 \leq \lambda_W^X \leq \lambda_D^X(\theta)$, from Lemma C.3(1), we have $U_W(\Lambda_1) \geq 0$ and $U_W(\lambda_W^X) = 0$; and from Lemma C.3(2), we have

$U_D(\Lambda_1) \geq 0$ and $U_D(\lambda_W^X) \geq 0$. This means that if all tech-savvy customers join (with rate $\Lambda_1$), walk-in customers have an incentive to join and obtain positive utility, until the total arrival rate to the system reaches $\lambda_W^X = \mu - \frac{c}{R-p}$, unless the tech-savvy customers' arrival rate reaches $\Lambda_1 = \lambda_W^X = \mu - \frac{c}{R-p}$ already. In this case (i) all tech-savvy customers will join and use the food delivery service, i.e., $\lambda_D = \Lambda_1$; (ii) walk-in customers will join with rate $\lambda_W = \mu - \frac{c}{R-p} - \Lambda_1$.

1.2 If $\theta > \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(3.3), we have $\lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta)$. Since $\Lambda_1 \leq \lambda_W^X < \lambda^X(\theta)$, from Lemma C.3(1) and (3), we have $U_D(\Lambda_1) < U_W(\Lambda_1)$ and $U_W(\Lambda_1) \geq 0$. This means that if only tech-savvy customers join, the food delivery service cannot attract customers; all $\Lambda_1$ tech-savvy customers will queue themselves and their utility of walk-in is $U_W(\Lambda_1) \geq 0$. Then walk-in customers may have an incentive to join and obtain positive utility until the total arrival rate to the system reaches $\lambda_W^X = \mu - \frac{c}{R-p}$, unless the tech-savvy customers' arrival rate is $\Lambda_1 = \lambda_W^X = \mu - \frac{c}{R-p}$ already. In this case, customers' equilibrium behavior is to all queue themselves with rate $\mu - \frac{c}{R-p}$; i.e., $\lambda_D = 0$ and $\lambda_W = \lambda_{1W} + \lambda_{0W} = \mu - \frac{c}{R-p}$.

2. If $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{c}{\mu} \leq R - \frac{\phi}{\mu-\Lambda_1}$, we have $0 \leq R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c} \leq R - p - \frac{\phi}{\mu}$, $\lambda_W^X = \mu - \frac{c}{R-p} < \Lambda_1 \leq \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$, and $\mu - \frac{c}{R-p} \geq 0$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.

2.1 If $0 < \theta \leq R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2) and (2.2), we have $U_D(\Lambda_1) \geq 0$. This means that if all tech-savvy customers join, their utility of using the food delivery service is non-negative, while the utility of walk-in is negative. Thus, tech-savvy customers will all join and use the food delivery service and all walk-in customers will balk; i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.

2.2 If $R - p - \frac{\phi}{\mu-\Lambda_1} < \theta \leq \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2), (2.2), and (3.3), we have $\lambda^X(\theta) \leq \lambda_W^X \leq \lambda_D^X(\theta) < \Lambda_1$ and $U_D(\Lambda_1) < 0$. This means that if all tech-savvy customers join, both options are unattractive. Then, some tech-savvy customers may balk to avoid negative utility, until the total arrival rate to the system drops to $\lambda_D^X(\theta)$, where $U_W(\lambda_D^X(\theta)) < U_D(\lambda_D^X(\theta)) = 0$ (using Lemma C.3(1) and (2)). They will not reduce the arrival rate further, because $U_D(\lambda) > 0$ if $\lambda \in (\lambda_D^X(\theta) - \epsilon, \lambda_D^X(\theta))$ (using Lemma C.3(2)). Thus, the tech-savvy customers join and use the food delivery service with rate $\lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$ and other customers balk: i.e., $\lambda_D = \lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$. In this case, all customers have zero utility.

2.3 If $\frac{(c-\phi)(R-p)}{c} < \theta \leq R - p - \frac{\phi}{\mu}$, similar to Case 1.2 above, we have $\lambda_D = 0$ and $\lambda_W = \mu - \frac{c}{R-p}$.

3. If $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu} < p \leq R - \frac{\phi}{\mu-\Lambda_1}$, we have $0 \leq R - p - \frac{\phi}{\mu-\Lambda_1} < R - p - \frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, $\lambda_W^X = \mu - \frac{c}{R-p} < \Lambda_1 \leq \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$, and $\mu - \frac{c}{R-p} < 0$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.

3.1 If $0 < \theta \leq R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2) and (2.2), we have $U_D(\Lambda_1) \geq 0$. This means that if all tech-savvy customers join, their utility of using the food delivery service is non-negative, while the utility of walk-in is negative. Thus, tech-savvy customers will all join and use the food delivery service and all walk-in customers will balk: i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.

3.2 If $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2 above, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.

4. If $R-\frac{c}{\mu} \le R-\frac{\phi}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1} < 0 < R-p-\frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$ and $\mu - \frac{c}{R-p} < 0$.

4.1 If $R-p-\frac{\phi}{\mu-\Lambda_1} < 0 < \theta \le R-p-\frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2 above, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.

Case 2: When $\Lambda_1 > \frac{c-\phi}{c}\mu$, we have $R-\frac{c}{\mu-\Lambda_1} < R-\frac{\phi}{\mu-\Lambda_1} < R-\frac{c}{\mu} < R-\frac{\phi}{\mu}$ by Lemma C.4.

1. If $p \le R-\frac{c}{\mu-\Lambda_1}$, we have $\Lambda_1 \le \lambda_W^X = \mu - \frac{c}{R-p}$. From Lemma C.3(1), when all customers join with rate $\Lambda \ge \mu > \lambda_W^X = \mu - \frac{c}{R-p}$, the utility of walk-in is negative, i.e., $U_W(\Lambda) < 0$.

1.1 If $\theta \le \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(3.3), we have $\lambda_W^X \le \lambda_D^X(\theta)$. Since $\Lambda_1 \le \lambda_W^X \le \lambda_D^X(\theta)$, from Lemma C.3(1), we have $U_W(\Lambda_1) \ge 0$ and $U_W(\lambda_W^X) = 0$; and from Lemma C.3(2), we have $U_D(\Lambda_1) \ge 0$ and $U_D(\lambda_W^X) \ge 0$. This means that if all tech-savvy customers join (with rate $\Lambda_1$), walk-in customers have an incentive to join and obtain positive utility, until the total arrival rate to the system reaches $\lambda_W^X = \mu - \frac{c}{R-p}$, unless the tech-savvy customers' arrival rate is $\Lambda_1 = \lambda_W^X = \mu - \frac{c}{R-p}$ already. In this case (i) all tech-savvy customers will join and use the food delivery service; i.e., $\lambda_D = \Lambda_1$; (ii) walk-in customers will join with rate $\lambda_W = \mu - \frac{c}{R-p} - \Lambda_1$.

1.2 If $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$, from Lemma C.3(3.3), we have $\lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta)$. Since $\Lambda_1 \le \lambda_W^X < \lambda^X(\theta)$, from Lemma C.3(1) and (3), we have $U_D(\Lambda_1) < U_W(\Lambda_1)$ and $U_W(\Lambda_1) \ge 0$. This means that if only tech-savvy customers join, (i) the food delivery service cannot attract any customers; (ii) all $\Lambda_1$ tech-savvy customers will queue themselves and their utility of walk-in is $U_W(\Lambda_1) \ge 0$. Then walk-in customers may have an incentive to join and obtain positive utility until the total arrival rate to the system reaches $\lambda_W^X = \mu - \frac{c}{R-p}$, unless the tech-savvy customers' arrival rate is $\Lambda_1 = \lambda_W^X = \mu - \frac{c}{R-p}$ already. In this case, customers' equilibrium behavior is to all queue themselves with rate $\mu - \frac{c}{R-p}$; i.e., $\lambda_D = 0$ and $\lambda_W = \mu - \frac{c}{R-p}$.

2. If $R-\frac{c}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu-\Lambda_1}$, we have $0 \le R-p-\frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$ and $\lambda_W^X = \mu - \frac{c}{R-p} < \Lambda_1 \le \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$. Note that $\mu - \frac{c}{R-p} > 0$, because $p \le R-\frac{\phi}{\mu-\Lambda_1} < R-\frac{c}{\mu}$.

2.1 If $0 < \theta \le R-p-\frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2) and (2.2), we have $U_D(\Lambda_1) \ge 0$. This means that if all tech-savvy customers join, their utility of using the food delivery service is non-negative, while the utility of walk-in is negative. Thus, all tech-savvy customers will join and use the food delivery service and all walk-in customers will balk: i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.

2.2 If $R-p-\frac{\phi}{\mu-\Lambda_1}<\theta\leq\frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2), (2.2), and (3.3), we have $\lambda^X(\theta)\leq\lambda_W^X\leq\lambda_D^X(\theta)<\Lambda_1$ and $U_D(\Lambda_1)<0$. This means that if all tech-savvy customers join, neither option is attractive. Then, some tech-savvy customers may balk to avoid negative utility, until the total arrival rate to the system drops to $\lambda_D^X(\theta)$, where $U_W(\lambda_D^X(\theta))<U_D(\lambda_D^X(\theta))=0$ (using Lemma C.3(1) and (2)). They will not reduce the arrival rate further, because $U_D(\lambda)>0$ if $\lambda\in(\lambda_D^X(\theta)-\epsilon,\lambda_D^X(\theta))$ (using Lemma C.3(2)). Thus, the tech-savvy customers join and use the food delivery service with rate $\lambda_D^X(\theta)=\mu-\frac{\phi}{R-p-\theta}$ and other customers balk; i.e., $\lambda_D=\lambda_D^X(\theta)=\mu-\frac{\phi}{R-p-\theta}$ and $\lambda_W=0$. In this case, all customers have zero utility.

2.3 If $\frac{(c-\phi)(R-p)}{c}<\theta\leq R-p-\frac{\phi}{\mu}$, similar to Case 2.2 above, we have $\lambda_D=0$ and $\lambda_W=\mu-\frac{c}{R-p}$.

3. If $R-\frac{\phi}{\mu-\Lambda_1}<p\leq R-\frac{c}{\mu}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1}<0<\frac{(c-\phi)(R-p)}{c}\leq R-p-\frac{\phi}{\mu}$ and $\mu-\frac{c}{R-p}\geq 0$.

3.1 If $R-p-\frac{\phi}{\mu-\Lambda_1}<0<\theta\leq\frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2 above, we have $\lambda_D=\mu-\frac{\phi}{R-p-\theta}$ and $\lambda_W=0$.

3.2 If $\frac{(c-\phi)(R-p)}{c}<\theta\leq R-p-\frac{\phi}{\mu}$, similar to Case 1.2 above, we have $\lambda_D=0$ and $\lambda_W=\mu-\frac{c}{R-p}$.

4. If $R-\frac{\phi}{\mu-\Lambda_1}<R-\frac{c}{\mu}<p\leq R-\frac{\phi}{\mu}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1}<0\leq R-p-\frac{\phi}{\mu}<\frac{(c-\phi)(R-p)}{c}$ and $\mu-\frac{c}{R-p}<0$.

4.1 If $R-p-\frac{\phi}{\mu-\Lambda_1}<0<\theta\leq R-p-\frac{\phi}{\mu}$, similar to Case 2.2 above, we have $\lambda_D=\mu-\frac{\phi}{R-p-\theta}$ and $\lambda_W=0$.

To summarize, in equilibrium, the joining rates of food-delivery and walk-in customers, $\lambda_D$ and $\lambda_W$, under the food price $p$ and delivery fee $\theta$, are

1. For $\Lambda_1\leq\frac{c-\phi}{c}\mu$,

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| 1.1 When $p\leq R-\frac{c}{\mu-\Lambda_1}$ | | |
| if $0<\theta\leq\frac{(c-\phi)(R-p)}{c}$ | $\Lambda_1$ | $\mu-\frac{c}{R-p}-\Lambda_1$ |
| if $\frac{(c-\phi)(R-p)}{c}<\theta\leq R-p-\frac{\phi}{\mu}$ | 0 | $\mu-\frac{c}{R-p}$ |
| 1.2 When $R-\frac{c}{\mu-\Lambda_1}<p\leq R-\frac{c}{\mu}$ | | |
| if $0<\theta\leq R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| if $R-p-\frac{\phi}{\mu-\Lambda_1}<\theta\leq\frac{(c-\phi)(R-p)}{c}$ | $\mu-\frac{\phi}{R-p-\theta}$ | 0 |
| if $\frac{(c-\phi)(R-p)}{c}<\theta\leq R-p-\frac{\phi}{\mu}$ | 0 | $\mu-\frac{c}{R-p}$ |
| 1.3 When $R-\frac{c}{\mu}<p\leq R-\frac{\phi}{\mu-\Lambda_1}$ | | |
| if $0<\theta\leq R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| if $R-p-\frac{\phi}{\mu-\Lambda_1}<\theta\leq R-p-\frac{\phi}{\mu}$ | $\mu-\frac{\phi}{R-p-\theta}$ | 0 |
| 1.4 When $R-\frac{\phi}{\mu-\Lambda_1}<p\leq R-\frac{\phi}{\mu}$ | | |
| if $0<\theta\leq R-p-\frac{\phi}{\mu}$ | $\mu-\frac{\phi}{R-p-\theta}$ | 0 |

2. For $\Lambda_1>\frac{c-\phi}{c}\mu$,

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| 2.1 When $p \le R - \frac{c}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\Lambda_1$ | $\mu - \frac{c}{R-p} - \Lambda_1$ |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | 0 | $\mu - \frac{c}{R-p}$ |
| 2.2 When $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\phi}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| if $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\mu - \frac{\phi}{R-p-\theta}$ | 0 |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | 0 | $\mu - \frac{c}{R-p}$ |
| 2.3 When $R - \frac{\phi}{\mu-\Lambda_1} < p \le R - \frac{c}{\mu}$ | | |
| if $0 < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\mu - \frac{\phi}{R-p-\theta}$ | 0 |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | 0 | $\mu - \frac{c}{R-p}$ |
| 2.4 When $R - \frac{c}{\mu} < p \le R - \frac{\phi}{\mu}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu}$ | $\mu - \frac{\phi}{R-p-\theta}$ | 0 |

This result directly leads to Lemma C.2. □

We have several observations from Lemma C.2. First, the tech-savvy customers are willing to pay at most $\frac{(c-\phi)(R-p)}{c}$ for the food delivery service. Otherwise, if the food delivery platform sets the delivery fee greater than $\frac{(c-\phi)(R-p)}{c}$, no tech-savvy customers would use the food delivery service and they would rather walk in. This implies that the food delivery platform should set the delivery fee no more than $\frac{(c-\phi)(R-p)}{c}$. If that is the case, we see from Lemma C.2 that no traditional customers choose to walk in unless all tech-savvy customers use the food delivery service.

Second, since we assume traditional customers' demand is abundant (i.e., $\Lambda_0 \ge \mu$), the restaurant cannot serve all traditional customers and some of them have to balk. Then our setting is equivalent to an alternative one where the arrival rate of all potential customers is fixed and sufficiently large. Among the population, a fraction of customers are tech-savvy customers and the rest are traditional customers. The monotonicity results in this paper with a growing $\Lambda_1$ and fixed $\Lambda_0$ is equivalent to those with a growing fraction of tech-savvy customers and fixed total customer arrival rate.

Third, when the food delivery platform sets a delivery fee of no more than $R-p-\frac{\phi}{\mu-\Lambda_1}$, all tech-savvy customers will use the food delivery service. Of course, when $p$ is sufficiently low, i.e., $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1} \ge \frac{(c-\phi)(R-p)}{c}$, so that even when the delivery fee is at the upper bound $\frac{(c-\phi)(R-p)}{c}$, all tech-savvy customers will use the food delivery service; moreover, when the food price $p$ is sufficiently high, i.e., $p > R - \frac{\phi}{\mu-\Lambda_1}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1} < 0$, so no matter what the delivery fee is, not all tech-savvy customers will use the food delivery service.

### C.1.2. Food Delivery Platform Strategy

The food delivery platform's profit per time unit $\pi$ is the product of the delivery fee $\theta$ and the joining rate of food-delivery customers $\lambda_D$. From Lemma C.2, we obtain the food delivery platform's profit.

Corollary C.1. *Under the food price $p$ and delivery fee $\theta$, the food delivery platform's profit $\pi(p,\theta)$ is*

| | $\pi(p,\theta)$ |
|---|---|
| *When* $p \le R - \frac{c}{\mu-\Lambda_1}$ | |
| *if* $\theta \le \frac{(c-\phi)(R-p)}{c}$ | $\theta\Lambda_1$ |
| *if* $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ |
| *When* $R-\frac{c}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu}$ | |
| *if* $\theta \le \max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right)$ | $\theta\Lambda_1$ |
| *if* $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right) < \theta \le \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ | $\theta\left(\mu-\frac{\phi}{R-p-\theta}\right)$ |
| *if* $\theta > \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ | $0$ |

From Corollary C.1, we see that the platform has no incentive to set the delivery fee $\theta$ too high because the platform would earn no profit otherwise. The platform will set the delivery fee such that either all tech-savvy customers will use the food delivery service, in which case the platform's profit is $\pi(p,\theta)=\theta\Lambda_1$, or a fraction of them will use the food delivery service, in which case the platform's profit is $\pi(p,\theta)=\theta\left(\mu-\frac{\phi}{R-p-\theta}\right)$. In the first case, the platform's profit $\pi(p,\theta)=\theta\Lambda_1$ increases in the delivery fee $\theta$, so the platform will charge the highest delivery fee in the corresponding interval of $\theta$ to obtain the maximum profit. In the second case, the platform's profit $\pi(p,\theta)=\theta\left(\mu-\frac{\phi}{R-p-\theta}\right)$ is a unimodal function of $\theta\in\left[-\infty,R-p-\frac{\phi}{\mu}\right]$ with a maximum at $\theta=R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}\in\left(0,R-p-\frac{\phi}{\mu}\right)$. Thus, the platform's optimal delivery fee depends on the comparison of $R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$ and the corresponding interval of $\theta$. Therefore, the platform will charge the optimal delivery fee

$$\theta^*=\min\left(\max\left(R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu},R-p-\frac{\phi}{\mu-\Lambda_1}\right),\frac{(c-\phi)(R-p)}{c}\right).$$

The following proposition characterizes the optimal delivery fee and resulting customers' behavior in equilibrium.

Proposition C.3 **(Food Delivery Platform Strategy)**. *Under the restaurant's food price $p$, the food delivery platform's best-response delivery fee $\theta^*(p)$, the joining rates of food-delivery and walk-in customers $\lambda_D(p,\theta^*(p))$ and $\lambda_W(p,\theta^*(p))$ under $\theta^*(p)$, are*

1. *For* $\Lambda_1 \le \frac{c-\phi}{c}\mu$,

| | $\theta^*(p)$ | $\lambda_D(p,\theta^*(p))$ | $\lambda_W(p,\theta^*(p))$ |
|---|---|---|---|
| *If* $p\le R-\frac{c}{\mu-\Lambda_1}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\Lambda_1$ | $\mu-\frac{c}{R-p}-\Lambda_1$ |
| *If* $R-\frac{c}{\mu-\Lambda_1}<p\le R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| *If* $R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}<p\le R-\frac{\phi}{\mu}$ | $R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\mu-\sqrt{\frac{\mu\phi}{R-p}}$ | $0$ |

2. *For $\Lambda_1 > \frac{c-\phi}{c}\mu$,*

| | $\theta^*(p)$ | $\lambda_D(p,\theta^*(p))$ | $\lambda_W(p,\theta^*(p))$ |
|---|---|---|---|
| *If* $p \le R - \frac{c}{\mu-\Lambda_1}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\Lambda_1$ | $\mu - \frac{c}{R-p} - \Lambda_1$ |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c^2}{\mu\phi}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\mu - \frac{c}{R-p}$ | $0$ |
| *If* $R - \frac{c^2}{\mu\phi} < p \le R - \frac{\phi}{\mu}$ | $R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\mu - \sqrt{\frac{\mu\phi}{R-p}}$ | $0$ |

*Moreover, the food delivery platform's profit under the platform's best-response delivery fee $\pi^*(p)$ is a decreasing function of the food price $p$.*

To prove Proposition C.3, we first derive some properties of the food delivery platform's profit $\pi(p,\theta) = \theta\lambda_D$ given various $\lambda_D$.

LEMMA C.5. *The food delivery platform's profit $\pi(p,\theta)$ depends on the demand rate for the food delivery service $\lambda_D$:*

1. *If $\lambda_D = \Lambda_1$, we have $\pi(p,\theta) = \theta\Lambda_1$, which strictly increases in $\theta$.*
2. *If $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$, we have $\pi(p,\theta) = \theta\left(\mu - \frac{\phi}{R-p-\theta}\right)$, which is a unimodal function of $\theta$ with a maximum at*

$$\theta_2 = R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu},$$

*which decreases in $p$.*

*2.1 $\theta_2$ intersects with $R-p-\frac{\phi}{\mu-\Lambda_1}$ once at $p = R - \frac{\phi\mu}{(\mu-\Lambda_1)^2}$.*

- *We have $\theta_2 \le R-p-\frac{\phi}{\mu-\Lambda_1} \Leftrightarrow p \le R - \frac{\phi\mu}{(\mu-\Lambda_1)^2}$, and $\theta_2 > R-p-\frac{\phi}{\mu-\Lambda_1} \Leftrightarrow p > R - \frac{\phi\mu}{(\mu-\Lambda_1)^2}$.*
- *We have*

$$R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} \ge R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 \le \frac{c-\phi}{c}\mu$$
$$R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} < R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 > \frac{c-\phi}{c}\mu$$

*where $R - \frac{c}{\mu-\Lambda_1}$ is the intersection point of $\frac{(c-\phi)(R-p)}{c}$ and $R-p-\frac{\phi}{\mu-\Lambda_1}$.*

- *We have $R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} < R - \frac{\phi}{\mu}$.*

*2.2 $\theta_2$ intersects with $\frac{(c-\phi)(R-p)}{c}$ once at $p = R - \frac{c^2}{\mu\phi}$.*

- *We have $\theta_2 \le \frac{(c-\phi)(R-p)}{c} \Leftrightarrow p \ge R - \frac{c^2}{\mu\phi}$, and $\theta_2 > \frac{(c-\phi)(R-p)}{c} \Leftrightarrow p < R - \frac{c^2}{\mu\phi}$.*
- *Moreover, we have*

$$R - \frac{c^2}{\mu\phi} \le R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 \le \frac{c-\phi}{c}\mu$$
$$R - \frac{c^2}{\mu\phi} > R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 > \frac{c-\phi}{c}\mu$$

*where $R - \frac{c}{\mu-\Lambda_1}$ is the intersection point of $\frac{(c-\phi)(R-p)}{c}$ and $R-p-\frac{\phi}{\mu-\Lambda_1}$.*

- *We have $R - \frac{c^2}{\mu\phi} < R - \frac{\phi}{\mu}$.*

*2.3 $\theta_2$ intersects with $R-p-\frac{\phi}{\mu}$ once at $p=R-\frac{\phi}{\mu}$.*

- *We have $0\le\theta_2\le R-p-\frac{\phi}{\mu}\Leftrightarrow p\le R-\frac{\phi}{\mu}$.*

*Proof of Lemma C.5.* 1. If $\lambda_D=\Lambda_1$, we have

$$\pi(p,\theta)=\theta\lambda_D=\theta\Lambda_1$$

which clearly increases in $\theta$.

2. If $\lambda_D=\mu-\frac{\phi}{R-p-\theta}$ for $0\le\theta\le R-p-\frac{\phi}{\mu}$, we have

$$\pi(p,\theta)=\theta\left(\mu-\frac{\phi}{R-p-\theta}\right).$$

Simple algebra shows that $\pi(p,\theta)$ is a unimodal function of $\theta$ with a maximum at $\theta_2=R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$, which decreases in $p$.

2.1 We derive

$$\theta_2-\left(R-p-\frac{\phi}{\mu-\Lambda_1}\right)=\frac{\phi}{\mu-\Lambda_1}-\frac{\sqrt{\mu\phi(R-p)}}{\mu},$$

which increases in $p$ and has a unique root $p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}$. Hence, we have: (i) $\theta_2$ intersects with $R-p-\frac{\phi}{\mu-\Lambda_1}$ only once at $p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}$; and (ii) $\theta_2\le R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}$, and $\theta_2>R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p>R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}$. At last, simple algebra gives $R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}\ge R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow\Lambda_1\le\frac{c-\phi}{c}\mu$ and $R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}<R-\frac{\phi}{\mu}$.

2.2 We derive

$$\theta_2-\frac{(c-\phi)(R-p)}{c}=\frac{\phi}{c}(R-p)-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$$

which has two roots $p=R-\frac{c^2}{\mu\phi}$ and $R$. Moreover, we have $\theta_2-\frac{(c-\phi)(R-p)}{c}=\frac{\phi-\sqrt{\phi c}}{\mu}<0$ when $p=R-\frac{c}{\mu}\in\left[R-\frac{c^2}{\mu\phi},R\right]$. Hence, we have: (i) $\theta_2$ intersects with $\frac{(c-\phi)(R-p)}{c}$ only once at $p=R-\frac{c^2}{\mu\phi}$; and (ii) $\theta_2\le\frac{(c-\phi)(R-p)}{c}\Leftrightarrow p\ge R-\frac{c^2}{\mu\phi}$, and $\theta_2>\frac{(c-\phi)(R-p)}{c}\Leftrightarrow p<R-\frac{c^2}{\mu\phi}$. At last, simple algebra gives $R-\frac{c^2}{\mu\phi}\le R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow\Lambda_1\le\frac{c-\phi}{c}\mu$ and $R-\frac{c^2}{\mu\phi}<R-\frac{\phi}{\mu}$.

2.3 We derive

$$\theta_2-\left(R-p-\frac{\phi}{\mu}\right)=\frac{\phi-\sqrt{\mu\phi(R-p)}}{\mu}$$

which increases in $p$ and has a unique root $p=R-\frac{\phi}{\mu}$. Moreover, we have $\theta_2=0$ at $p=R-\frac{\phi}{\mu}$. Hence, we have: (i) $\theta_2$ intersects with $R-p-\frac{\phi}{\mu}$ only once at $p=R-\frac{\phi}{\mu}$; and (ii) $\theta_2\le R-p-\frac{\phi}{\mu}\Leftrightarrow p\le R-\frac{\phi}{\mu}$. □

Next, we prove Proposition C.3.

When $\Lambda_1\le\frac{c-\phi}{c}\mu$, we have $R-\frac{c^2}{\mu\phi}\le R-\frac{c}{\mu-\Lambda_1}\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}\le R-\frac{\phi}{\mu}$ by Lemma C.5(2.1) and (2.2). Figure C.11 illustrates the intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1\le\frac{c-\phi}{c}\mu$.

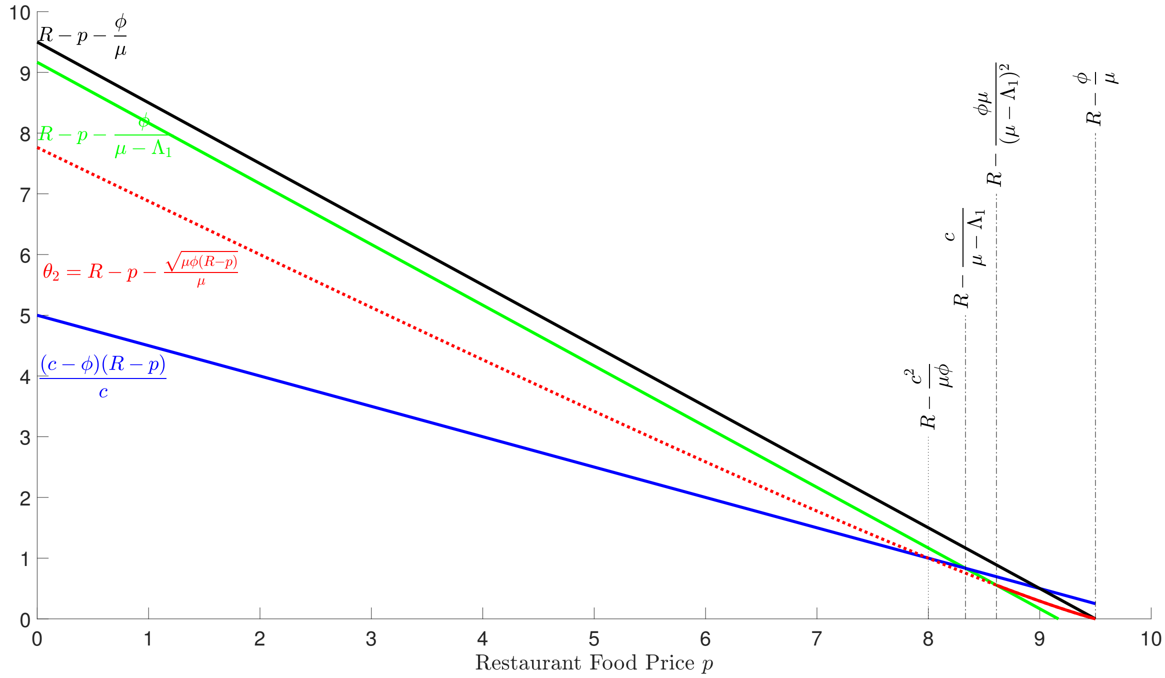


**Figure C.11** **The intersection points of** $\frac{(c-\phi)(R-p)}{c}$**,** $R-p-\frac{\phi}{\mu-\Lambda_1}$**,** $\theta_2 = R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$**, and** $R-p-\frac{\phi}{\mu}$**, when** $\Lambda_1 \le \frac{c-\phi}{c}\mu$**, for the parameter setting** $R=10$**,** $\Lambda_0=\mu=c=1$**, and** $\phi=0.5$**.**

1. If $p \le R-\frac{c}{\mu-\Lambda_1}$, we have $\frac{(c-\phi)(R-p)}{c} \le R-p-\frac{\phi}{\mu-\Lambda_1} < R-p-\frac{\phi}{\mu}$. From Lemma C.2, we see that tech-savvy customers are willing to pay a delivery fee of at most $\frac{(c-\phi)(R-p)}{c}$ for the food delivery service, and the demand for the food delivery service is $\lambda_D = \Lambda_1$ when $\theta \le \frac{(c-\phi)(R-p)}{c}$. By Lemma C.5(1), the food delivery platform will charge the highest delivery fee $\theta^* = \frac{(c-\phi)(R-p)}{c}$ to maximize its profit, and we have $\lambda_D = \Lambda_1$, and $\lambda_W = \mu-\frac{c}{R-p}-\Lambda_1$. The food delivery platform's profit $\pi^*(p) = \frac{(c-\phi)(R-p)}{c}\Lambda_1$ is clearly a decreasing function of food price $p$.
2. If $R-\frac{c}{\mu-\Lambda_1} \le p \le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$. Furthermore, we have $\theta_2 \le R-p-\frac{\phi}{\mu-\Lambda_1}$, by Lemma C.5(2.1). From Lemma C.2, the demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$. Then, from Lemma C.5(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $R-p-\frac{\phi}{\mu-\Lambda_1}$ for the food delivery service. From Lemma C.2, the demand for the food delivery service is $\lambda_D = \mu-\frac{\phi}{R-p-\theta}$ for $R-p-\frac{\phi}{\mu-\Lambda_1} \le \theta \le \frac{(c-\phi)(R-p)}{c}$. In this case, the food delivery platform's profit $\pi(p,\theta)$ decreases in $\theta$ for $\theta \ge R-p-\frac{\phi}{\mu-\Lambda_1} \ge \theta_2$ by Lemma C.5(2). Thus, the food delivery platform will charge $\theta^* = R-p-\frac{\phi}{\mu-\Lambda_1}$ as the delivery fee, and we have $\lambda_D = \Lambda_1$, and $\lambda_W = 0$. The food delivery platform's profit $\pi^*(p) = \left(R-p-\frac{\phi}{\mu-\Lambda_1}\right)\Lambda_1$ is clearly a decreasing function of food price $p$.
3. If $R-\frac{\phi\mu}{(\mu-\Lambda_1)^2} \le p \le R-\frac{\phi}{\mu}$, we have $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right) \le \theta_2 \le \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$, by Lemma C.5(2). From Lemma C.2, the demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \le \max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right)$. Then, from Lemma C.5(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right)$

for the food delivery service. From Lemma C.2, the demand for the food delivery service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right) < \theta \le \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$. In this case, the food delivery platform's profit $\pi(p,\theta)$ has its maximum at $\theta_2$ on $\left[\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right), \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)\right]$ by Lemma C.5(2). Thus, the food delivery platform will charge $\theta^* = \theta_2 = R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$ as the delivery fee, and we have $\lambda_D = \mu - \sqrt{\frac{\mu\phi}{R-p}}$, and $\lambda_W = 0$. The food delivery platform's profit is $\pi^*(p) = \theta^*\lambda_D = (R-p)\mu + \phi - 2\sqrt{\mu\phi(R-p)}$. The first derivative of $\theta^*\lambda_D$ is

$$\frac{\partial \pi^*(p)}{\partial p} = \frac{\sqrt{\mu\phi} - \mu\sqrt{R-p}}{\sqrt{R-p}},$$

which is negative, because it increases in $p$ and reaches zero when $p = R - \phi/\mu$. Thus, $\pi^*(p) = \theta^*\lambda_D$ is a decreasing function of food price $p$.

When $\Lambda_1 > \frac{c-\phi}{c}\mu$, we have $R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} < R - \frac{c}{\mu-\Lambda_1} < R - \frac{c^2}{\mu\phi} < R - \frac{\phi}{\mu}$ by Lemma C.5(2.1) and (2.2). Figure C.12 illustrates the intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1 > \frac{c-\phi}{c}\mu$.

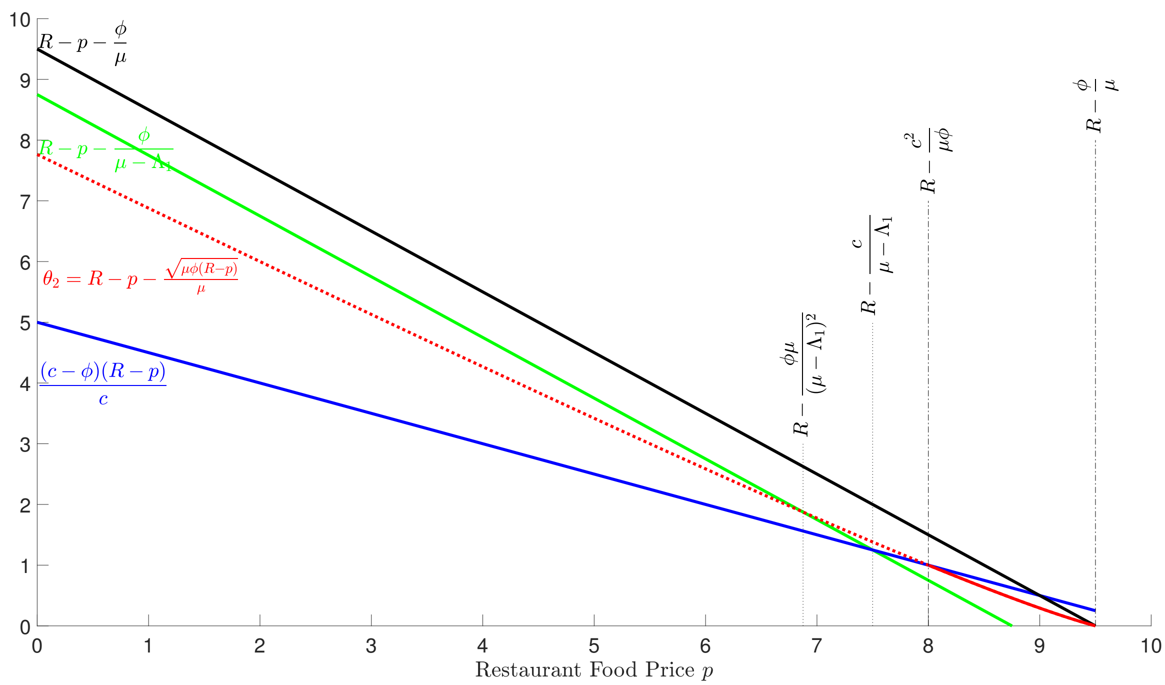


**Figure C.12** **The intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2 = R-p-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1 > \frac{c-\phi}{c}\mu$, for the parameter setting $R=10$, $\Lambda_0=\mu=c=1$, and $\phi=0.5$.**

1. If $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $\frac{(c-\phi)(R-p)}{c} \le R-p-\frac{\phi}{\mu-\Lambda_1} < R-p-\frac{\phi}{\mu}$. From Lemma C.2, we see that tech-savvy customers are willing to pay a delivery fee of at most $\frac{(c-\phi)(R-p)}{c}$ for the food delivery service, and the demand for the food delivery service is $\lambda_D = \Lambda_1$ when $\theta \le \frac{(c-\phi)(R-p)}{c}$. By Lemma C.5(1), the food delivery platform will charge the highest delivery fee $\theta^* = \frac{(c-\phi)(R-p)}{c}$ to

maximize its profit, and we have $\lambda_D = \Lambda_1$, and $\lambda_W = \mu - \frac{c}{R-p} - \Lambda_1$. The food delivery platform's profit $\pi^*(p) = \frac{(c-\phi)(R-p)}{c}\Lambda_1$ is clearly a decreasing function of food price $p$.

2. If $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c^2}{\mu\phi}$, we have $R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$. Furthermore, we have $\theta_2 \ge \frac{(c-\phi)(R-p)}{c}$, by Lemma C.5(2). From Lemma C.2, the demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \le R - p - \frac{\phi}{\mu-\Lambda_1}$. Then, from Lemma C.5(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $R - p - \frac{\phi}{\mu-\Lambda_1}$ for the food delivery service. From Lemma C.2, the demand for the food delivery service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $R - p - \frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$. In this case, the food delivery platform's profit $\pi(p,\theta)$ increases in $\theta$ for $\theta \le \frac{(c-\phi)(R-p)}{c} \le \theta_2$ by Lemma C.5(2). Thus, the food delivery platform will charge $\theta^* = \frac{(c-\phi)(R-p)}{c}$ as the delivery fee, and we have $\lambda_D = \mu - \frac{c}{R-p}$, and $\lambda_W = 0$. The food delivery platform's profit $\pi^*(p) = \frac{(c-\phi)(R-p)}{c}\left(\mu - \frac{c}{R-p}\right)$ is clearly a decreasing function of food price $p$.
3. If $R - \frac{c^2}{\mu\phi} < p \le R - \frac{\phi}{\mu}$, we have $\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right) \le \theta_2 \le \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$, by Lemma C.5(2). From Lemma C.2, the demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \le \max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right)$. Then, from Lemma C.5(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right)$ for the food delivery service. From Lemma C.2, the demand for the food delivery service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \le \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$. In this case, the food delivery platform's profit $\pi(p,\theta)$ has its maximum at $\theta_2$ on $\left[\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right), \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)\right]$ by Lemma C.5(2). Thus, the food delivery platform will charge $\theta^* = \theta_2 = R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ as the delivery fee, and we have $\lambda_D = \mu - \sqrt{\frac{\mu\phi}{R-p}}$, and $\lambda_W = 0$. The food delivery platform's profit $\pi^*(p) = (R-p)\mu + \phi - 2\sqrt{\mu\phi(R-p)}$ is a decreasing function of food price $p$, following a discussion similar to the one in the $\Lambda_1 \le \frac{c-\phi}{c}\mu$ case. $\square$

### C.1.3. Restaurant Strategy

The restaurant's profit $\Pi$ per time unit is the product of the food price $p$ and the throughput rate, which is the sum of the joining rates of food-delivery and walk-in customers, i.e., $\lambda_D + \lambda_W$. From Lemma C.2, we obtain the restaurant's profit.

Corollary C.2. *Under the food price $p$, the restaurant's profit $\Pi(p)$ is*

1. *For $\Lambda_1 \le \frac{c-\phi}{c}\mu$,*

| | $\Pi(p)$ |
|---|---|
| *If* $p \le R - \frac{c}{\mu-\Lambda_1}$ | $p\left(\mu - \frac{c}{R-p}\right)$ |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $p\Lambda_1$ |
| *If* $p > R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ |

2. *For* $\Lambda_1 > \frac{c-\phi}{c}\mu$,

| | $\Pi(p)$ |
|---|---|
| *If* $p \le R - \frac{c^2}{\mu\phi}$ | $p\left(\mu - \frac{c}{R-p}\right)$ |
| *If* $p > R - \frac{c^2}{\mu\phi}$ | $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ |

The following proposition characterizes the restaurant's optimal strategy as a Stackelberg leader.

Proposition C.4 **(Restaurant Strategy)**. *There exist threshold values $\phi_1$, $\lambda_1$, and $\lambda_2$, such that, in equilibrium, the restaurant's optimal price $p^*$, profit $\Pi^*$, and throughput $\lambda_D^* + \lambda_W^*$ are*

| | $p^*(\Lambda_1)$ | $\Pi^*(\Lambda_1)$ | $\lambda_D^* + \lambda_W^*$ |
|---|---|---|---|
| *When* $\phi \le \phi_1$ | | | |
| *if* $0 < \Lambda_1 \le \lambda_1$ | $R - \sqrt{\frac{c}{\mu}R}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $\mu - \sqrt{\frac{c\mu}{R}}$ |
| *if* $\lambda_1 < \Lambda_1 \le \lambda_2$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ | $\Lambda_1$ |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\mu - \chi$ |
| *When* $\phi > \phi_1$ | $R - \sqrt{\frac{c}{\mu}R}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $\mu - \sqrt{\frac{c\mu}{R}}$ |

*where $\chi$ is the unique real root of $-R\chi^3 - \mu\phi\chi + 2\mu^2\phi = 0$ in $[0,\mu]$. Moreover, we have*

*(i)* $p^*(\Lambda_1)|_{\Lambda_1 \le \lambda_1} < p^*(\Lambda_1)|_{\Lambda_1 > \lambda_1}$ *when* $\phi \le \phi_1$.

*(ii)* $\Pi^*(\Lambda_1)$ *is a weakly increasing function of* $\Lambda_1$.

*(iii)* $\mu - \sqrt{\frac{c\mu}{R}} < \lambda_2$ *if* $\phi < \frac{c}{2\sqrt{R\mu/c-1}} \le \phi_1$.

From Corollary C.2, the restaurant's profit $\Pi(p)$ depends critically on the interplay of three functions $p\Lambda_1$, $p\left(\mu - \frac{c}{R-p}\right)$, and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$. We first summarize some properties of the intersection points of these three curves in Lemma C.6, C.7, and C.8. These results will help us prove Proposition C.4.

Lemma C.6. *Some properties of $p\left(\mu - \frac{c}{R-p}\right)$:*

1. *It intersects with the x-axis at $p = R - \frac{c}{\mu}$.*
2. *It is a unimodal function of $p \in \left(0, R - \frac{\phi}{\mu}\right]$ and*

$$\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$$
$$\arg\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) = R - \sqrt{\frac{c}{\mu}}R.$$

3. *We have*

$$R - \sqrt{\frac{c}{\mu}R} \le R - \frac{c^2}{\mu\phi} \Leftrightarrow \phi \ge \sqrt{\frac{c^3}{R\mu}}$$
$$R - \sqrt{\frac{c}{\mu}R} \ge R - \frac{c^2}{\mu\phi} \Leftrightarrow \phi \le \sqrt{\frac{c^3}{R\mu}}.$$

*Proof of Lemma C.6.* 1. This conclusion can be reached immediately by solving $p\left(\mu - \frac{c}{R-p}\right) = 0$.

2. The first derivative of $p\left(\mu-\frac{c}{R-p}\right)$ is

$$\frac{\partial\left(p\left(\mu-\frac{c}{R-p}\right)\right)}{\partial p}=\mu-\frac{cR}{(R-p)^2},$$

which is $\frac{\mu}{c^3}\left(c^3-\mu\phi^2R\right)$ at $p=R-\frac{c^2}{\mu\phi}$ and $\mu-\frac{R}{c}\left(\mu-\Lambda_1\right)^2$ at $p=p=R-\frac{c}{\mu-\Lambda_1}$.

Solving $\partial\left(p\left(\mu-\frac{c}{R-p}\right)\right)/\partial p=0$ gives $p^*=R-\sqrt{\frac{c}{\mu}R}$. Then, replacing $p$ with $p^*$ in $p\left(\mu-\frac{c}{R-p}\right)$ leads to the maximum $\left(\sqrt{R\mu}-\sqrt{c}\right)^2$. Moreover, $\partial\left(p\left(\mu-\frac{c}{R-p}\right)\right)/\partial p$ decreases in $p$, $\partial\left(p\left(\mu-\frac{c}{R-p}\right)\right)/\partial p\geq 0$ if $p\leq R-\sqrt{\frac{c}{\mu}R}$ and $\partial\left(p\left(\mu-\frac{c}{R-p}\right)\right)/\partial p\leq 0$ if $p\geq R-\sqrt{\frac{c}{\mu}R}$. This means that $p\left(\mu-\frac{c}{R-p}\right)$ is a unimodal function of $p\in\left(0,R-\frac{\phi}{\mu}\right]$.

3. We have

$$R-\sqrt{\frac{c}{\mu}R}\leq R-\frac{c^2}{\mu\phi}\Leftrightarrow\sqrt{\frac{c}{\mu}R}\geq\frac{c^2}{\mu\phi}\Leftrightarrow\phi\geq\sqrt{\frac{c^3}{R\mu}}.$$

Similarly, we have $R-\sqrt{\frac{c}{\mu}R}>R-\frac{c^2}{\mu\phi}\Leftrightarrow\phi<\sqrt{\frac{c^3}{R\mu}}$. □

Lemma C.7. *Some properties of* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$*:*

1. *It intersects with the x-axis at* $p=R-\frac{\phi}{\mu}$.
2. *It is a unimodal function of* $p\in\left(0,R-\frac{\phi}{\mu}\right]$ *and*

$$\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R\left(\mu-\chi\right)^2}{2\mu-\chi},$$

$$\arg\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R\left(\mu-\chi\right)}{2\mu-\chi},$$

*where* $\chi$ *is the unique real root of* $-R\chi^3-\mu\phi\chi+2\mu^2\phi$ *in* $[0,\mu]$*, and* $\chi$ *increases in* $\phi$*. Moreover,* $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ *decreases with* $\phi$*.*

3. *We have*

$$\frac{2R\left(\mu-\chi\right)}{2\mu-\chi}\geq R-\frac{c^2}{\mu\phi}\Leftrightarrow 0\leq\phi\leq\frac{\sqrt{c^3\left(c+8R\mu\right)}-c^2}{2R\mu}$$
$$\frac{2R\left(\mu-\chi\right)}{2\mu-\chi}\leq R-\frac{c^2}{\mu\phi}\Leftrightarrow\frac{\sqrt{c^3\left(c+8R\mu\right)}-c^2}{2R\mu}<\phi\leq c,$$

*where* $\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}\geq\sqrt{\frac{c^3}{R\mu}}$.

4. $\arg\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)}{2\mu-\chi}>\arg\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\frac{c}{R-p}\right)=R-\sqrt{\frac{c}{\mu}R}$ *for* $\phi\leq\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$.

*Proof of Lemma C.7.* 1. This conclusion is immediate when we use $p=R-\frac{\phi}{\mu}$ in $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$.

2. Simple algebra gives

$$\frac{\partial\left(p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)\right)}{\partial p}=\frac{2\mu\left(R-p\right)-\sqrt{\mu\phi\left(R-p\right)}-R\sqrt{\frac{\mu\phi}{R-p}}}{2\left(R-p\right)}\overset{x=\sqrt{\frac{\mu\phi}{R-p}}}{=}\frac{1}{2\mu\phi}\left(-Rx^3+\left(2\mu^2-\mu x\right)\phi\right)$$

$$\overset{p=R-\frac{c^2}{\mu\phi}}{=} \frac{\mu}{2c^3}\left(-R\mu\phi^2 - c^2\phi + 2c^3\right)$$
$$\overset{p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}}{=} \frac{1}{2\mu\phi}\left(-R\left(\mu-\Lambda_1\right)^3 - \mu\phi\left(\mu-\Lambda_1\right) + 2\mu^2\phi\right)$$

We can obtain the discriminant of the cubic function $-Rx^3 - \mu\phi x + 2\mu^2\phi$ by using $a=-R$, $b=0$, $c=-\mu\phi$, and $d=2\mu^2\phi$ in $\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$:

$$\Delta = -4R\mu^3\phi^2\left(\phi + 27R\mu\right) < 0.$$

This means that $-Rx^3 - \mu\phi x + 2\mu^2\phi$ has only *one* real root. Let $\chi$ denote this real root. Furthermore, because $-Rx^3 - \mu\phi x + 2\mu^2\phi|_{x=0} = 2\mu^2\phi > 0$ and $-Rx^3 - \mu\phi x + 2\mu^2\phi|_{x=\mu} = -\mu^3\left(R - \frac{\phi}{\mu}\right) < 0$, this real root $\chi$ is in $[0,\mu]$; and $\partial\left(p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\right)/\partial p \geq 0$ if $x \leq \chi$ and $\partial\left(p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\right)/\partial p \leq 0$ if $x \geq \chi$. This means that $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ is a unimodal function of $p \in \left(0, R - \frac{\phi}{\mu}\right]$.

From $-Rx^3 + \left(2\mu^2 - \mu x\right)\phi = 0$, we have the correspondence between $\phi$ and $\chi$:

$$\phi = \frac{R\chi^3}{\mu\left(2\mu - \chi\right)} = \frac{R\chi^2}{\mu\left(\frac{2\mu}{\chi} - 1\right)},$$

which increases in $\chi$. This means that $\chi$ increases with $\phi$.

Solving $\partial\left(p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\right)/\partial p = 0$ gives $\sqrt{\frac{\mu\phi}{R-p^*}} = \chi \Rightarrow p^* = R - \frac{\mu\phi}{\chi^2} \Rightarrow p^* = \frac{2R(\mu-\chi)}{2\mu-\chi}$ where $\chi$ is the unique real root of $-R\chi^3 - \mu\phi\chi + 2\mu^2\phi$ in $[0,\mu]$. Then, substituting $p^*$ for $p$ in $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ leads to the maximum value $\left(R - \frac{\mu\phi}{\chi^2}\right)(\mu - \chi) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$.

Clearly, $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ decreases with $\phi$, so $\max p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ decreases with $\phi$ too.

3. We have

$$\frac{2R\left(\mu-\chi\right)}{2\mu-\chi} = R - \frac{\mu\phi}{\chi^2} \leq R - \frac{c^2}{\mu\phi} \Leftrightarrow \chi \leq \frac{\mu\phi}{c} \Leftrightarrow -Rx^3 - \mu\phi x + 2\mu^2\phi\big|_{x=\frac{\mu\phi}{c}} \leq 0 \Leftrightarrow \frac{\sqrt{c^3\left(c+8R\mu\right)} - c^2}{2R\mu} < \phi \leq c.$$

Similarly, we have $R - \frac{\mu\phi}{\chi^2} > R - \frac{c^2}{\mu\phi} \Leftrightarrow 0 \leq \phi \leq \frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$. We next prove

$$\begin{aligned}\sqrt{\frac{c^3}{R\mu}} &\leq \frac{\sqrt{c^3\left(c+8R\mu\right)} - c^2}{2R\mu}\\ 2\sqrt{v}+1 &\leq \sqrt{1+8v} \text{ (let } v = \frac{R\mu}{c}\text{)}\\ -4\sqrt{v}\left(\sqrt{v}-1\right) &\leq 0,\end{aligned}$$

which is clearly true since $v = \frac{R\mu}{c} \geq 1$.

4. We next prove, if $\phi \leq \frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, $\frac{2R(\mu-\chi)}{2\mu-\chi} > R - \sqrt{\frac{c}{\mu}R} \Leftrightarrow \chi < \frac{2\mu}{\sqrt{\frac{R\mu}{c}}+1}$. Since $-Rx^3 - \mu\phi x + 2\mu^2\phi$ has only *one* real root $\chi \in [0,\mu]$, $-Rx^3 - \mu\phi x + 2\mu^2\phi|_{x=0} = 2\mu^2\phi > 0$, and $-Rx^3 - \mu\phi x + 2\mu^2\phi|_{x=\mu} = -\mu^3\left(R - \frac{\phi}{\mu}\right) < 0$, we have

$$\chi < \frac{2\mu}{\sqrt{\frac{R\mu}{c}}+1} \Leftrightarrow -Rx^3 - \mu\phi x + 2\mu^2\phi\big|_{x=\frac{2\mu}{\sqrt{\frac{R\mu}{c}}+1}} < 0$$

$$\Leftrightarrow \phi < \frac{4c\frac{R\mu}{c}}{\left(2\frac{R\mu}{c} + \left(1+\frac{R\mu}{c}\right)\sqrt{\frac{R\mu}{c}}\right)},$$

which is clearly true since $\phi \le \frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu} = c\frac{\sqrt{1+8\frac{R\mu}{c}}-1}{2\frac{R\mu}{c}} < \frac{4c\frac{R\mu}{c}}{\left(2\frac{R\mu}{c}+\left(1+\frac{R\mu}{c}\right)\sqrt{\frac{R\mu}{c}}\right)}$; the last inequality can be verified by replacing $\frac{R\mu}{c}$ with $v$; and note that $\frac{R\mu}{c} > 1$. □

Lemma C.8. *We have the following properties regarding the intersection points of the three curves* $p\Lambda_1$, $p\left(\mu-\frac{c}{R-p}\right)$, *and* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$.

1. $p\left(\mu-\frac{c}{R-p}\right)$ *and* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ *intersect at point* $\left(R-\frac{c^2}{\mu\phi}, \mu\left(R-\frac{c^2}{\mu\phi}\right)\left(\frac{c-\phi}{c}\right)\right)$. *Moreover, we have* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) \le p\left(\mu-\frac{c}{R-p}\right)$, *if* $p \le R-\frac{c^2}{\mu\phi}$, *and* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) > p\left(\mu-\frac{c}{R-p}\right)$, *if* $p > R-\frac{c^2}{\mu\phi}$.
2. $p\Lambda_1$ *and* $p\left(\mu-\frac{c}{R-p}\right)$ *intersect at point* $\left(R-\frac{c}{\mu-\Lambda_1}, \Lambda_1\left(R-\frac{c}{\mu-\Lambda_1}\right)\right)$, *which is* $\left(R-\sqrt{\frac{c}{\mu}R}, \left(\mu-\sqrt{\frac{c\mu}{R}}\right)\left(R-\sqrt{\frac{c}{\mu}R}\right)\right)$ *when* $\Lambda_1 = \mu-\sqrt{\frac{c\mu}{R}}$.
   - *When* $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right) = \left(\sqrt{R\mu}-\sqrt{c}\right)^2 > \max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = 2R\frac{(\mu-\chi)^2}{2\mu-\chi}$,
   $\lambda\left(R-\frac{c}{\mu-\lambda}\right) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ *has two roots* $\lambda_3$ *and* $\lambda_3'$ *in* $[0,\mu]$ *such that* $\lambda_3' \le \lambda_3$.
   - *Moreover,* $\frac{c-\phi}{c}\mu < \lambda_3 \le \mu-\sqrt{\frac{c\mu}{R}}$.
3. $p\Lambda_1$ *and* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ *intersect at point* $\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}, \Lambda_1\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\right)$.
   - $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$ *is a unimodal function with* $\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = 2R\frac{(\mu-\chi)^2}{2\mu-\chi}$ *and*
   $\arg\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = \lambda_2 \equiv \mu-\chi$.
   - *When* $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right) = \left(\sqrt{R\mu}-\sqrt{c}\right)^2 < \max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = 2R\frac{(\mu-\chi)^2}{2\mu-\chi}$,
   $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = \left(\sqrt{R\mu}-\sqrt{c}\right)^2$ *has two roots* $\lambda_1$ *and* $\lambda_1'$ *in* $[0,\mu]$ *such that* $\lambda_1 \le \lambda_1'$.
   - *Moreover,* $\lambda_1 < \lambda_2$.
4. *When* $\Lambda_1 = \frac{c-\phi}{c}\mu$, *the three curves* $p\Lambda_1$, $p\left(\mu-\frac{c}{R-p}\right)$, *and* $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ *intersect at the same point* $\left(R-\frac{c^2}{\mu\phi}, \mu\left(R-\frac{c^2}{\mu\phi}\right)\left(\frac{c-\phi}{c}\right)\right)$. *Moreover,*
   - $\Lambda_1 \le \frac{c-\phi}{c}\mu \Leftrightarrow R-\frac{c}{\mu-\Lambda_1} \le R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$; *and* $\Lambda_1 \ge \frac{c-\phi}{c}\mu \Leftrightarrow R-\frac{c}{\mu-\Lambda_1} \ge R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$.
   - *If* $\phi \le \frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, *we have* $\lambda_2 \le \frac{c-\phi}{c}\mu$; *and if* $\phi > \frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, *we have* $\lambda_2 > \frac{c-\phi}{c}\mu$.

*Proof of Lemma C.8.* 1. Solving $p\left(\mu-\frac{c}{R-p}\right) = p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ gives $p = R-\frac{c^2}{\mu\phi}$. Using $p = R-\frac{c^2}{\mu\phi}$ in $p\left(\mu-\frac{c}{R-p}\right)$ gives the value $\mu\left(R-\frac{c^2}{\mu\phi}\right)\left(\frac{c-\phi}{c}\right)$.

2. Similarly, we have $p = R-\frac{c}{\mu-\Lambda_1}$ by solving $p\Lambda_1 = p\left(\mu-\frac{c}{R-p}\right)$. Using $p = R-\frac{c}{\mu-\Lambda_1}$ in $p\left(\mu-\frac{c}{R-p}\right)$ gives the value $\Lambda_1\left(R-\frac{c}{\mu-\Lambda_1}\right)$. Clearly, when $\Lambda_1 = \mu-\sqrt{\frac{c\mu}{R}}$, we have $\left(R-\frac{c}{\mu-\Lambda_1}, \Lambda_1\left(R-\frac{c}{\mu-\Lambda_1}\right)\right) = \left(R-\sqrt{\frac{c}{\mu}R}, \left(\mu-\sqrt{\frac{c\mu}{R}}\right)\left(R-\sqrt{\frac{c}{\mu}R}\right)\right)$.

Further, when $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right) = \left(\sqrt{R\mu}-\sqrt{c}\right)^2 > \max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right) = 2R\frac{(\mu-\chi)^2}{2\mu-\chi}$,

- We know that $\lambda\left(R-\frac{c}{\mu-\lambda}\right)$ is a unimodal function maximized at $\lambda = \mu-\sqrt{\frac{c\mu}{R}}$, $\lambda\left(R-\frac{c}{\mu-\lambda}\right)\Big|_{\lambda=0} = 0$, and $\lim_{\lambda\to\mu}\lambda\left(R-\frac{c}{\mu-\lambda}\right) = -\infty$, , and $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ is a constant regarding $\Lambda_1$. Hence, the equation $\lambda\left(R-\frac{c}{\mu-\lambda}\right)=\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ has two roots if $\frac{2R(\mu-\chi)^2}{2\mu-\chi}<\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$.

- We should have $\lambda_3\geq\frac{c-\phi}{c}\mu$; otherwise, if $\lambda_3<\frac{c-\phi}{c}\mu$, we will be able to reach the conclusion that $\frac{2R(\mu-\chi)^2}{2\mu-\chi}>\lambda_3\left(R-\frac{c}{\mu-\lambda_3}\right)$, which controdicts the definition of $\lambda_3$. If $\Lambda_1=\mu-\sqrt{\frac{c\mu}{R}}$, we have $\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)>\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)^2}{2\mu-\chi}$, which implies $\lambda_3\leq\mu-\sqrt{\frac{c\mu}{R}}$. Therefore, given $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2>\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=2R\frac{(\mu-\chi)^2}{2\mu-\chi}$, we have $\frac{c-\phi}{c}\mu<\lambda_3\leq\mu-\sqrt{\frac{c\mu}{R}}$.

3. Similarly, we have $p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ by solving $p\Lambda_1=p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$. Using $p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ in $p\Lambda_1$ gives the value $\Lambda_1\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$.
   - Note that $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)\Big|_{\lambda=\mu-\sqrt{\frac{\mu\phi}{R-p}}}=p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$, so we can obtain the unimodality of $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$ and its maximum by applying results from Lemma C.7. We see that $\lambda=\mu-\sqrt{\frac{\mu\phi}{R-p}}$ decreases in $p$ and $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)\Big|_{\lambda=\mu-\sqrt{\frac{\mu\phi}{R-p}}}=p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ is a unimodal function by Lemma C.7(2), so $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$ is also a unimodal function. Using $p=\arg\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=2R\frac{\mu-\chi}{2\mu-\chi}$ and $\phi=\frac{R\chi^3}{\mu(2\mu-\chi)}$ in $\lambda=\mu-\sqrt{\frac{\mu\phi}{R-p}}$ gives $\lambda_2=\mu-\chi$. Clearly, $\lambda_2$ is the root of $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=\frac{2R(\mu-\chi)^2}{2\mu-\chi}$. Moreover, we have $\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=2R\frac{(\mu-\chi)^2}{2\mu-\chi}$.
   - Because $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$ is a unimodal function of $\lambda$, $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)\Big|_{\lambda=0}=0$, and $\lim_{\lambda\to\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=-\infty$, we have that $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$ has two roots if $\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=2R\frac{(\mu-\chi)^2}{2\mu-\chi}>\left(\sqrt{R\mu}-\sqrt{c}\right)^2$.
   - Because $\lambda_1$ is the smallest $\lambda$ satisfying $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$ and $\lambda_2=\arg\max_{0<\lambda<\mu}\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$, we have $\lambda_1<\lambda_2$.
4. When substituting $p=R-\frac{c^2}{\mu\phi}$ for $p$ in $p\Lambda_1|_{\Lambda_1=\frac{c-\phi}{c}\mu}$, $p\left(\mu-\frac{c}{R-p}\right)$, and $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$, we obtain the same value $\mu\left(R-\frac{c^2}{\mu\phi}\right)\left(\frac{c-\phi}{c}\right)$. Hence, the three curves intersect at the same point $\left(R-\frac{c^2}{\mu\phi},\mu\left(R-\frac{c^2}{\mu\phi}\right)\left(\frac{c-\phi}{c}\right)\right)$ when $\Lambda_1=\frac{c-\phi}{c}\mu$. Simple algebra gives $\Lambda_1\leq\frac{c-\phi}{c}\mu\Leftrightarrow R-\frac{c}{\mu-\Lambda_1}\leq R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ and $\Lambda_1\geq\frac{c-\phi}{c}\mu\Leftrightarrow R-\frac{c}{\mu-\Lambda_1}\geq R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$.

   When $\phi=\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, we have $\lambda_2=\frac{c-\phi}{c}\mu$. Three curves $p\Lambda_1$, $p\left(\mu-\frac{c}{R-p}\right)$, $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ intersect at $\lambda=R-\frac{c^2}{\mu\phi}$, and this intersection point is the maximum point of $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ by Lemma C.8(2) and (4). If $\phi\leq\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, we have $\lambda_1<\lambda_2\leq\frac{c-\phi}{c}\mu$; if $\phi>\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, we have $\lambda_2>\frac{c-\phi}{c}\mu$. $\square$

Next, we prove Proposition C.4.

Recall from Lemma C.8(4) that (i) When $\Lambda_1\leq\frac{c-\phi}{c}\mu$, the restaurant's profit $\Pi(p)$ increases on $\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right]$, so we only need to compare $\max_{p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]} p\left(\mu-\frac{c}{R-p}\right)$

and $\max_{p\in\left[R-\frac{\mu\phi}{(\mu-\Lambda_1)^2},R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ to determine the restaurant's maximum profit $\Pi^*$. (ii) When $\Lambda_1 > \frac{c-\phi}{c}\mu$, we only need to compare $\max_{p\in\left(0,R-\frac{c^2}{\mu\phi}\right]} p\left(\mu-\frac{c}{R-p}\right)$ and $\max_{p\in\left[R-\frac{c^2}{\mu\phi},R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ to determine the restaurant's maximum profit $\Pi^*$.

We next discuss the relationship between $p\left(\mu-\frac{c}{R-p}\right)$ and $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$. By Lemma C.7(2), we have that $\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ decreasing in $\phi$.

When $\phi\le\sqrt{\frac{c^3}{R\mu}}$, we have $\arg\max p\left(\mu-\frac{c}{R-p}\right)=R-\sqrt{\frac{c}{\mu}R}\ge R-\frac{c^2}{\mu\phi}$ by Lemma C.6(3), which leads to $\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)\ge\max p\left(\mu-\frac{c}{R-p}\right)$.

When $\phi\ge\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, we have $\arg\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)}{2\mu-\chi}\le R-\frac{c^2}{\mu\phi}$ by Lemma C.7(3), which leads to $\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)\le\max p\left(\mu-\frac{c}{R-p}\right)$.

When $\phi$ increases to $\phi_1=\frac{R\chi_1^3}{\mu(2\mu-\chi_1)}\in\left[\sqrt{\frac{c^3}{R\mu}},\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}\right]$ where

$$\chi_1=\frac{3R\mu-\left(c-2\sqrt{Rc\mu}\right)-\sqrt{\left(3R\mu-\left(c-2\sqrt{Rc\mu}\right)\right)^2+16R\mu\left(c-2\sqrt{Rc\mu}\right)}}{4R}$$

is the unique solution of $\frac{2R(\mu-\chi)^2}{2\mu-\chi}=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$ in $[0,\mu]$, we have $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)$. Here, we can prove $\phi_1\in\left[\sqrt{\frac{c^3}{R\mu}},\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}\right]$.

1. If $\phi\le\phi_1\Leftrightarrow\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=2R\frac{(\mu-\chi)^2}{2\mu-\chi}\ge\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$. The maximum point of $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ is on the right-hand side of $R-\frac{c^2}{\mu\phi}$ the intersection point of $p\left(\mu-\frac{c}{R-p}\right)$ and $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$; i.e., $2R\frac{\mu-\chi}{2\mu-\chi}>R-\frac{c^2}{\mu\phi}$. Otherwise, if $2R\frac{\mu-\chi}{2\mu-\chi}\le R-\frac{c^2}{\mu\phi}$, we have $2R\frac{(\mu-\chi)^2}{2\mu-\chi}<\left(\sqrt{R\mu}-\sqrt{c}\right)^2$ by Lemma C.8(1), which contradicts $\phi\le\phi_1$.

   Figure C.13 illustrates the restaurant's profit as a function of $p$ under different $\Lambda_1$.

   - If $0<\Lambda_1\le\lambda_1$, the maximum point of $p\left(\mu-\frac{c}{R-p}\right)$ must be in $\left(0,R-\frac{c}{\mu-\Lambda_1}\right]$; otherwise there exists a $\lambda'<\lambda_1$ such that $\lambda'\left(R-\frac{\mu\phi}{(\mu-\lambda')^2}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$, which violates the definition of $\lambda_1$. Then, as Figure C.13(1) shows, we have $\Pi^*(\Lambda_1)=\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2\ge\max_{p\in\left[R-\frac{\mu\phi}{(\mu-\Lambda_1)^2},R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ and $p^*(\Lambda_1)=\arg\max\limits_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right)=R-\sqrt{\frac{c}{\mu}R}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.
   - If $\lambda_1<\Lambda_1\le\lambda_2$, we have $\max_{p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]} p\left(\mu-\frac{c}{R-p}\right)<\max_{p\in\left[R-\frac{\mu\phi}{(\mu-\Lambda_1)^2},R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}}=\Lambda_1\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$, as illustrated in Figure C.13(2). In this case, $\Pi^*(\Lambda_1)=\Lambda_1\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ and $p^*(\Lambda_1)=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$. Recall from Lemma C.8(3) that $\lambda\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)$ is a unimodal function with the maximum at $\lambda_2$, so $\Pi^*(\Lambda_1)$ increases in $\Lambda_1$ on $(\lambda_1,\lambda_2]$.
   - If $\lambda_2<\Lambda_1\le\frac{c-\phi}{c}\mu$, the maximum point of $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ is in $\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2},R-\frac{\phi}{\mu}\right]$, by the definition of $\lambda_2$. Then, we have $\max_{p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]} p\left(\mu-\frac{c}{R-p}\right)<\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=$

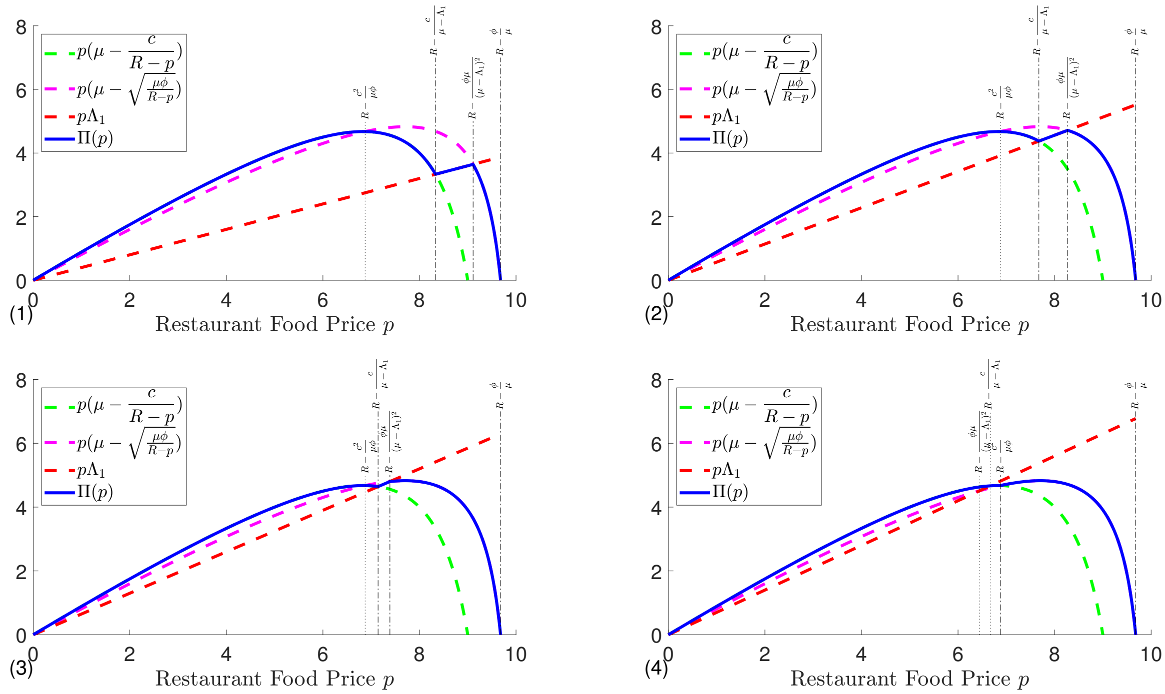


**Figure C.13** **The restaurant's profit $\Pi$ as a function of $p$, when $\Lambda_1 = 0.4$, $0.57$, $0.65$, and $0.7$, for the parameter setting $R = 10$, $\Lambda_0 = \mu = c = 1$, and $\phi = 0.32$.**

$2R\frac{(\mu-\chi)^2}{2\mu-\chi}$, as illustrated in Figure C.13(3). In this case, $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} \in \left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}, R - \frac{\phi}{\mu}\right]$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.

- If $\Lambda_1 > \frac{c-\phi}{c}\mu$, $\max_{p\in\left(0, R-\frac{c^2}{\mu\phi}\right]} p\left(\mu - \frac{c}{R-p}\right) \le \max_{p\in\left[R-\frac{c^2}{\mu\phi}, R-\frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \max_{p\in\left(0, R-\frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ $2R\frac{(\mu-\chi)^2}{2\mu-\chi}$, as illustrated in Figure C.13(4). In this case, $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} \in \left(R - \frac{c^2}{\mu\phi}, R - \frac{\phi}{\mu}\right]$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.

To summarize, the restaurant's maximum profit $\Pi^*$ and the profit-maximizing price $p^*$ are

| | $\Pi^*(\Lambda_1)$ | $p^*(\Lambda_1)$ |
|---|---|---|
| if $0 < \Lambda_1 \le \lambda_1$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $R - \sqrt{\frac{c}{\mu}R}$ |
| if $\lambda_1 < \Lambda_1 \le \lambda_2$ | $\Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ |
| if $\Lambda_1 > \lambda_2$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ |

By Lemma C.7(2), we have $p^*(\Lambda_1)|_{\Lambda_1>\lambda_2} = \frac{2R(\mu-\chi)}{2\mu-\chi} > p^*(\Lambda_1)|_{\Lambda_1\le\lambda_1} = R - \sqrt{\frac{c}{\mu}R}$ when $\phi \le \phi_1$. Then, because $p^*(\Lambda_1)|_{\lambda_1<\Lambda_1\le\lambda_2} = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ decreases in $\Lambda_1$, we have $p^*(\Lambda_1)|_{\lambda_1<\Lambda_1\le\lambda_2} = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} > p^*(\Lambda_1)|_{\Lambda_1\le\lambda_1} = R - \sqrt{\frac{c}{\mu}R}$. Thus, $p^*(\Lambda_1)|_{\Lambda_1\le\lambda_1} < p^*(\Lambda_1)|_{\Lambda_1>\lambda_1}$ when $\phi \le \phi_1$.

An observation here is that the intersection point of $p\lambda_1$ and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ is on the right-hand side of the maximum point of $p\left(\mu - \frac{c}{R-p}\right)$; i.e., $R - \sqrt{\frac{c}{\mu}R} < R - \frac{\mu\phi}{(\mu-\lambda_1)^2}$. Otherwise, if $R - \frac{\mu\phi}{(\mu-\lambda_1)^2} \le R - \sqrt{\frac{c}{\mu}R}$, from the definition of $\lambda_1$ we have $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) \le \left(\sqrt{R\mu} - \sqrt{c}\right)^2$ for $p \ge R - \frac{\mu\phi}{(\mu-\lambda_1)^2}$. This suggests that there is another intersection point of $p\left(\mu - \frac{c}{R-p}\right)$ and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ on the interval $\left[R - \frac{\mu\phi}{(\mu-\lambda_1)^2}, R - \sqrt{\frac{c}{\mu}R}\right]$. Recall that point $\left(R - \frac{\mu\phi}{(\mu-\lambda_1)^2}, \lambda_1\left(R - \frac{\mu\phi}{(\mu-\lambda_1)^2}\right)\right)$,

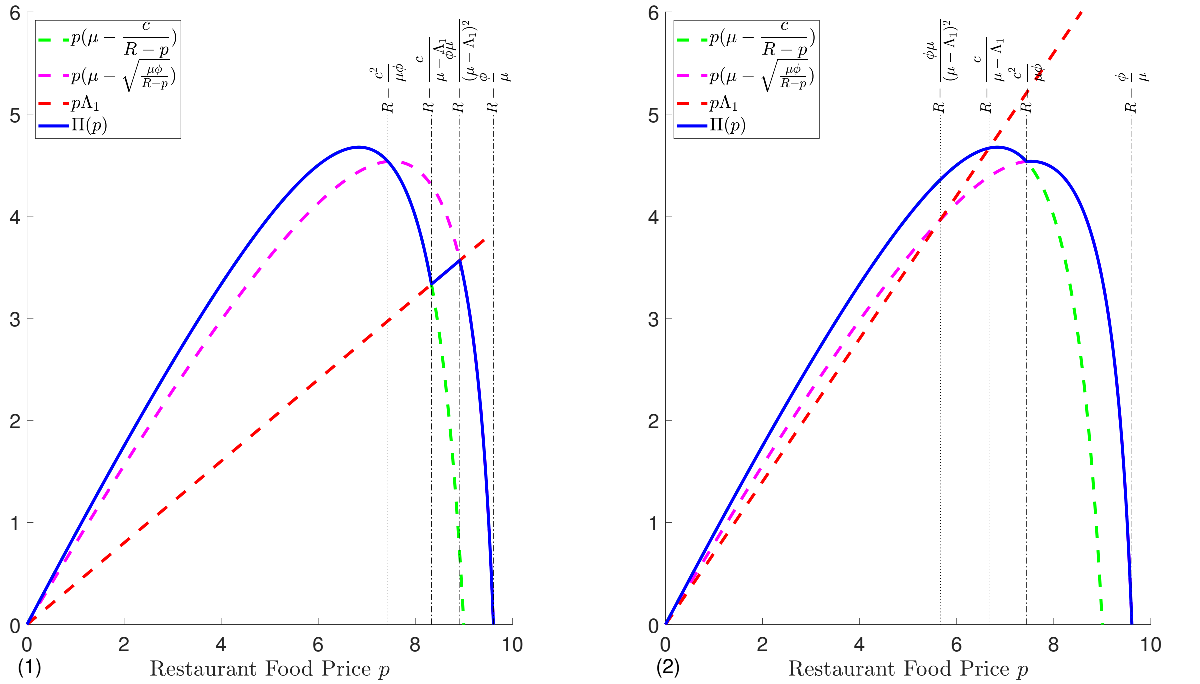


**Figure C.14** **The restaurant's profit $\Pi$ as a function of $p$, when $\Lambda_1 = 0.4$ and $0.7$, for the parameter setting $R = 10$, $\Lambda_0 = \mu = c = 1$, and $\phi = 0.39$.**

which is the intersection point of $p\lambda_1$ and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$, is on the right-hand side of the maximum point of $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$. This contradicts our result above that the maximum point of $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ is on the right-hand side of the intersection point of $p\left(\mu - \frac{c}{R-p}\right)$ and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$. Thus, we have $p^*\left(\Lambda_1\right)|_{\Lambda_1 \leq \lambda_1} = R - \sqrt{\frac{c}{\mu}R} < p^*\left(\Lambda_1\right)|_{\Lambda_1 \searrow \lambda_1} = R - \frac{\mu\phi}{(\mu-\lambda_1)^2}$.

Moreover, we note that if $\Lambda_1 \leq \lambda_1$, the restaurant's throughput is $\mu - \sqrt{\frac{c\mu}{R}}$, which is independent of $\phi$; if $\lambda_1 < \Lambda_1 \leq \lambda_2$, the restaurant's throughput is $\Lambda_1$; and if $\Lambda_1 > \lambda_2$, the restaurant's throughput is $\lambda_2 = \mu - \chi$, which decreases in $\phi$ by Lemma C.7(2). Solving $\mu - \sqrt{\frac{c\mu}{R}} = \lambda_2$ gives $\phi = \frac{c}{2\sqrt{\frac{\mu R}{c}} - 1} \leq \sqrt{\frac{c^3}{R\mu}} \leq \phi_1$. Thus, if $\phi < \frac{c}{2\sqrt{\frac{\mu R}{c}} - 1}$, we have $\mu - \sqrt{\frac{c\mu}{R}} < \lambda_2$, which implies that for $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, the delivery service increases the restaurant's throughput from $\mu - \sqrt{\frac{c\mu}{R}}$ to $\min\left(\Lambda_1, \lambda_2\right)$.

2. If $\phi > \phi_1 \Leftrightarrow \max\limits_{p \in \left(0, R - \frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = 2R\frac{(\mu-\chi)^2}{2\mu-\chi} < \max\limits_{p \in \left(0, R - \frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$. The maximum point of $p\left(\mu - \frac{c}{R-p}\right)$ is on the left-hand side of the intersection point of $p\left(\mu - \frac{c}{R-p}\right)$ and $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$; i.e., $R - \sqrt{\frac{c}{\mu}R} < R - \frac{c^2}{\mu\phi}$. Otherwise, if $R - \sqrt{\frac{c}{\mu}R} \geq R - \frac{c^2}{\mu\phi}$, we have $\left(\sqrt{R\mu} - \sqrt{c}\right)^2 \leq 2R\frac{(\mu-\chi)^2}{2\mu-\chi}$ by Lemma C.8(1), which contradicts $\phi > \phi_1$. Figure C.14 illustrates the restaurant's profit as a function of $p$ under different $\Lambda_1$.

Following the same discussion as that for the $\phi \leq \phi_1$ case, the restaurant's maximum profit $\Pi^*$ and the profit-maximizing price $p^*$ are $\Pi^* = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$ at $p^* = R - \sqrt{\frac{c}{\mu}R}$. Note that $\Pi^*\left(\Lambda_1\right)$ here is a constant regarding $\Lambda_1$.

At last, the restaurant's throughput $\lambda_D^* + \lambda_W^*$ can be readily derived using $\Pi^*(\Lambda_1)/p^*(\Lambda_1)$. $\square$

Given the restaurant's optimal food price $p^*(\Lambda_1)$ in Proposition C.4, we can use Proposition C.3 to obtain the platform's optimal delivery fee $\theta^*(p^*)$ and corresponding profit $\pi^*(p^*)$ in equilibrium of the Stackelberg game.

Corollary C.3 **(Food Delivery Platform's Profit)**. *There exist threshold values $\phi_1$, $\lambda_1$, and $\lambda_2$, such that the food delivery platform's equilibrium profit $\pi^*$, delivery fee $\theta^*$, the joining rates of food-delivery and walk-in customers $\lambda_D^*$ and $\lambda_W^*$ under the restaurant's equilibrium price $p^*$ are*

| | $\theta^*(p^*(\Lambda_1))$ | $\pi^*(p^*(\Lambda_1))$ | $\lambda_D^*$ | $\lambda_W^*$ |
|---|---|---|---|---|
| *When* $\phi \le \phi_1$ | | | | |
| *if* $0 < \Lambda_1 \le \lambda_1$ | $(c-\phi)\sqrt{\frac{R}{\mu c}}$ | $\Lambda_1(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $\Lambda_1$ | $\mu - \sqrt{\frac{c\mu}{R}} - \Lambda_1$ |
| *if* $\lambda_1 < \Lambda_1 \le \lambda_2$ | $\frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$ | $\frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}$ | $\Lambda_1$ | $0$ |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$ | $\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ | $\mu-\chi$ | $0$ |
| *When* $\phi > \phi_1$ | | | | |
| *if* $0 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $\Lambda_1(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $\Lambda_1$ | $\mu - \sqrt{\frac{c\mu}{R}} - \Lambda_1$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $(c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right)$ | $\mu - \sqrt{\frac{c\mu}{R}}$ | $0$ |

*where $\chi$ is the unique real root of $-R\chi^3 - \mu\phi\chi + 2\mu^2\phi = 0$ in $[0,\mu]$. Moreover, we have*

*(i) $\pi^*(p^*)|_{\Lambda_1 \le \lambda_1} > \pi^*(p^*)|_{\Lambda_1 \searrow \lambda_1}$ when $\phi \le \phi_1$.*

*(ii) $\pi^*(p^*)$ is a weakly increasing function of $\Lambda_1$ when $\phi > \phi_1$.*

*Proof of Corollary C.3.* (i) When $\phi \le \phi_1$, similar to the $\phi \le \phi_1$ case in Proposition C.4, we have $\lambda_1 \le \lambda_2 \le \frac{c-\phi}{c}\mu$.

- If $0 < \Lambda_1 \le \lambda_1$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R} \le R - \frac{c}{\mu - \Lambda_1}$ by Proposition C.4. From Proposition C.3(1), the food delivery platform's profit $\pi$ and profit maximizing delivery fee $\theta^*$ are $\pi(p^*) = \Lambda_1 \frac{(c-\phi)(R-p)}{c}\Big|_{p=R-\sqrt{\frac{c}{\mu}R}} = \Lambda_1(c-\phi)\sqrt{\frac{R}{c\mu}} = \Lambda_1\left(c - \frac{R\chi^3}{\mu(2\mu-\chi)}\right)\sqrt{\frac{R}{c\mu}}$ and $\theta^*(p^*) = (c-\phi)\sqrt{\frac{R}{\mu c}} = \left(c - \frac{R\chi^3}{\mu(2\mu-\chi)}\right)\sqrt{\frac{R}{\mu c}}$.
- If $\lambda_1 < \Lambda_1 \le \lambda_2 \le \frac{c-\phi}{c}\mu$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ by Proposition C.4. From Proposition C.3(1), the food delivery platform's profit $\pi$ and profit maximizing delivery fee $\theta^*$ are $\pi(p^*) = \Lambda_1\left(R - p - \frac{\phi}{\mu-\Lambda_1}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}} = \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2} = \frac{(\Lambda_1)^2R\chi^3}{\mu(2\mu-\chi)(\mu-\Lambda_1)^2}$ and $\theta^*(p^*) = \frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2} = \frac{\Lambda_1R\chi^3}{\mu(2\mu-\chi)(\mu-\Lambda_1)^2}$.
- If $\lambda_2 < \Lambda_1 \le \frac{c-\phi}{c}\mu$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} > R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ by Proposition C.4. From Proposition C.3(1), the food delivery platform's profit $\pi$ and profit maximizing delivery fee $\theta^*$ are $\pi(p^*) = \left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\left(R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}\right)\Big|_{p=\frac{2R(\mu-\chi)}{2\mu-\chi}} = \frac{\phi(\mu-\chi)^2}{\chi^2} = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ and $\theta^*(p^*) = \frac{\phi(\mu-\chi)}{\chi^2} = \frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$.

- If $\Lambda_1 > \frac{c-\phi}{c}\mu$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} > R - \frac{c^2}{\mu\phi}$ by Proposition C.4. From Proposition C.3(2), the food delivery platform's profit $\pi$ and profit maximizing delivery fee $\theta^*$ are $\pi(p^*) = \left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\left(R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}\right)\Big|_{p=\frac{2R(\mu-\chi)}{2\mu-\chi}} = \frac{\phi(\mu-\chi)^2}{\chi^2} = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ and $\theta^*(p^*) = \frac{\phi(\mu-\chi)}{\chi^2} = \frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$.

From Proposition C.3, it is easy to verify that when $\Lambda_1 \le \frac{c-\phi}{c}\mu$ the food delivery platform's profit $\pi = \theta^*\lambda_D$ decreases in the restaurant's price $p$ for $p \in \left[0, R - \frac{\phi\mu}{(\mu-\Lambda_1)^2}\right]$. When $\Lambda_1$ increases to $\lambda_1$, the restaurant's profit-maximizing price increases from $R - \sqrt{\frac{c}{\mu}R}$ to $R - \frac{\mu\phi}{(\mu-\lambda_1)^2}$ by Proposition C.4, so the food delivery platform's profit decreases. Thus, we have $\Lambda_1(c-\phi)\sqrt{\frac{R}{c\mu}}\Big|_{\Lambda_1=\lambda_1} > \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}\Big|_{\Lambda_1=\lambda_1}$. When $\phi = \phi_1$, we have $\frac{2R(\mu-\chi_1)^2}{2\mu-\chi_1} = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$, so $\lambda_1 = \lambda_2 = \mu - \chi_1$.

(ii) When $\phi > \phi_1$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R}$ by Proposition C.4.

Recall from Lemma C.8(2) that $p\Lambda_1$ intersects with $p\left(\mu - \frac{c}{R-p}\right)$ at $p = R - \sqrt{\frac{c}{\mu}R}$ when $\Lambda_1 = \mu - \sqrt{\frac{c\mu}{R}}$.

- If $0 < \Lambda_1 \le \frac{c-\phi}{c}\mu$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R} \le R - \frac{c}{\mu-\Lambda_1}$ by Proposition C.4. From Proposition C.3(1), the food delivery platform's profit is $\Lambda_1\frac{(c-\phi)(R-p)}{c}\Big|_{p=R-\sqrt{\frac{c}{\mu}R}} = \frac{\Lambda_1}{\mu}(c-\phi)\sqrt{\frac{R\mu}{c}}$.
- If $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R} \le R - \frac{c}{\mu-\Lambda_1}$ by Proposition C.4. From Proposition C.3(2), the food delivery platform's profit is $\Lambda_1\frac{(c-\phi)(R-p)}{c}\Big|_{p=R-\sqrt{\frac{c}{\mu}R}} = \frac{\Lambda_1}{\mu}(c-\phi)\sqrt{\frac{R\mu}{c}}$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, the restaurant's profit-maximizing price is $R - \frac{c}{\mu-\Lambda_1} < p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R} \le R - \frac{c^2}{\mu\phi}$ by Proposition C.4. From Proposition C.3(2), the food delivery platform's profit is $\left(\mu - \frac{c}{R-p}\right)\frac{(c-\phi)(R-p)}{c}\Big|_{p=R-\sqrt{\frac{c}{\mu}R}} = (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right)$.

Clearly, $\pi^*(p^*)$ is a weakly increasing function of $\Lambda_1$ when $\phi > \phi_1$.

Next, we investigate the food delivery platform's profit under the restaurant's profit-maximizing price $p^*$, $\pi(p^*)$, when the tech-savvy customers' arrival rate is sufficiently large; i.e., $\Lambda_1 \nearrow \mu$. If $\phi \le \phi_1$, we have $\pi(p^*) = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$. From the above discussion, we have $\arg\max_{\chi\in[0,\mu]}\pi(p^*) = \frac{3-\sqrt{5}}{2}\mu$. Correspondingly, $\phi = \frac{R\chi^3}{\mu(2\mu-\chi)}\Big|_{\chi=\frac{3-\sqrt{5}}{2}\mu} = \frac{13\sqrt{5}-29}{2}R\mu$ and $\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}\Big|_{\chi=\frac{3-\sqrt{5}}{2}\mu} = \frac{5\sqrt{5}-11}{2}R\mu$. If $\phi > \phi_1$, we have $\pi(p^*) = (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right)$ which clearly decreases in $\phi$.

At last, the joining rate of food-delivery customers $\lambda_D^*$ can be readily derived using $\pi^*(p^*)/\theta^*(p^*)$. The joining rate of walk-in customers $\lambda_W^*$ is the difference between $\lambda_D^* + \lambda_W^*$ in Proposition C.4 and $\lambda_D^*$. $\square$

### C.1.4. Social Welfare

We next investigate social welfare, which is defined as the sum of the restaurant's equilibrium profit in Proposition C.4 and the platform's equilibrium profit in Corollary C.3 under the restaurant's optimal food price $p^*(\Lambda_1)$ in Proposition C.4 and the food delivery platform's best-response delivery fee $\theta^*(p^*)$ in Corollary C.3. Note that customers have zero utility in equilibrium; otherwise, either the restaurant or the food delivery platform could raise the price without changing the throughput, which would lead to a higher profit.

COROLLARY C.4 **(Social Welfare)**. *Social welfare under the restaurant's optimal food price $p^*$ in Proposition C.4 and the food delivery platform's best-response delivery fee $\theta^*(p^*)$ in Corollary C.3 is*

| | $S^*(\Lambda_1)$ |
|---|---|
| *When* $\phi \le \phi_1$ | |
| *if* $0 < \Lambda_1 \le \lambda_1$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2 + \Lambda_1 (c-\phi)\sqrt{\frac{R}{c\mu}}$ |
| *if* $\lambda_1 < \Lambda_1 \le \lambda_2$ | $R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$ |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$ |
| *When* $\phi > \phi_1$ | |
| *if* $0 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2 + \Lambda_1 (c-\phi)\sqrt{\frac{R}{c\mu}}$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $\phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$ |

*where $\chi$ is the unique real root of $-R\chi^3 - \mu\phi\chi + 2\mu^2\phi = 0$ in $[0,\mu]$. Moreover, we have*

*(i) $S^*(\Lambda_1)|_{\Lambda_1 \le \lambda_1} > S^*(\Lambda_1)|_{\Lambda_1 \searrow \lambda_1}$ when $\phi \le \phi_1$.*

*(ii) $S^*(\Lambda_1)$ is a weakly increasing function of $\Lambda_1$ when $\phi > \phi_1$.*

*Proof of Corollary C.4.* From Proposition C.4 and Corollary C.3, we have

(i) When $\phi \le \phi_1$,

- If $0 < \Lambda_1 \le \lambda_1$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2 + \Lambda_1 (c-\phi)\sqrt{\frac{R}{c\mu}}$.
- If $\lambda_1 < \Lambda_1 \le \lambda_2 \le \frac{c-\phi}{c}\mu$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right) + \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2} = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\Lambda_1 > \lambda_2$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \frac{2R(\mu-\chi)^2}{2\mu-\chi} + \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)} = \frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$.

We have $\Pi^*(\Lambda_1)$ is a weakly increasing function of $\Lambda_1$ from Proposition C.4 and $\pi^*(p^*)|_{\Lambda_1 \le \lambda_1} > \pi^*(p^*)|_{\Lambda_1 \searrow \lambda_1}$ from Corollary C.3. When $\Lambda_1$ increases to $\lambda_1$, the restaurant's profit-maximizing price increases from $R - \sqrt{\frac{c}{\mu}R}$ to $R - \frac{\mu\phi}{(\mu-\lambda_1)^2}$ by Proposition C.4, the restaurant's profit stays constant while the platform's profit decreases, i.e., $\left(\sqrt{R\mu} - \sqrt{c}\right)^2 = \Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Big|_{\Lambda_1=\lambda_1}$ and $\Lambda_1 (c-\phi)\sqrt{\frac{R}{c\mu}}\Big|_{\Lambda_1=\lambda_1} > \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}\Big|_{\Lambda_1=\lambda_1}$. Therefore, we have $S(\Lambda_1)|_{\Lambda_1 \le \lambda_1} > S(\Lambda_1)|_{\Lambda_1 \searrow \lambda_1}$ when $\phi \le \phi_1$.

(ii) When $\phi > \phi_1$,

- If $0 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2 + \Lambda_1 (c-\phi)\sqrt{\frac{R}{c\mu}}$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2 + (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right) = \phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$.

From Proposition C.4 and Corollary C.3, we have $\Pi^*(\Lambda_1)$ and $\pi^*(p^*)$ are weakly increasing functions of $\Lambda_1$. Therefore, $S(\Lambda_1)$ is a weakly increasing function of $\Lambda_1$ when $\phi > \phi_1$.

We have Proposition 1 from Proposition C.4, Corollary C.3 and Corollary C.4. $\square$

### C.2. Proof of Lemma 1

Due to our unobservable queue assumption and customers' homogeneity in their service reward and marginal waiting cost, the centralized owner can extract all customer surplus as profit by setting the food price and delivery fee (see Chap. 3 of Hassin and Haviv 2003 for a single-segment problem). Thus, the optimal monopoly food price $p^o$ and delivery fee $\theta^o$ maximize not only the aggregated profit but also social welfare. Here, the centralized owner's goal of maximizing the aggregated profit aligns with a social planner's goal of maximizing social welfare.

We first derive (i) the socially optimal joining rates of food-delivery and walk-in customers, and (ii) the expected utility of food-delivery and that of walk-in customers under the socially optimal joining rates and zero food price and delivery fee.

Proposition C.5 **(Social Optimization)**. *The maximum social welfare and socially optimal joining rates of food-delivery and walk-in customers $\lambda_D^o$ and $\lambda_W^o$ are*

| | $S^o$ | $\lambda_D^o$ | $\lambda_W^o$ |
|---|---|---|---|
| *If* $0 < \Lambda_1 \le \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R}$ | $R\mu + c - 2\sqrt{R(c\mu - \Lambda_1(c-\phi))}$ | $\Lambda_1$ | $\mu - \sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}} - \Lambda_1$ |
| *If* $\frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} < \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\Lambda_1\left(R - \frac{\phi}{\mu - \Lambda_1}\right)$ | $\Lambda_1$ | 0 |
| *If* $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\left(\sqrt{R\mu} - \sqrt{\phi}\right)^2$ | $\mu - \sqrt{\frac{\phi\mu}{R}}$ | 0 |

*Moreover, we have:*

*(i) The optimal social welfare $S^o$ is a weakly increasing function of $\Lambda_1$.*

*(ii) The socially optimal throughput $\lambda_D^o + \lambda_W^o$ is a weakly increasing function of $\Lambda_1$.*

*Proof of Proposition C.5.* 1. When $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$, from Hassin and Haviv (2003), it is socially optimal for tech-savvy customers to join with rate $\mu - \sqrt{\frac{\phi\mu}{R}}$, and optimal social welfare is $\left(\sqrt{R\mu} - \sqrt{\phi}\right)^2$. If we can increase social welfare further by letting some walk-in customers join, then it would be more socially beneficial to switch these walk-in customers to tech-savvy customers, which contradicts the result from Hassin and Haviv (2003). In this case, the optimal social welfare $\left(\sqrt{R\mu} - \sqrt{\phi}\right)^2$ and the socially optimal throughput $\mu - \sqrt{\frac{\phi\mu}{R}}$ are both constant regarding $\Lambda_1$.

2. When $\Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$, all tech-savvy customers join using the food delivery service. Let $\lambda$ denote the total arrival rate, then $\lambda - \Lambda_1$ is the arrival rate of walk-in customers. Social welfare is

$$S(\lambda) = \Lambda_1\left(R - \frac{\phi}{\mu - \lambda}\right) + (\lambda - \Lambda_1)\left(R - \frac{c}{\mu - \lambda}\right)$$

$$= \Lambda_1 \frac{c-\phi}{\mu-\lambda} + \lambda\left(R - \frac{c}{\mu-\lambda}\right) \text{ for } \lambda \geq \Lambda_1,$$

whose first derivative is

$$\frac{\partial S(\lambda)}{\partial \lambda} = \frac{R(\lambda-\mu)^2 + \Lambda_1(c-\phi) - c\mu}{(\lambda-\mu)^2}.$$

Clearly, we have $\frac{\partial S}{\partial \lambda} \leq 0$ if $\lambda \leq \lambda^o = \mu - \sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}}$, and $\frac{\partial S}{\partial \lambda} > 0$ if $\lambda > \lambda^o$. Hence, social welfare is a unimodal function with a unique maximum at $\lambda^o$.

2.1 When $\Lambda_1 \leq \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} \Leftrightarrow \lambda^o \geq \Lambda_1$, it is socially optimal to have $\lambda_D = \Lambda_1$ and $\lambda_W = \lambda^o - \Lambda_1 = \mu - \sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}} - \Lambda_1$. Here, the throughput is $\lambda_D + \lambda_W = \mu - \sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}}$, which clearly increases in $\Lambda_1$. The optimal social welfare is

$$S(\lambda^o) = R\mu + c - 2\sqrt{R(c\mu - \Lambda_1(c-\phi))},$$

which is clearly an increasing function of $\Lambda_1$.

2.2 When $\frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} \leq \Lambda_1 \leq \mu - \sqrt{\frac{\phi\mu}{R}} \Leftrightarrow R(\Lambda_1)^2 + (c - \phi - 2R\mu)\Lambda_1 + \mu(R\mu - c) \leq 0 \Leftrightarrow \lambda^o \leq \Lambda_1$, which is true because $R(\Lambda_1)^2 + (c - \phi - 2R\mu)\Lambda_1 + \mu(R\mu - c)$ is a quadratic equation and $R(\Lambda_1)^2 + (c - \phi - 2R\mu)\Lambda_1 + \mu(R\mu - c)\Big|_{\Lambda_1 = \mu - \sqrt{\frac{\phi\mu}{R}}} = -(c-\phi)\sqrt{\frac{\phi\mu}{R}} < 0$, it is socially optimal to have $\lambda_D = \Lambda_1$ and $\lambda_W = 0$. In this case, the throughput is $\Lambda_1$, which clearly increases in $\Lambda_1$. The optimal social welfare is $\Lambda_1\left(R - \frac{\phi}{\mu - \Lambda_1}\right)$, which is a unimodal function with maximum point $\Lambda_1^* = \mu - \sqrt{\frac{\phi\mu}{R}}$; hence, it is an increasing function of $\Lambda_1$ when $\Lambda_1 \leq \mu - \sqrt{\frac{\phi\mu}{R}}$. $\square$

Next we study how to achieve the socially optimal joining rates of food-delivery and walk-in customers, $\lambda_D^o$ and $\lambda_W^o$, characterized in Proposition C.5. Let $p^o$ and $\theta^o$ denote the optimal monopoly food price and delivery fee that induce the socially optimal joining rates $\lambda_D^o$ and $\lambda_W^o$.

We first discuss the $\Lambda_1 \leq \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R}$ case, where both tech-savvy customers and traditional customers join. Our intuition suggests that all customers should expect zero utility when $p^o$ and $\theta^o$ are offered; otherwise, if any customers expect a positive utility, these customers have the incentive to join the system more often, or the food price or delivery fee should be increased when all customers have already joined the system. Following this rationale, we set $p = \theta = 0$ and $\lambda = \lambda_D^o + \lambda_W^o$, where $\lambda_D^o$ and $\lambda_W^o$ are from Proposition C.5, in (C.1) and (C.2) to obtain the utility of food-delivery customers $U_D(\lambda_D^o, \lambda_W^o)$ and that of walk-in customers $U_W(\lambda_D^o, \lambda_W^o)$ under the socially optimal joining rates.

Corollary C.5. *The expected utility of food-delivery and that of walk-in customers under the socially optimal joining rates and* $p = \theta = 0$ *are*

| | $U_D(\lambda_D^o,\lambda_W^o)$ | $U_W(\lambda_D^o,\lambda_W^o)$ |
|---|---|---|
| *If* $0<\Lambda_1\le\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$ | $R-\frac{\phi}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}$ | $R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}$ |
| *If* $\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}<\Lambda_1\le\mu-\sqrt{\frac{\phi\mu}{R}}$ | $R-\frac{\phi}{\mu-\Lambda_1}$ | $R-\frac{c}{\mu-\Lambda_1}$ |
| *If* $\Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}$ | $R-\phi\sqrt{\frac{R}{\phi\mu}}$ | $R-c\sqrt{\frac{R}{\phi\mu}}$ |

We expect that the centralized owner can set the food price and delivery fee as $p^o=U_W(\lambda_D^o,\lambda_W^o)=R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}$ and $\theta^o=U_D(\lambda_D^o,\lambda_W^o)-U_W(\lambda_D^o,\lambda_W^o)=\frac{c-\phi}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}$ where $U_D(\lambda_D^o,\lambda_W^o)$ and $U_W(\lambda_D^o,\lambda_W^o)$ are given in Corollary C.5, to induce the socially optimal joining rates $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5 and extract all the surpluses from customers. Here, we have $p^o=R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\le R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow R(\mu-\Lambda_1)^2-(c-\phi)(\mu-\Lambda_1)-\phi\mu\ge 0$, which is clearly true since $\Lambda_1\le\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$. By Proposition C.3, the platform's best response is $\theta^o=\frac{(c-\phi)(R-p^o)}{c}=\frac{c-\phi}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}$. Also, by Lemma C.2, we have, under food price $p^o$ and delivery fee $\theta^o$, customers' joining rates are $\lambda_D=\Lambda_1$ and $\lambda_W=\mu-\frac{c}{R-p^o}-\Lambda_1=\mu-\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}-\Lambda_1$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5.

We next discuss the $\Lambda_1>\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$ case. From Proposition C.5, all traditional customers balk, the centralized owner operates a system that only serves the tech-savvy customers. We expect that the solution of $p+\theta^*(p)=p_m$, where $\theta^*(p)$ is the platform's best response delivery fee from Proposition D.7 and

$$p_m=\begin{cases} R-\frac{\phi}{\mu-\Lambda_1} & \text{if } \Lambda_1\le\mu-\sqrt{\frac{\phi\mu}{R}}\\ R-\sqrt{\frac{\phi R}{\mu}} & \text{if } \Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}\end{cases}.$$

is the social optimal price in an unobservable M/M/1 with waiting cost $\phi$ (see, e.g., Hassin and Haviv (2003)), is the socially optimal food price $p^o$, and the platform's best response to $p^o$ is $\theta^o$; i.e., $\theta^*(p^o)=\theta^o$. We next verify this conjecture.

From Proposition C.3, we have the sum of food price $p$ and platform's best response delivery fee $\theta^*(p)$:

1. For $\Lambda_1\le\frac{c-\phi}{c}\mu$,

| | $p+\theta^*(p)$ | |
|---|---|---|
| If $p\le R-\frac{c}{\mu-\Lambda_1}$ | $R-\frac{\phi}{c}(R-p)$ | $\in\left[\frac{c-\phi}{c}R,R-\frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R-\frac{c}{\mu-\Lambda_1}<p\le R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $R-\frac{\phi}{\mu-\Lambda_1}$ | $\in\left(R-\frac{\phi}{\mu-\Lambda_1},R-\frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}<p\le R-\frac{\phi}{\mu}$ | $R-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\in\left(R-\frac{\phi}{\mu-\Lambda_1},R-\frac{\phi}{\mu}\right]$ |

2. For $\Lambda_1>\frac{c-\phi}{c}\mu$,

| | $p+\theta^*(p)$ | |
|---|---|---|
| If $p\le R-\frac{c}{\mu-\Lambda_1}$ | $R-\frac{\phi}{c}(R-p)$ | $\in\left[\frac{c-\phi}{c}R,R-\frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R-\frac{c}{\mu-\Lambda_1}<p\le R-\frac{c^2}{\mu\phi}$ | $R-\frac{\phi}{c}(R-p)$ | $\in\left(R-\frac{\phi}{\mu-\Lambda_1},R-\frac{c}{\mu}\right]$ |
| If $R-\frac{c^2}{\mu\phi}<p\le R-\frac{\phi}{\mu}$ | $R-\frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\in\left(R-\frac{c}{\mu},R-\frac{\phi}{\mu}\right]$ |

When $\phi \le \frac{c^2}{R\mu}$, we have $\mu - \sqrt{\frac{\phi\mu}{R}} \le \frac{c-\phi}{c}\mu$, $R - \sqrt{\frac{\phi R}{\mu}} \ge R - \frac{c}{\mu}$, and $R - \frac{c^2}{\mu\phi} \le 0$.

- If $\frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} < \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$, the solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\phi}{c}(R-p) = R - \frac{\phi}{\mu - \Lambda_1}$ is $p^o = R - \frac{c}{\mu-\Lambda_1}$. Also, by Proposition C.3, we have $\theta^o = \theta^*(p^o) = \frac{(c-\phi)(R-p^o)}{c} = \frac{c-\phi}{\mu-\Lambda_1}$. By Lemma C.2, we have $\lambda_D = \Lambda_1$ and $\lambda_W = \mu - \frac{c}{R-p^o} - \Lambda_1 = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5.
- If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$, we have $R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$, $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < 0$, and $R - \frac{c}{\mu-\Lambda_1} < 0$. The solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\sqrt{\mu\phi(R-p)}}{\mu} = R - \sqrt{\frac{\phi R}{\mu}}$ is $p^o = 0$. By Proposition C.3, the food delivery platform's best response is $\theta^o = R - p^o - \frac{\sqrt{\mu\phi(R-p^o)}}{\mu} = R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$. By Lemma C.2, we have $\lambda_D = \mu - \frac{\phi}{R-p^o-\theta^o} = \mu - \sqrt{\frac{\mu\phi}{R}}$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5.

When $\phi > \frac{c^2}{R\mu}$, we have $\frac{c-\phi}{c}\mu < \mu - \sqrt{\frac{\phi\mu}{R}}$, and $R - \sqrt{\frac{\phi R}{\mu}} < R - \frac{c}{\mu}$.

- If $\frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} < \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$, the solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\phi}{c}(R-p) = R - \frac{\phi}{\mu - \Lambda_1}$ is $p^o = R - \frac{c}{\mu-\Lambda_1}$. Also, by Proposition C.3, we have $\theta^o = \theta^*(p^o) = \frac{(c-\phi)(R-p^o)}{c} = \frac{c-\phi}{\mu-\Lambda_1}$. By Lemma C.2, we have $\lambda_D = \Lambda_1$ and $\lambda_W = \mu - \frac{c}{R-p^o} - \Lambda_1 = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5.
- If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$, we have $R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$. The solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\phi}{c}(R-p) = R - \sqrt{\frac{\phi R}{\mu}}$ is $p^o = R - c\sqrt{\frac{R}{\phi\mu}} \in \left(R - \frac{c}{\mu-\Lambda_1}, R - \frac{c^2}{\mu\phi}\right)$. By Proposition C.3(2), the food delivery platform's best response is $\theta^o = \frac{(c-\phi)(R-p^o)}{c} = (c-\phi)\sqrt{\frac{R}{\phi\mu}} > R - p^o - \frac{\phi}{\mu-\Lambda_1} = c\sqrt{\frac{R}{\phi\mu}} - \frac{\phi}{\mu-\Lambda_1}$. By Lemma C.2, we have $\lambda_D = \mu - \frac{\phi}{R-p^o-\theta^o} = \mu - \sqrt{\frac{\mu\phi}{R}}$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5.

We expect that the centralized owner of the food service chain can set the food price and delivery fee as

$$p^o = \begin{cases} R - \frac{c}{\sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}}} & \text{if } 0 < \Lambda_1 \le \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} \\ R - \frac{c}{\mu - \Lambda_1} & \text{if } \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} < \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}} \\ 0 & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi \le \frac{c^2}{R\mu} \\ R - c\sqrt{\frac{R}{\phi\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi > \frac{c^2}{R\mu} \end{cases} \tag{C.3}$$

and

$$\theta^o = \begin{cases} \frac{c-\phi}{\sqrt{\frac{c\mu - \Lambda_1(c-\phi)}{R}}} & \text{if } 0 < \Lambda_1 \le \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} \\ \frac{c-\phi}{\mu - \Lambda_1} & \text{if } \frac{2R\mu - c + \phi - \sqrt{4R\mu\phi + (c-\phi)^2}}{2R} < \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}} \\ R - \sqrt{\frac{\phi R}{\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi \le \frac{c^2}{R\mu} \\ (c-\phi)\sqrt{\frac{R}{\phi\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi > \frac{c^2}{R\mu} \end{cases}, \tag{C.4}$$

to induce the socially optimal joining rates $\lambda_D^o$ and $\lambda_W^o$ in Proposition C.5 and extract all the surpluses from customers.

It is easy to verify that $p^o$ and $p^o+\theta^o$ are weakly decreasing functions of $\Lambda_1$, and that $\theta^o$ is a weakly increasing function of $\Lambda_1$.

The restaurant's corresponding profit is

| | $\Pi^o(\Lambda_1)=p^o(\lambda_D^o+\lambda_W^o)$ |
|---|---|
| If $0<\Lambda_1\le\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$ | $\left(R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\right)\left(\mu-\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}\right)$ |
| If $\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}<\Lambda_1\le\mu-\sqrt{\frac{\phi\mu}{R}}$ | $\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $\Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}$ and $\phi\le\frac{c^2}{R\mu}$ | 0 |
| If $\Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}$ and $\phi>\frac{c^2}{R\mu}$ | $\left(R-c\sqrt{\frac{R}{\phi\mu}}\right)\left(\mu-\sqrt{\frac{\mu\phi}{R}}\right)$ |

- We consider

$$\begin{aligned}&\left(R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\right)\left(\mu-\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}\right)\\ &=R\mu+c-c\mu\left(\frac{R}{c\mu}\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}+\frac{1}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\right)\\ &=R\mu+c-c\mu\left(\frac{R}{c\mu}x+\frac{1}{x}\right)\qquad(\text{by } x=\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}\le\sqrt{\frac{c\mu}{R}})\end{aligned}$$

whose first derivative is

$$\frac{\partial\left(R\mu+c-c\mu\left(\frac{R}{c\mu}x+\frac{1}{x}\right)\right)}{\partial x}=\frac{c\mu-Rx^2}{x^2}\ge 0.$$

Thus, $\left(R-\frac{c}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\right)\left(\mu-\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}\right)$ is an increasing function of $x$ and a decreasing function of $\Lambda_1$.

- Next we consider $\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1$, whose first derivative is

$$\frac{\partial\left(\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1\right)}{\partial\Lambda_1}=\frac{R(\mu-\Lambda_1)^2-\mu c}{(\mu-\Lambda_1)^2},$$

which is negative if $\Lambda_1\ge\mu-\sqrt{\frac{\mu c}{R}}$. We can prove $\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}\ge\mu-\sqrt{\frac{\mu c}{R}}$ by showing that $\left.\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}\right|_{\phi=c}=\mu-\sqrt{\frac{\mu c}{R}}$ and that $\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$ decreases in $\phi$, which can be proved by simple algebra. Thus, $\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ is a decreasing function of $\Lambda_1$.

- Finally, 0 and $\left(R-c\sqrt{\frac{R}{\phi\mu}}\right)\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)$ are constants regarding $\Lambda_1$.

Thus, $p^o(\lambda_D^o+\lambda_W^o)$ is a weakly decreasing function of $\Lambda_1$.

The platform's corresponding profit is

| | $\pi^o\left(p^o\left(\Lambda_1\right)\right)=\theta^o\lambda_D^o$ |
|---|---|
| If $0<\Lambda_1\leq\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}$ | $\frac{c-\phi}{\sqrt{\frac{c\mu-\Lambda_1(c-\phi)}{R}}}\Lambda_1$ |
| If $\frac{2R\mu-c+\phi-\sqrt{4R\mu\phi+(c-\phi)^2}}{2R}<\Lambda_1\leq\mu-\sqrt{\frac{\phi\mu}{R}}$ | $\frac{c-\phi}{\mu-\Lambda_1}\Lambda_1$ |
| If $\Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}$ and $\phi\leq\frac{c^2}{R\mu}$ | $\left(R-\sqrt{\frac{\phi R}{\mu}}\right)\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)$ |
| If $\Lambda_1>\mu-\sqrt{\frac{\phi\mu}{R}}$ and $\phi>\frac{c^2}{R\mu}$ | $(c-\phi)\sqrt{\frac{R}{\phi\mu}}\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)$ |

which is clearly a weakly increasing function of $\Lambda_1$. $\square$

### C.3. Proof of Proposition 2

We discuss the one-way and two-way RS contracts separately.

1. We first discuss the one-way RS contract with a price ceiling. In a decentralized system under the platform's best-response delivery fee $\theta^*(p)$, social welfare can be derived as

1.1 For $\Lambda_1\leq\frac{c-\phi}{c}\mu$,

| | $\Pi(p)+\pi^*(p)$ |
|---|---|
| If $p\leq R-\frac{c}{\mu-\Lambda_1}$ | $R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}$ |
| If $R-\frac{c}{\mu-\Lambda_1}<p\leq R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\left(R-\frac{\phi}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}<p\leq R-\frac{\phi}{\mu}$ | $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ |

1.2 For $\Lambda_1>\frac{c-\phi}{c}\mu$,

| | $\Pi(p)+\pi^*(p)$ |
|---|---|
| If $p\leq R-\frac{c}{\mu-\Lambda_1}$ | $R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}$ |
| If $R-\frac{c}{\mu-\Lambda_1}<p\leq R-\frac{c^2}{\mu\phi}$ | $R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ |
| If $R-\frac{c^2}{\mu\phi}<p\leq R-\frac{\phi}{\mu}$ | $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ |

We first derive

- $\frac{\partial\left(R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}\right)}{\partial p}=\mu-\left(1-\frac{\phi}{c}\right)\Lambda_1-R\frac{c}{(R-p)^2}$, which is zero when $p=R-c\sqrt{\frac{R}{c\mu-(c-\phi)\Lambda_1}}$; $R-c\sqrt{\frac{R}{c\mu-(c-\phi)\Lambda_1}}\leq R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow\Lambda_1\leq\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}$; and $\frac{\partial^2\left(R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}\right)}{\partial p^2}=\frac{-2Rc}{(R-p)^3}<0$.
- $\frac{\partial\left(R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)\right)}{\partial p}=\frac{\mu\phi}{c}-\frac{Rc}{(R-p)^2}$, which is zero when $p=R-c\sqrt{\frac{R}{\mu\phi}}$; $R-c\sqrt{\frac{R}{\mu\phi}}\leq R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow\Lambda_1\leq\mu-\sqrt{\frac{\mu\phi}{R}}$; $R-c\sqrt{\frac{R}{\mu\phi}}<R-\frac{c^2}{\mu\phi}\Leftrightarrow\phi>\frac{c^2}{R\mu}$, and $\frac{\partial^2\left(R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)\right)}{\partial p^2}=\frac{-2Rc}{(R-p)^3}<0$.
- $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases in the food price $p$, because $\frac{\partial\left(R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)\right)}{\partial p}=-\frac{p\sqrt{\mu\phi}}{2(R-p)^{\frac{3}{2}}}<0$.

It is clear that $\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}\leq\mu-\sqrt{\frac{\mu\phi}{R}}$.

- When $\Lambda_1\leq\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}\leq\mu-\sqrt{\frac{\mu\phi}{R}}$, we have

  — $R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}$ is a unimodal function on $p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]$ with the maximum at $p=R-c\sqrt{\frac{R}{c\mu-\Lambda_1(c-\phi)}}$;

—$R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ decreases with $p$ for $p\in\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]$;

—$R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases with $p$.

Thus, $\Pi(p)+\pi^*(p)$ is a unimodal function of $p$ with the maximum at $p^o=R-c\sqrt{\frac{R}{c\mu-\Lambda_1(c-\phi)}}$.

- When $\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}\le\Lambda_1\le\mu-\sqrt{\frac{\mu\phi}{R}}$, we have

—$R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}$ is an increasing function of $p$ for $p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]$;

—$R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ decreases with $p$ for $p\in\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]$;

—$R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases with $p$.

Thus, $\Pi(p)+\pi^*(p)$ is a unimodal function of $p$ with the maximum at $p^o=R-\frac{c}{\mu-\Lambda_1}$.

- When $\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}\le\mu-\sqrt{\frac{\mu\phi}{R}}\le\Lambda_1$ and $\phi\le\frac{c^2}{R\mu}$, we have $\mu-\sqrt{\frac{\phi\mu}{R}}\le\frac{c-\phi}{c}\mu$. Clearly, $\Pi(p)+\pi^*(p)$ is a constant for $p\in[0,p^o]$ where $p^o=0$.

- When $\mu-\frac{(c-\phi)+\sqrt{(c-\phi)^2+4R\mu\phi}}{2R}\le\mu-\sqrt{\frac{\mu\phi}{R}}\le\Lambda_1$ and $\phi>\frac{c^2}{R\mu}$, we have $\frac{c-\phi}{c}\mu<\mu-\sqrt{\frac{\phi\mu}{R}}$.

—$R\mu+c+\left(\left(1-\frac{\phi}{c}\right)\Lambda_1-\mu\right)(R-p)-\frac{Rc}{R-p}$ is an increasing function of $p$ for $p\in\left(0,R-\frac{c}{\mu-\Lambda_1}\right]$;

—$R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ is a unimodal function of $p$ with the maximum at $p=R-c\sqrt{\frac{R}{\mu\phi}}$ for $p\in\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]$;

—$R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases with $p$.

Thus, $\Pi(p)+\pi^*(p)$ is a unimodal function of $p$ with the maximum at $p^o=R-c\sqrt{\frac{R}{\mu\phi}}$.

In all cases, $p^o$ here matches the result in Lemma 1, and if the RS contract sets a price ceiling at $p^o$, the restaurant will increase the food price to $p^o$ to maximize its profit.

When the platform shares $\gamma_1$ fraction of its profit with the restaurant, the restaurant's profit is $\Pi(p)+\gamma_1\pi^*(p)=(\Pi(p)+\pi^*(p))-(1-\gamma_1)\pi^*(p)$. We have shown above that $\Pi(p)+\pi^*(p)$ increases in $p$ for $p\in[0,p^o]$, and $\pi^*(p)$ decreases in $p$ by Proposition C.3. Thus, the restaurant's profit $\Pi(p)+\gamma_1\pi^*(p)$ increases for $p\in[0,p^o]$, which means that the restaurant will set the food price $p=p^o$ to maximize its profit. By Lemma 1(i), given the food price $p^o$, the platform's best-response delivery fee is $\theta^o$. Hence, the price-ceiling one-way revenue-sharing contract proposed in Proposition 2 successfully induces the restaurant and the platform to behave in the socially optimal manner for $\forall\gamma_1\in[0,1]$.

For the decentralized system, we have the restaurant's profit $\Pi^*(\Lambda_1)$ from Proposition C.4, the platform's profit $\pi^*(p^*(\Lambda_1))$ from Corollary C.3, and social welfare $S^*(\Lambda_1)$ from Corollary C.4 in equilibrium. Note that $\Pi^*(\Lambda_1)$ and $\pi^*(p^*(\Lambda_1))$ are the minimum profits the restaurant and the delivery platform target. In the centralized system, under the optimal monopoly food price $p^o$ and delivery fee $\theta^o$, we have the restaurant's profit $\Pi^o(\Lambda_1)$, the platform's profit $\pi^o(\Lambda_1)$, and social welfare $S^o(\Lambda_1)$ from Lemma 1. The range of sharing fraction

$$\gamma_1(\Lambda_1)\in\left[\frac{\Pi^*(\Lambda_1)-\Pi^o(\Lambda_1)}{\pi^o(p^o(\Lambda_1))},1-\frac{\pi^*(p^*(\Lambda_1))}{\pi^o(p^o(\Lambda_1))}\right]\tag{C.5}$$

makes sure that the restaurant and the platform can reach a win-win.

Specifically, in the decentralized system, when the demand rate of tech-savvy customers is sufficiently large, i.e., $\Lambda_1 \geq \mu$, the restaurant's profit is $\Pi^* = \begin{cases} \frac{2R(\mu-\chi)^2}{2\mu-\chi} & \text{if } \phi \leq \phi_1 \\ \left(\sqrt{R\mu}-\sqrt{c}\right)^2 & \text{if } \phi > \phi_1 \end{cases}$ and the platform's profit is $\pi^* = \begin{cases} \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)} & \text{if } \phi \leq \phi_1 \\ (c-\phi)\left(\sqrt{\frac{R\mu}{c}}-1\right) & \text{if } \phi > \phi_1 \end{cases}$, which are the minimum profits the restaurant and the delivery platform aim at. In the centralized system, as Lemma 1 suggests, the maximum aggregated total profit the service system can obtain is $S^o = \left(\sqrt{R\mu}-\sqrt{\phi}\right)^2$, and the food sales and delivery profit under the socially optimal food price $p^o$ and delivery fee $\theta^o$ are $p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)$ and $\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2 - p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)$, respectively. Hence, the range of sharing fraction

$$\gamma_1 \in \begin{cases} \left[\frac{\frac{2R(\mu-\chi)^2}{2\mu-\chi}-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}, 1-\frac{\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}\right] & \text{if } \phi \leq \phi_1 \\ \left[\frac{\left(\sqrt{R\mu}-\sqrt{c}\right)^2-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}, 1-\frac{(c-\phi)\left(\sqrt{\frac{R\mu}{c}}-1\right)}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2-p^o\left(\mu-\sqrt{\frac{\phi\mu}{R}}\right)}\right] & \text{if } \phi > \phi_1 \end{cases},$$

where $\chi$ is given in Proposition C.4, makes sure that the restaurant and the platform can reach a win-win.

2. The two-way revenue-sharing contract turns the restaurant's profit function into an affine transformation of the aggregated profit of the whole service system. Thus, the restaurant will set the food price as the socially optimal one $p^o$. Then, from Lemma 1, the food delivery platform's best response is to set the delivery fee as $\theta^o$. This contract coordinates the whole system in the socially optimal fashion. From a discussion similar to the one of the price-ceiling one-way RS contract, the range of sharing fraction

$$\gamma_2(\Lambda_1) \in \left[\frac{\Pi^*(\Lambda_1)}{S^o(\Lambda_1)}, 1-\frac{\pi^*(p^*(\Lambda_1))}{S^o(\Lambda_1)}\right] \tag{C.6}$$

makes sure that the restaurant and the platform can reach a win-win. In the extreme case $\Lambda_1 \geq \mu$, we have

$$\gamma_2 \in \begin{cases} \left[\frac{\frac{2R(\mu-\chi)^2}{2\mu-\chi}}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2}, 1-\frac{\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2}\right] & \text{if } \phi \leq \phi_1 \\ \left[\frac{\left(\sqrt{R\mu}-\sqrt{c}\right)^2}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2}, 1-\frac{(c-\phi)\left(\sqrt{\frac{R\mu}{c}}-1\right)}{\left(\sqrt{R\mu}-\sqrt{\phi}\right)^2}\right] & \text{if } \phi > \phi_1 \end{cases},$$

where $\chi$ is given in Proposition C.4, makes sure that the restaurant and the platform can reach a win-win. $\square$

### C.4. Proof of Proposition 3

Under the uniformly distributed opportunity cost assumption, when the food delivery platform sets the delivery wage at $w \in [0, \beta]$ per unit of time, the expected supply of delivery workers is

$$\nu(w) = N \cdot F(w) = \frac{w}{\beta} N. \tag{C.7}$$

REMARK 1. For analytical convenience, we use an hourly rate for the delivery wage; see also Cui et al. (2020). One could also use a piece rate instead of an hourly rate for the delivery wage. Say the platform sets the delivery wage as $l$ per order. When the demand for the food delivery service is less than the supply of delivery workers, each delivery worker has equal probability of being assigned a food delivery order. Then, the supply of delivery workers $v(l)$ satisfies $v(l) = \frac{N}{\beta}\frac{l\min(\lambda_D, v(l))}{v(l)}$, with solution $v(l) = \begin{cases} \frac{N}{\beta}l & \text{if } l \le \frac{\beta\lambda_D}{N} \\ \sqrt{\frac{N}{\beta}l\lambda_D} & \text{if } l > \frac{\beta\lambda_D}{N} \end{cases}$. This expression of delivery-worker supply using a piece rate is more complicated than that using an hourly rate. We focus on a parsimonious setting with delivery workers getting an hourly rate to generate the supply of delivery workers.

The total opportunity cost of $\nu(w)$ delivery workers is $\frac{1}{2}w \cdot \nu(w)$. Thus, under delivery fee $\theta$ and delivery wage $w$, the platform's profit is

$$\pi(p,\theta,w) = \theta \cdot \min(\lambda_D, \nu(w)) - w \cdot \nu(w) = \theta \cdot \min\left(\lambda_D, \frac{w}{\beta}N\right) - \frac{w^2}{\beta}N, \tag{C.8}$$

and the delivery workers' total utility is

$$u_D(\theta,w) = w \cdot \nu(w) - N\int_0^w xf(x)\,\mathrm{d}x = \frac{1}{2}\frac{w^2}{\beta}N. \tag{C.9}$$

When the supply of delivery workers is less than the tech-savvy customers' unconstrained demand for the food delivery service, i.e., $v(w) < \lambda_D$, the number of tech-savvy customers who walk in by themselves depends on the comparison of the delivery-worker supply $v(w)$ and the join-up-to level $\mu - \frac{c}{R-p}$, in the classical unobservable queue with only traditional customers. If $v(w) < \mu - \frac{c}{R-p}$, the tech-savvy customers will walk in with rate $\min\left(\lambda_D, \mu - \frac{c}{R-p}\right) - v(w)$; otherwise, if $v(w) \ge \mu - \frac{c}{R-p}$, no tech-savvy customers will walk in. With both cases combined, the joining rate of walk-in tech-savvy customers is $\lambda_{1W}(\theta) = \max\left(0, \min\left(\lambda_D, \mu - \frac{c}{R-p}\right) - v(w)\right)$.

The food delivery platform should not hire more delivery workers than required by the tech-savvy customers' unconstrained demand for the food delivery service, $\lambda_D$, i.e., $\nu(w) \le \lambda_D \Leftrightarrow w \le \frac{\beta\lambda_D}{N}$. Otherwise, the food delivery platform can reduce the delivery wage while it still manages to fulfill all demand, which increases its profit.

LEMMA C.9. *Given the delivery fee $\theta$, the equilibrium delivery wage is*

$$w^*(\theta) = \begin{cases} \frac{\theta}{2} & \text{if } \theta \le \frac{2\beta\lambda_D}{N} \\ \frac{\beta\lambda_D}{N} & \text{if } \theta > \frac{2\beta\lambda_D}{N} \end{cases}.$$

*Moreover, under the equilibrium wage $w^*(\theta)$, the food delivery platform's profit is*

$$\pi(p,\theta,w^*(\theta)) = \begin{cases} \frac{N}{4\beta}\theta^2 & \text{if } \theta \le \frac{2\beta\lambda_D}{N} \\ \theta\lambda_D - \frac{\beta}{N}\lambda_D^2 & \text{if } \theta > \frac{2\beta\lambda_D}{N} \end{cases},$$

*and the joining rates of food-delivery and walk-in tech-savvy customers, $\lambda_D^*(\theta)$ and $\lambda_{1W}^*(\theta)$, are*

$$\lambda_D^*(\theta) = \begin{cases} \frac{\theta}{2\beta}N & \text{if } \theta \le \frac{2\beta\lambda_D}{N} \\ \lambda_D & \text{if } \theta > \frac{2\beta\lambda_D}{N} \end{cases} \text{ and } \lambda_{1W}^*(\theta) = \begin{cases} \max\left(0, \min\left(\lambda_D, \mu - \frac{c}{R-p}\right) - \frac{\theta}{2\beta}N\right) & \text{if } \theta \le \frac{2\beta\lambda_D}{N} \\ 0 & \text{if } \theta > \frac{2\beta\lambda_D}{N} \end{cases}.$$

*Proof of Lemma C.9.* First, given any delivery fee $\theta$, $\pi(p,\theta,w)$ as a function of $w$ in (C.8) can be written as

$$\pi(p,\theta,w)=\begin{cases}\frac{N}{\beta}\left(\theta w-w^2\right) & \text{if } \frac{w}{\beta}N\le\lambda_D\Leftrightarrow w\le\frac{\beta\lambda_D}{N}\\ \theta\lambda_D-\frac{N}{\beta}w^2 & \text{if } \frac{w}{\beta}N>\lambda_D\Leftrightarrow w>\frac{\beta\lambda_D}{N}\end{cases},$$

and its first derivative regarding $w$ is

$$\frac{\partial}{\partial w}\pi(p,\theta,w)=\begin{cases}\frac{N}{\beta}\left(\theta-2w\right) & \text{if } \frac{w}{\beta}N\le\lambda_D\Leftrightarrow w\le\frac{\beta\lambda_D}{N}\\ -2\frac{N}{\beta}w & \text{if } \frac{w}{\beta}N>\lambda_D\Leftrightarrow w>\frac{\beta\lambda_D}{N}\end{cases}.$$

When the number of participating drivers is more than sufficient to serve all tech-savvy customers' no-supply-constrained demand, i.e., $\frac{w}{\beta}N\ge\lambda_D$, the food delivery platform's profit decreases in $w$, so the platform should set wage $w$ at no more than $\frac{\beta\lambda_D}{N}$. For $w\le\frac{\beta\lambda_D}{N}$, we have $\pi(p,\theta,w)=\frac{N}{\beta}\left(\theta w-w^2\right)$, whose maximum is at $w=\frac{\theta}{2}$. If $\frac{\theta}{2}\le\frac{\beta\lambda_D}{N}$, the food delivery platform should set the wage at $w^*(\theta)=\frac{\theta}{2}$ to induce drivers to join with rate $\frac{\theta}{2\beta}N$; otherwise, if $\frac{\theta}{2}>\frac{\beta\lambda_D}{N}$, the food delivery platform should set the wage at $w^*(\theta)=\frac{\beta\lambda_D}{N}$ to induce drivers to join with rate $\lambda_D$:

$$w^*(\theta)=\begin{cases}\frac{\theta}{2} & \text{if } \theta\le\frac{2\beta\lambda_D}{N}\Leftrightarrow\frac{\theta}{2}\le\frac{\beta\lambda_D}{N}\\ \frac{\beta\lambda_D}{N} & \text{if } \theta>\frac{2\beta\lambda_D}{N}\Leftrightarrow\frac{\theta}{2}>\frac{\beta\lambda_D}{N}\end{cases}.$$

Correspondingly, the joining rate of food-delivery customers is

$$\lambda_D^*(\theta)=\begin{cases}\frac{\theta}{2\beta}N & \text{if } \theta\le\frac{2\beta\lambda_D}{N}\\ \lambda_D & \text{if } \theta>\frac{2\beta\lambda_D}{N}\end{cases},$$

the joining rate of walk-in tech-savvy customers is

$$\lambda_{1W}^*(\theta)=\begin{cases}\max\left(0,\min\left(\lambda_D,\mu-\frac{c}{R-p}\right)-\frac{\theta}{2\beta}N\right) & \text{if } \theta\le\frac{2\beta\lambda_D}{N}\\ 0 & \text{if } \theta>\frac{2\beta\lambda_D}{N}\end{cases},$$

and the food delivery platform's profit is

$$\pi(p,\theta,w^*(\theta))=\begin{cases}\frac{\theta^2}{4\beta}N & \text{if } \theta\le\frac{2\beta\lambda_D}{N}\\ \theta\lambda_D-\frac{\beta}{N}\lambda_D^2 & \text{if } \theta>\frac{2\beta\lambda_D}{N}\end{cases}. \quad\square$$

As in the base model, the platform's profit is characterized by two functions: (C.10) and (C.11) in Lemma C.10 of the Online Appendix. In the first case, (C.10) is an increasing function of delivery fee $\theta$, so the food delivery platform will charge the highest delivery fee in the corresponding interval to obtain the maximum profit. In the second case, (C.11) is a unimodal function of $\theta$ with a maximum at $\theta_2\in\left(0,R-p-\frac{\phi}{\mu}\right)$ which is given in Lemma C.10. Then, the platform will charge an optimal delivery fee $\theta^*=\min\left(\max\left(\theta_2,R-p-\frac{\phi}{\mu-\bar{\Lambda}_1}\right),\frac{(c-\phi)(R-p)}{c}\right)$. The following proposition gives the platform's equilibrium strategy and the corresponding joining rates of food-delivery and walk-in customers.

Proposition C.6 **(Food Delivery Platform Strategy)**. *If $\mu^2\beta\le cN$, there exists a threshold value $\bar{\Lambda}_1$ as given in (C.12) and $\theta_2$ as given in Lemma C.10 of the Online Appendix, such that for the food price $p$, we have the optimal delivery fee $\theta^*$, delivery wage $w^*$, and the joining rates of food-delivery and walk-in customers, $\lambda_D^*$ and $\lambda_W^*$, as:*

1. *For* $\Lambda_1 \leq \bar{\Lambda}_1$,

| | $\theta^*$ | $w^*(\theta^*)$ | $\lambda_D^*(\theta^*)$ | $\lambda_W^*(\theta^*)$ |
|---|---|---|---|---|
| *If* $p \leq R - \frac{c}{\mu-\Lambda_1}$ | | | | |
| *if* $\frac{(c-\phi)(R-p)}{c} \leq \frac{2\beta\Lambda_1}{N}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{(c-\phi)(R-p)}{2c}$ | $\frac{N(c-\phi)(R-p)}{2\beta c}$ | $\mu - \frac{c}{R-p} - \frac{N(c-\phi)(R-p)}{2\beta c}$ |
| *if* $\frac{(c-\phi)(R-p)}{c} > \frac{2\beta\Lambda_1}{N}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{\beta\Lambda_1}{N}$ | $\Lambda_1$ | $\mu - \frac{c}{R-p} - \Lambda_1$ |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N}$ | $R - p - \frac{\phi}{\mu-\Lambda_1}$ | $\frac{\beta\Lambda_1}{N}$ | $\Lambda_1$ | $0$ |
| *If* $R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N} < p \leq R - \frac{\phi}{\mu}$ | $\theta_2$ | $\frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ | $\mu - \frac{\phi}{R-p-\theta_2}$ | $0$ |

2. *For* $\Lambda_1 > \bar{\Lambda}_1$,

| | $\theta^*$ | $w^*(\theta^*)$ | $\lambda_D^*(\theta^*)$ | $\lambda_W^*(\theta^*)$ |
|---|---|---|---|---|
| *If* $p \leq R - \frac{c}{\mu-\Lambda_1}$ | | | | |
| *if* $\frac{(c-\phi)(R-p)}{c} \leq \frac{2\beta\Lambda_1}{N}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{(c-\phi)(R-p)}{2c}$ | $\frac{N(c-\phi)(R-p)}{2\beta c}$ | $\mu - \frac{c}{R-p} - \frac{N(c-\phi)(R-p)}{2\beta c}$ |
| *if* $\frac{(c-\phi)(R-p)}{c} > \frac{2\beta\Lambda_1}{N}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{\beta\Lambda_1}{N}$ | $\Lambda_1$ | $\mu - \frac{c}{R-p} - \Lambda_1$ |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \leq \bar{p}$ | | | | |
| *if* $\frac{(c-\phi)(R-p)}{c} \leq \theta_1$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{(c-\phi)(R-p)}{2c}$ | $\frac{N(c-\phi)(R-p)}{2\beta c}$ | $\mu - \frac{c}{R-p} - \frac{N(c-\phi)(R-p)}{2\beta c}$ |
| *if* $\frac{(c-\phi)(R-p)}{c} > \theta_1$ | $\frac{(c-\phi)(R-p)}{c}$ | $\frac{\beta}{N}\left(\mu - \frac{c}{R-p}\right)$ | $\mu - \frac{c}{R-p}$ | $0$ |
| *If* $\bar{p} < p \leq R - \frac{\phi}{\mu}$ | $\theta_2$ | $\frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ | $\mu - \frac{\phi}{R-p-\theta_2}$ | $0$ |

To prove Proposition C.6, we first prove a Lemma.

LEMMA C.10. *Under the equilibrium driver wage* $w^*(\theta)$*, the food delivery platform's profit* $\pi(p,\theta,w^*(\theta))$*, arrival rates of food-delivery customers* $\lambda_D^*(\theta)$*, and those who join the queue themselves* $\lambda_{1W}^*(\theta^*)$*, depend on the no-supply-constrained demand rate for the food delivery service* $\lambda_D$*:*

1. *When* $\lambda_D = \Lambda_1$*, we have*

$$\begin{aligned} w^*(\theta) &= \begin{cases} \frac{\theta}{2} & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ \frac{\beta\Lambda_1}{N} & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}, \\ \pi(p,\theta,w^*(\theta)) &= \begin{cases} \frac{N}{4\beta}\theta^2 & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ \theta\Lambda_1 - \frac{\beta}{N}(\Lambda_1)^2 & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}, \\ \lambda_D^*(\theta) &= \begin{cases} \frac{\theta}{2\beta}N & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ \Lambda_1 & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}, \\ \text{and } \lambda_{1W}^*(\theta^*) &= \begin{cases} \Lambda_1 - \frac{\theta}{2\beta}N & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ 0 & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}. \end{aligned} \tag{C.10}$$

*In this case,* $\pi(p,\theta,w^*(\theta))$ *is an increasing function of* $\theta$*.*

2. *When* $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$*, we have*

$$\begin{aligned} w^*(\theta) &= \begin{cases} \frac{\theta}{2} & \text{if } \theta \leq \theta_1 \\ \frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta}\right) & \text{if } \theta > \theta_1 \end{cases}, \\ \pi(p,\theta,w^*(\theta)) &= \begin{cases} \frac{N}{4\beta}\theta^2 & \text{if } \theta \leq \theta_1 \\ \theta\left(\mu - \frac{\phi}{R-p-\theta}\right) - \frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta}\right)^2 & \text{if } \theta > \theta_1 \end{cases}, \\ \lambda_D^*(\theta) &= \begin{cases} \frac{\theta}{2\beta}N & \text{if } \theta \leq \theta_1 \\ \mu - \frac{\phi}{R-p-\theta} & \text{if } \theta > \theta_1 \end{cases}, \end{aligned} \tag{C.11}$$

$$\text{and } \lambda_{1W}^*(\theta^*) = \begin{cases} \mu - \frac{\phi}{R-p-\theta} - \frac{\theta}{2\beta}N & \text{if } \theta \le \theta_1 \\ 0 & \text{if } \theta > \theta_1 \end{cases},$$

*where* $\theta_1 = \frac{1}{2}\left(R-p+\frac{2\beta\mu}{N}-\sqrt{\left(R-p-\frac{2\beta\mu}{N}\right)^2+\frac{8\beta\phi}{N}}\right)$ *denotes the unique root of* $\frac{\theta}{\beta}N - 2\left(\mu-\frac{\phi}{R-p-\theta}\right)=0$ *on* $\left(0,R-p-\frac{\phi}{\mu}\right)$. *In this case,* $\pi(p,\theta,w^*(\theta))$ *is a unimodal function of* $\theta$ *with a maximum at* $\theta_2$, *where* $\theta_2>\theta_1$ *is the unique solution of* $G(\theta)\equiv\mu-\frac{\phi}{R-p-\theta}-\left(\theta-2\frac{\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\right)\frac{\phi}{(R-p-\theta)^2}=0$ *on* $\left[\theta_1,R-p-\frac{\phi}{\mu}\right]$.

*We have some properties of* $\theta_1$ *and* $\theta_2$*:*

*2.1* $\theta_1$ *decreases in* $N$ *and* $p$.

*2.2* $\lim_{N\to\infty}\theta_1=0$.

*2.3* $\theta_1\le R-p-\frac{\phi}{\mu}$. *The equality only holds when* $p=R-\frac{\phi}{\mu}$, *where* $\theta_1=R-p-\frac{\phi}{\mu}=0$ *at* $p=R-\frac{\phi}{\mu}$.

*2.4* $\theta_1$ *intersects with* $R-p-\frac{\phi}{\mu-\Lambda_1}$ *at* $p=R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$. *We have* $\theta_1\le R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p\le R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$, *and* $\theta_1>R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p>R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$.

*2.5 We have* $\theta_1\le\frac{(c-\phi)(R-p)}{c}\Leftrightarrow 2\left(\mu-\frac{c}{R-p}\right)\le\frac{N}{\beta}\frac{(c-\phi)(R-p)}{c}$, *and* $\theta_1\ge\frac{(c-\phi)(R-p)}{c}\Leftrightarrow 2\left(\mu-\frac{c}{R-p}\right)\ge\frac{N}{\beta}\frac{(c-\phi)(R-p)}{c}$.

*2.6* $\theta_2$ *decreases in* $N$, $\phi$, *and* $p$.

*2.7* $\theta_2\le R-p-\frac{\phi}{\mu}$. *The equality only holds when* $p=R-\frac{\phi}{\mu}$, *where* $\theta_2=R-p-\frac{\phi}{\mu}=0$ *at* $p=R-\frac{\phi}{\mu}$.

*2.8* $\theta_2$ *intersects with* $R-p-\frac{\phi}{\mu-\Lambda_1}$ *at* $p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$, *which decreases in* $\Lambda_1$.

- *We have* $\theta_2\le R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$, *and* $\theta_2>R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow p>R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$.

*2.9 When* $\frac{\beta\mu^2}{cN}\in(0,1)$, $\theta_2$ *intersects with* $\frac{(c-\phi)(R-p)}{c}$ *at* $p=\bar{p}\equiv R-c\left(\Xi-\frac{\frac{2}{3\phi}\frac{\beta}{N}-\frac{c^2}{9\mu^2\phi^2}}{\Xi}+\frac{c}{3\mu\phi}\right)\in\left(R-\frac{c^2}{\phi\mu},R-\frac{c}{\mu}\right)$, *where* $\Xi=\sqrt[3]{\sqrt{\frac{8}{27\phi^3}\frac{\beta^3}{N^3}+\frac{1}{\mu^2}\left(\frac{1}{\phi^2}-\frac{c^2}{27\phi^4}-\frac{2c}{3\phi^3}\right)\frac{\beta^2}{N^2}+\frac{2c^3}{27\mu^4\phi^4}\frac{\beta}{N}}+\frac{1}{\mu}\left(\frac{1}{\phi}-\frac{c}{3\phi^2}\right)\frac{\beta}{N}+\frac{c^3}{27\mu^3\phi^3}}$.

- *We have* $\theta_2\le\frac{(c-\phi)(R-p)}{c}\Leftrightarrow p\ge\bar{p}$, *and* $\theta_2>\frac{(c-\phi)(R-p)}{c}\Leftrightarrow p<\bar{p}$.

*Proof of Lemma C.10.* 1. If $\lambda_D=\Lambda_1$, using it in Lemma C.9 gives $w^*(\theta)$, $\lambda_D^*(\theta)$, $\lambda_{1W}^*(\theta^*)$, and

$$\pi(p,\theta,w^*(\theta))=\begin{cases}\frac{\theta^2}{4\beta}N & \text{if } \theta\le\frac{2\beta\Lambda_1}{N}\\ \theta\Lambda_1-\frac{\beta}{N}(\Lambda_1)^2 & \text{if } \theta>\frac{2\beta\Lambda_1}{N}\end{cases},$$

whose first derivative is

$$\frac{\partial}{\partial\theta}\pi(p,\theta,w^*(\theta))=\begin{cases}\frac{\theta}{2\beta}N & \text{if } \theta\le\frac{2\beta\Lambda_1}{N}\\ \Lambda_1 & \text{if } \theta>\frac{2\beta\Lambda_1}{N}\end{cases}.$$

Clearly, we have $\frac{\partial}{\partial\theta}\pi(p,\theta,w^*(\theta))>0$, so $\pi(p,\theta,w^*(\theta))$ is an increasing function of $\theta$.

2. If $\lambda_D=\mu-\frac{\phi}{R-p-\theta}$ for $0\le\theta\le R-p-\frac{\phi}{\mu}$, using it in Lemma C.9 gives $w^*(\theta)$, $\lambda_D^*(\theta)$, $\lambda_{1W}^*(\theta^*)$, and

$$\pi(p,\theta,w^*(\theta))=\begin{cases}\frac{\theta^2}{4\beta}N & \text{if } \theta\le\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\\ \theta\left(\mu-\frac{\phi}{R-p-\theta}\right)-\frac{\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)^2 & \text{if } \theta>\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\end{cases},$$

whose first derivative is

$$\frac{\partial}{\partial\theta}\pi\left(p,\theta,w^*\left(\theta\right)\right)=\begin{cases}\frac{N}{2\beta}\theta>0 & \text{if } \theta\le\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\\ G\left(\theta\right)\equiv\mu-\frac{\phi}{R-p-\theta}-\left(\theta-2\frac{\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\right)\frac{\phi}{(R-p-\theta)^2} & \text{if } \theta>\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\end{cases}.$$

Let $\theta_1=\frac{1}{2}\left(R-p+\frac{2\beta\mu}{N}-\sqrt{\left(R-p-\frac{2\beta\mu}{N}\right)^2+\frac{8\beta\phi}{N}}\right)$ denote the unique root of $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)=0$ on $\left(0,R-p-\frac{\phi}{\mu}\right)$.

- $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)$ increases in $\theta$, $N$ and $p$.
- $\theta_1$ decreases in $N$ and $p$, because $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)$ increases in $N$ and $p$.
- $\lim_{N\to\infty}\theta_1=0$.
- $\theta\le\theta_1\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\le 0$ and $\theta>\theta_1\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)>0$, because $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)$ increases in $\theta$.
- $\theta_1$ intersects with $R-p-\frac{\phi}{\mu}$ at $p=R-\frac{\phi}{\mu}$, and we have $\theta_1=R-p-\frac{\phi}{\mu}=0$ at $p=R-\frac{\phi}{\mu}$. We have $\theta_1<R-p-\frac{\phi}{\mu}$ for $p<R-\frac{\phi}{\mu}$, because $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)$ increases in $p$ and $\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\Big|_{\theta=R-p-\frac{\phi}{\mu}}=R-p-\frac{\phi}{\mu}>0$.
- $\theta_1$ intersects with $R-p-\frac{\phi}{\mu-\Lambda_1}$ at $p=R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$. We have $\theta_1\le R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\Big|_{\theta=R-p-\frac{\phi}{\mu-\Lambda_1}}\ge 0\Leftrightarrow p\le R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$, and $\theta_1>R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\Big|_{\theta=R-p-\frac{\phi}{\mu-\Lambda_1}}<0\Leftrightarrow p>R-\frac{\phi}{\mu-\Lambda_1}-\frac{2\beta\Lambda_1}{N}$.
- $\theta_1\le\frac{(c-\phi)(R-p)}{c}\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\Big|_{\theta=\frac{(c-\phi)(R-p)}{c}}\ge 0\Leftrightarrow 2\left(\mu-\frac{c}{R-p}\right)\le\frac{N}{\beta}\frac{(c-\phi)(R-p)}{c}$, and $\theta_1\ge\frac{(c-\phi)(R-p)}{c}\Leftrightarrow\theta-\frac{2\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)\Big|_{\theta=\frac{(c-\phi)(R-p)}{c}}<0\Leftrightarrow 2\left(\mu-\frac{c}{R-p}\right)\ge\frac{N}{\beta}\frac{(c-\phi)(R-p)}{c}$.

Clearly, $G\left(\theta\right)$ decreases in $N$, $\theta$, $\phi$, and $p$. Let $\theta_2$ denote the solution of the equation $G\left(\theta\right)=0\Leftrightarrow R-p-\frac{\mu(R-p-\theta)^2}{\phi}=2\frac{\beta}{N}\left(\mu-\frac{\phi}{R-p-\theta}\right)$.

- $\theta_2$ is unique on $\left[\theta_1,R-p-\frac{\phi}{\mu}\right]$, because $G\left(\theta\right)$ decreases in $\theta$, $G\left(0\right)=\frac{\phi}{(R-p)^2}\left(\frac{\mu}{\phi}\left(R-p\right)\left(R-p-\frac{\phi}{\mu}\right)+2\frac{\beta}{N}\left(\mu-\frac{\phi}{R-p}\right)\right)>0$, $G\left(\theta_1\right)=\mu-\frac{\phi}{R-p-\theta_1}=\frac{1}{2}\frac{\theta}{\beta}N>0$, and $G\left(R-p-\frac{\phi}{\mu}\right)=-\frac{\mu^2}{\phi}\left(R-p-\frac{\phi}{\mu}\right)<0$.
- $\theta_2$ is decreasing in $p$, because $G\left(\theta\right)$ decreases in $p$.
- $\theta_2$ is decreasing in $N$, because $G\left(\theta\right)$ decreases in $N$.
- $\theta_2$ is decreasing in $\phi$, because $G\left(\theta\right)$ decreases in $\phi$ and $\theta$.
- $\theta_2$ intersects with $R-p-\frac{\phi}{\mu}$ at $p=R-\frac{\phi}{\mu}$, and we have $\theta_2=R-p-\frac{\phi}{\mu}=0$ at $p=R-\frac{\phi}{\mu}$. If $p<R-\frac{\phi}{\mu}$, we have $\theta_2<R-p-\frac{\phi}{\mu}$, because $G\left(R-p-\frac{\phi}{\mu}\right)=-\frac{\mu^2}{\phi}\left(R-p-\frac{\phi}{\mu}\right)<0$.
- To find the intersection point of $\theta_2$ and $R-p-\frac{\phi}{\mu-\Lambda_1}$, we solve $\theta_2=R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow G\left(R-p-\frac{\phi}{\mu-\Lambda_1}\right)=0\Leftrightarrow p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$, which clearly decreases in $\Lambda_1$. From simple algebra, we have $\theta_2\le R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow G\left(R-p-\frac{\phi}{\mu-\Lambda_1}\right)\le 0\Leftrightarrow p\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$, and $\theta_2>R-p-\frac{\phi}{\mu-\Lambda_1}\Leftrightarrow G\left(R-p-\frac{\phi}{\mu-\Lambda_1}\right)>0\Leftrightarrow p>R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$.

• To find the intersection point of $\theta_2$ and $\frac{(c-\phi)(R-p)}{c}$, we solve $\theta_2 = \frac{(c-\phi)(R-p)}{c} \Leftrightarrow G\left(\frac{(c-\phi)(R-p)}{c}\right) = 0 \Leftrightarrow 2\frac{\beta}{N}\left(\mu - \frac{c}{R-p}\right) = R - p - \frac{\phi\mu(R-p)^2}{c^2} \Longleftrightarrow \phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N} = 0$ where $x = \frac{R-p}{c}$. We have

$$\begin{aligned} \left.\phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N}\right|_{x=\frac{1}{\mu}} &= -\frac{1}{\mu^2}(c-\phi) < 0 \\ \left.\phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N}\right|_{x=\frac{c}{\phi\mu}} &= \frac{2}{\phi}y(c-\phi) > 0 \end{aligned}$$

and the discriminant of $\phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N}$ is $\Delta = -\frac{4c^3\beta}{N}\left(8\alpha y^2 + (27\alpha^2 - 10\alpha - 1)y + 2\right)$, where $\alpha = \frac{\phi}{c} \in (0,1)$ and $y = \frac{\beta\mu^2}{cN}$. If we can show that $f(y) = 8\alpha y^2 + (27\alpha^2 - 10\alpha - 1)y + 2 > 0$ for $y \in (0,1)$, then we have $\Delta < 0 \Leftrightarrow \phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N}$ has a unique real root for $y \in (0,1)$. Before we prove this result, let $\alpha_0$ denote the unique root of the discriminant of $f(y)$: $\Delta_f = (27\alpha^2 - 10\alpha - 1)^2 - 64\alpha$ in $\left(0, \frac{2\sqrt{13}+5}{27}\right)$. One can numerically verify that $\Delta_f \geq 0$ if $\alpha \in [0, \alpha_0]$, and $\Delta_f \leq 0$ if $\alpha \in \left[\alpha_0, \frac{2\sqrt{13}+5}{27}\right]$.

—If $\alpha \geq \frac{2\sqrt{13}+5}{27} \Leftrightarrow 27\alpha^2 - 10\alpha - 1 \geq 0$, then we clearly have $f(y) > 0$.

—If $\alpha_0 < \phi < \frac{2\sqrt{13}+5}{27}$, we have $\Delta_f \leq 0$, which means that $f(y) > 0$ for $\forall y \in (0,1)$.

—If $\phi \leq \alpha_0$, we have $\Delta_f \geq 0$, which means that $f(y) = 0$ has two real roots:

$$\begin{aligned} y_1 &= \frac{-(27\alpha^2 - 10\alpha - 1) - \sqrt{(27\alpha^2 - 10\alpha - 1)^2 - 64\alpha}}{16\alpha} \\ y_2 &= \frac{-(27\alpha^2 - 10\alpha - 1) + \sqrt{(27\alpha^2 - 10\alpha - 1)^2 - 64\alpha}}{16\alpha}, \end{aligned}$$

where $y_1 < y_2$. We have

$$\begin{aligned} y_1 &> 1 \Leftrightarrow -\left(27\alpha^2 - 10\alpha - 1\right) - 16\alpha > \sqrt{(27\alpha^2 - 10\alpha - 1)^2 - 64\alpha} \\ &\Leftrightarrow 32\alpha\left(27\alpha^2 - 2\alpha + 1\right) > 0, \end{aligned}$$

which is clearly true. Thus, we have $f(y) > 0$ for $\forall y \in (0,1)$.

(a) Thus, $\theta_2 = \frac{(c-\phi)(R-p)}{c} \Leftrightarrow 2\frac{\beta}{N}\left(\mu - \frac{c}{R-p}\right) = R - p - \frac{\phi\mu(R-p)^2}{c^2} \overset{x=\frac{R-p}{c}}{\Leftrightarrow} \phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N} = 0$ has a unique root in $\left(\frac{c}{\phi\mu}, \frac{1}{\mu}\right)$ if $y = \frac{\beta\mu^2}{cN} \in (0,1)$. This is equivalent to $\theta_2 = \frac{(c-\phi)(R-p)}{c}$ having a unique root in $\left(R - \frac{c^2}{\phi\mu}, R - \frac{c}{\mu}\right)$ if $y = \frac{\beta\mu^2}{cN} \in (0,1)$:

$$\bar{p} = R - c\left(\Xi - \frac{\frac{2}{3\phi}\frac{\beta}{N} - \frac{c^2}{9\mu^2\phi^2}}{\Xi} + \frac{c}{3\mu\phi}\right)$$

where $\Xi = \sqrt[3]{\sqrt{\frac{8}{27\phi^3}\frac{\beta^3}{N^3} + \frac{1}{\mu^2}\left(\frac{1}{\phi^2} - \frac{c^2}{27\phi^4} - \frac{2c}{3\phi^3}\right)\frac{\beta^2}{N^2}} + \frac{2c^3}{27\mu^4\phi^4}\frac{\beta}{N} + \frac{1}{\mu}\left(\frac{1}{\phi} - \frac{c}{3\phi^2}\right)\frac{\beta}{N} + \frac{c^3}{27\mu^3\phi^3}}$.

Since $G(\theta)$ decreases in $\theta$, we have $\theta_2 \leq \frac{(c-\phi)(R-p)}{c} \Leftrightarrow G\left(\frac{(c-\phi)(R-p)}{c}\right) \leq 0 \Leftrightarrow 2\frac{\beta}{N}\left(\mu - \frac{c}{R-p}\right) \leq R - p - \frac{\phi\mu(R-p)^2}{c^2} \overset{x=\frac{R-p}{c}}{\Leftrightarrow} \phi\mu x^3 - cx^2 + 2\mu\frac{\beta}{N}x - 2\frac{\beta}{N} \leq 0 \Leftrightarrow p \geq \bar{p}$. Thus, we have $\theta_2 \leq \frac{(c-\phi)(R-p)}{c} \Leftrightarrow p \geq \bar{p}$, and $\theta_2 > \frac{(c-\phi)(R-p)}{c} \Leftrightarrow p < \bar{p}$.

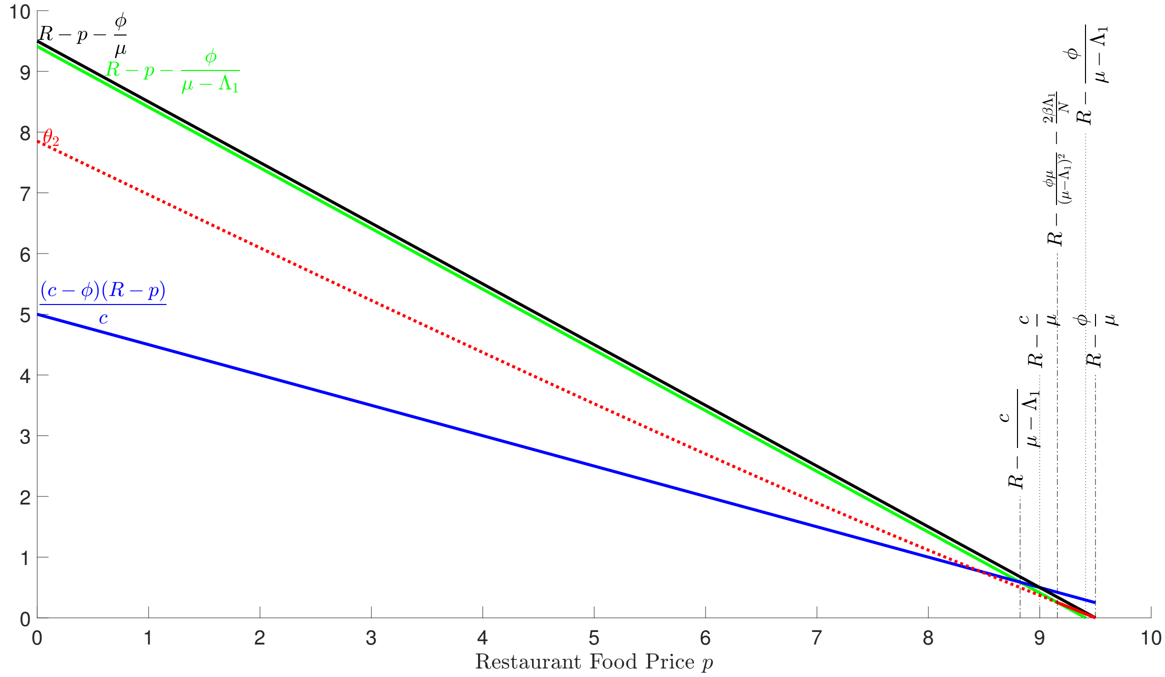


**Figure C.15** **The intersection points of** $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, **and** $R-p-\frac{\phi}{\mu}$, **when** $\Lambda_1 \le \bar{\Lambda}_T$, **for the parameter setting** $R=10$, $\Lambda=\mu=c=1$, $\phi=0.5$, $\Lambda_1=0.15$, **and** $N=20$.

In this case, $\pi\left(p,\theta,w^*\left(\theta\right)\right)$ is a unimodal function with a maximum at $\theta_2$. $\square$

Next, we prove Proposition C.6.

Recall that $\frac{(c-\phi)(R-p)}{c}$ intersects $R-p-\frac{\phi}{\mu-\Lambda_1}$ at $p=R-\frac{c}{\mu-\Lambda_1}$, and $\theta_2$ intersects $R-p-\frac{\phi}{\mu-\Lambda_1}$ at $p=R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$. Then, we have $R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}=R-\frac{c}{\mu-\Lambda_1} \Leftrightarrow g\left(\Lambda_1\right)\equiv 2\beta\Lambda_1^3-4\beta\mu\Lambda_1^2+\left(2\beta\mu^2+Nc\right)\Lambda_1-N\mu\left(c-\phi\right)=0$. Note that the discriminant of $g\left(\Lambda_1\right)$ is $\Delta_g=-4N^3\beta c^3\left(27y\alpha^2+\left(8y^2-18y\right)\alpha+2-y\right)$, where $y=\frac{\beta\mu^2}{cN}$ and $\alpha=\frac{\phi}{c}$. When $y\le 1\le 3/2$, we have $27y\alpha^2+\left(8y^2-18y\right)\alpha+2-y\ge 0 \Leftrightarrow \Delta_g\le 0$. Thus, $g\left(\Lambda_1\right)$ has a unique real root

$$\bar{\Lambda}_1=\frac{2}{3}\mu+\Gamma+\frac{\frac{1}{9}\mu^2-\frac{1}{6}N\frac{c}{\beta}}{\Gamma} \tag{C.12}$$

$$\text{where } \Gamma=\sqrt[3]{\frac{\mu\left(9\left(c-3\phi\right)N-4\beta\mu^2\right)}{108\beta}+\sqrt{\frac{N\left(2c^3N^2+\mu^2\left(27\phi^2-18c\phi-c^2\right)\beta N+8\beta^2\mu^4\phi\right)}{432\beta^3}}}.$$

Thus, when $\Lambda_1\le\bar{\Lambda}_1$, we have $\bar{p}\le R-\frac{c}{\mu-\Lambda_1}\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}$; and when $\Lambda_1>\bar{\Lambda}_1$, we have $R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}\le R-\frac{c}{\mu-\Lambda_1}\le\bar{p}$.

If $\Lambda_1\le\bar{\Lambda}_1$, we have $\bar{p}\le R-\frac{c}{\mu-\Lambda_1}\le R-\frac{\phi\mu}{(\mu-\Lambda_1)^2}-\frac{2\beta\Lambda_1}{N}\le R-\frac{\phi}{\mu}$. Figure C.15 illustrates the intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1\le\bar{\Lambda}_1$.

1. If $p\le R-\frac{c}{\mu-\Lambda_1}\Leftrightarrow\Lambda_1\le\mu-\frac{c}{R-p}$, we have $\frac{(c-\phi)(R-p)}{c}\le R-p-\frac{\phi}{\mu-\Lambda_1}<R-p-\frac{\phi}{\mu}$. From Lemma C.2, we see that tech-savvy customers are willing to pay a delivery fee of at most $\frac{(c-\phi)(R-p)}{c}$ for the food delivery service, and the no-supply-constrained demand for the food delivery service is $\lambda_D=\Lambda_1$ when $\theta\le\frac{(c-\phi)(R-p)}{c}$. By Lemma C.10(1), the food delivery platform will

charge the highest delivery fee $\theta^* = \frac{(c-\phi)(R-p)}{c}$ to maximize its profit, and we have $\lambda_D^*(\theta) = \begin{cases} \frac{N(c-\phi)(R-p)}{2\beta c} & \text{if } \frac{(c-\phi)(R-p)}{c} \leq \frac{2\beta\Lambda_1}{N} \\ \Lambda_1 & \text{if } \frac{(c-\phi)(R-p)}{c} > \frac{2\beta\Lambda_1}{N} \end{cases}$, and $\lambda_W^*(\theta^*) = \mu - \frac{c}{R-p} - \lambda_D^*(\theta)$. From Lemma C.10(1), we have $w^*(\theta) = \begin{cases} \frac{(c-\phi)(R-p)}{2c} & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ \frac{\beta\Lambda_1}{N} & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}$.

2. If $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N}$, we have $\theta_2 \leq R-p-\frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$ by Lemma C.10(2.8) and $\Lambda_1 > \mu - \frac{c}{R-p}$. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \leq R-p-\frac{\phi}{\mu-\Lambda_1}$. Then, from Lemma C.10(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $R-p-\frac{\phi}{\mu-\Lambda_1}$ for the food delivery service. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $R-p-\frac{\phi}{\mu-\Lambda_1} \leq \theta \leq \frac{(c-\phi)(R-p)}{c}$. In this case, the food delivery platform's profit $\pi(p,\theta,w^*(\theta))$ decreases in $\theta$ for $\theta \geq R-p-\frac{\phi}{\mu-\Lambda_1} \geq \theta_2$ by Lemma C.10(2). Thus, the food delivery platform will charge $\theta^* = R-p-\frac{\phi}{\mu-\Lambda_1}$ (which is greater than $\frac{2\beta\Lambda_1}{N}$ because $p \leq R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N}$) as the delivery fee, and we have $\lambda_D^*(\theta^*) = \Lambda_1$, and $\lambda_W^*(\theta^*) = 0$. From Lemma C.10(1), we have $w^*(\theta) = \frac{\beta\Lambda_1}{N}$.
3. If $R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N} < p \leq R - \frac{\phi}{\mu}$, we have $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right) \leq \theta_2 \leq \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ by Lemma C.10(2.7) and (2.8). We also have $\Lambda_1 > \mu - \frac{c}{R-p}$ here. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D = \Lambda_1$ if $\theta \leq \max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right)$. Then, from Lemma C.10(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right)$ for the food delivery service. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \leq \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$. In this case, the food delivery platform's profit $\pi(p,\theta,w^*(\theta))$ has its maximum at $\theta_2$ on $\left[\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right), \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)\right]$ by Lemma C.5(2). Thus, the food delivery platform will charge $\theta^* = \theta_2$ as the delivery fee, and we have $\lambda_D^*(\theta^*) = \mu - \frac{\phi}{R-p-\theta_2}$, which is greater than $\mu - \frac{c}{R-p}$ because $\theta_2 \leq \frac{(c-\phi)(R-p)}{c}$, and $\lambda_W^*(\theta^*) = 0$. From Lemma C.10(2), we have $w^*(\theta) = \frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$.

If $\Lambda_1 > \bar{\Lambda}_1$, we have $R - \frac{\phi\mu}{(\mu-\Lambda_1)^2} - \frac{2\beta\Lambda_1}{N} \leq R - \frac{c}{\mu-\Lambda_1} \leq \bar{p} < R - \frac{\phi}{\mu}$. Figure C.16 illustrates the intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1 > \bar{\Lambda}_1$.

1. If $p \leq R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 \leq \mu - \frac{c}{R-p}$, we have $\frac{(c-\phi)(R-p)}{c} \leq R-p-\frac{\phi}{\mu-\Lambda_1} < R-p-\frac{\phi}{\mu}$. From Lemma C.2, we see that tech-savvy customers are willing to pay a delivery fee of at most $\frac{(c-\phi)(R-p)}{c}$ for the food delivery service, and the no-supply-constrained demand for the food delivery service is $\lambda_D = \Lambda_1$ when $\theta \leq \frac{(c-\phi)(R-p)}{c}$. By Lemma C.10(1), the food delivery platform will charge the highest delivery fee $\theta^* = \frac{(c-\phi)(R-p)}{c}$ to maximize its profit, and we have $\lambda_D^*(\theta) = \begin{cases} \frac{N(c-\phi)(R-p)}{2\beta c} & \text{if } \frac{(c-\phi)(R-p)}{c} \leq \frac{2\beta\Lambda_1}{N} \\ \Lambda_1 & \text{if } \frac{(c-\phi)(R-p)}{c} > \frac{2\beta\Lambda_1}{N} \end{cases}$, and $\lambda_W^*(\theta^*) = \mu - \frac{c}{R-p} - \lambda_D^*(\theta)$. From Lemma C.10(1), we have $w^*(\theta) = \begin{cases} \frac{(c-\phi)(R-p)}{2c} & \text{if } \theta \leq \frac{2\beta\Lambda_1}{N} \\ \frac{\beta\Lambda_1}{N} & \text{if } \theta > \frac{2\beta\Lambda_1}{N} \end{cases}$.

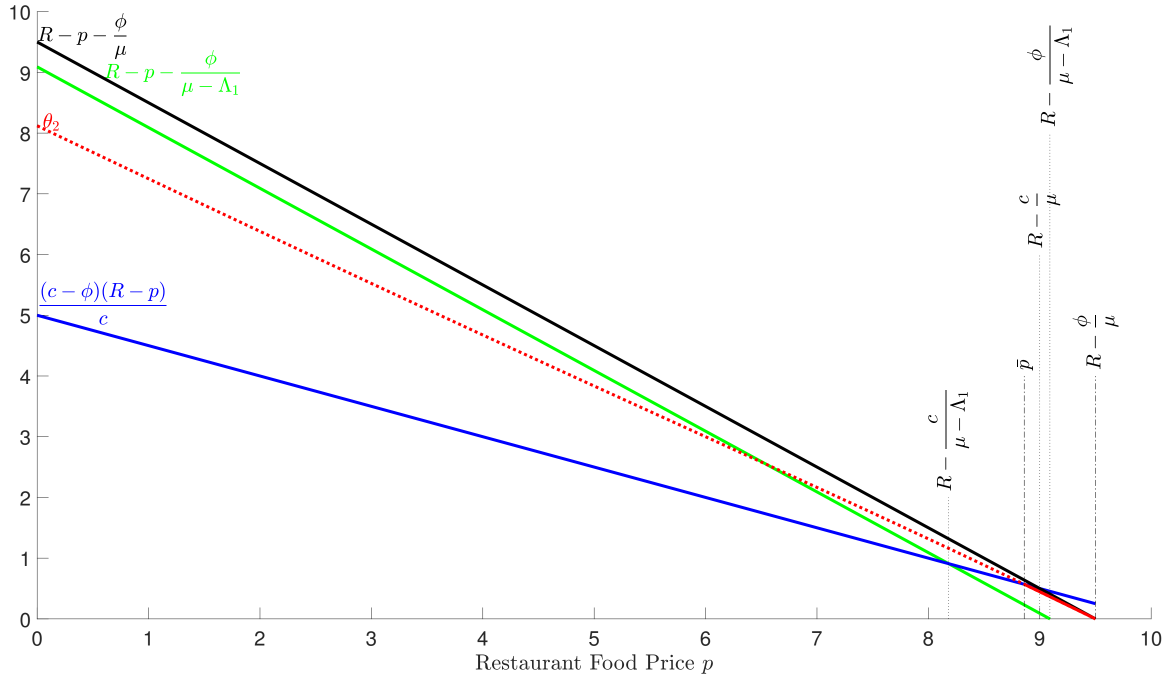


**Figure C.16** **The intersection points of $\frac{(c-\phi)(R-p)}{c}$, $R-p-\frac{\phi}{\mu-\Lambda_1}$, $\theta_2$, and $R-p-\frac{\phi}{\mu}$, when $\Lambda_1 > \bar{\Lambda}_T$, for the parameter setting $R=10$, $\Lambda=\mu=c=1$, $\phi=0.5$, $\Lambda_1=0.45$, and $N=20$.**

2. If $R-\frac{c}{\mu-\Lambda_1} < p \le \bar{p}$, we have $R-p-\frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c} \le \theta_2$, by Lemma C.5(2.9) and $\Lambda_1 > \mu-\frac{c}{R-p}$. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D=\Lambda_1$ if $\theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$. Then, from Lemma C.10(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $R-p-\frac{\phi}{\mu-\Lambda_1}$ for the food delivery service. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D=\mu-\frac{\phi}{R-p-\theta}$ for $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$. In this case, the food delivery platform's profit $\pi(p,\theta,w^*(\theta))$ increases in $\theta$ for $\theta \le \frac{(c-\phi)(R-p)}{c} \le \theta_2$ by Lemma C.10(2). Thus, the food delivery platform will charge $\theta^*=\frac{(c-\phi)(R-p)}{c}$ as the delivery fee, and we have $\lambda_D^*(\theta) = \begin{cases} \frac{N(c-\phi)(R-p)}{2\beta c} & \text{if } \frac{(c-\phi)(R-p)}{c} \le \theta_1 \\ \mu-\frac{c}{R-p} & \text{if } \frac{(c-\phi)(R-p)}{c} > \theta_1 \end{cases}$, and $\lambda_{1W}^*(\theta^*) = \begin{cases} \mu-\frac{c}{R-p}-\frac{N(c-\phi)(R-p)}{2\beta c} & \text{if } \frac{(c-\phi)(R-p)}{c} \le \theta_1 \\ 0 & \text{if } \frac{(c-\phi)(R-p)}{c} > \theta_1 \end{cases}$. (Note that we have $\mu-\frac{c}{R-p} \ge \frac{N(c-\phi)(R-p)}{2\beta c}$ if $\frac{(c-\phi)(R-p)}{c} \le \theta_1$, by Lemma C.10(2.5).) Then, the number of customers who join the queue themselves is $\lambda_{1W}^*(\theta^*)$. From Lemma C.10(1), we have $w^*(\theta) = \begin{cases} \frac{(c-\phi)(R-p)}{2c} & \text{if } \frac{(c-\phi)(R-p)}{c} \le \theta_1 \\ \frac{\beta}{N}\left(\mu-\frac{c}{R-p}\right) & \text{if } \frac{(c-\phi)(R-p)}{c} > \theta_1 \end{cases}$.
3. If $\bar{p} < p \le R-\frac{\phi}{\mu}$, we have $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right) \le \theta_2 \le \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ by Lemma C.10(2.7) and (2.9). We also have $\Lambda_1 > \mu-\frac{c}{R-p}$ here. From Lemma C.2, the no-supply-constrained demand for the food delivery service is $\lambda_D=\Lambda_1$ if $\theta \le \max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right)$. Then, from Lemma C.10(1), the food delivery platform's profit increases in $\theta$. Thus, the platform will charge at least $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1},0\right)$ for the food delivery service. From Lemma C.2, the no-supply-constrained demand for the food delivery

service is $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ for $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \leq \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$. In this case, the food delivery platform's profit $\pi\left(p, \theta, w^*(\theta)\right)$ has its maximum at $\theta_2$ on $\left[\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right), \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)\right]$ by Lemma C.10(2). Thus, the food delivery platform will charge $\theta^* = \theta_2$ as the delivery fee, and we have $\lambda_D^*(\theta^*) = \mu - \frac{\phi}{R-p-\theta_2}$, which is greater than $\mu - \frac{c}{R-p}$ because $\theta_2 \leq \frac{(c-\phi)(R-p)}{c}$, and $\lambda_W^*(\theta^*) = 0$. From Lemma C.10(1), we have $w^*(\theta) = \frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$. □

From Proposition C.6, we have the restaurant's profit Π as a function of the food price $p$ when $\Lambda_1 > \bar{\Lambda}_1$:

| | Restaurant's Profit Π |
|---|---|
| If $p \leq \bar{p}$ | $p\left(\mu - \frac{c}{R-p}\right)$ |
| If $\bar{p} < p \leq R - \frac{\phi}{\mu}$ | $p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ |

The restaurant's strategy is decided by the interplay of $p\left(\mu - \frac{c}{R-p}\right)$ and $p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$. Clearly, if $\max_p p\left(\mu - \frac{c}{R-p}\right) \geq \max_p p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$, then the restaurant's equilibrium price is $p^* = \arg\max p\left(\mu - \frac{c}{R-p}\right) = R - \sqrt{\frac{c}{\mu}R}$ and its equilibrium profit is $\Pi^* = \max_p p\left(\mu - \frac{c}{R-p}\right) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$.

Note from Lemma C.10(2.6) that $\theta_2$ decreases in $N$, so $p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ increases in $N$. This means that there exists an $\bar{N}$ such that $\max_p p\left(\mu - \frac{c}{R-p}\right) = \max_p p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ for $N = \bar{N}$, and $\max_p p\left(\mu - \frac{c}{R-p}\right) \geq \max_p p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ for $N \leq \bar{N}$. In this case, the restaurant's equilibrium price stays at $p^* = R - \sqrt{\frac{c}{\mu}R}$.

We next prove that $p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ decreases in $\phi$. Recall that $\theta_2$ satisfies $G(\theta_2) = 0 \Leftrightarrow R - p - \phi\frac{\mu(R-p-\theta_2)^2}{\phi^2} = 2\frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$. If $\frac{\phi}{R-p-\theta_2}$ decreases in $\phi$, we will have $R - p - \phi\frac{\mu(R-p-\theta_2)^2}{\phi^2}$ decreasing in $\phi$ while $2\frac{\beta}{N}\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ increases in $\phi$, which contradicts the definition of $\theta_2$. Hence, we must have $\frac{\phi}{R-p-\theta_2}$ increasing in $\phi$, which leads to $p\left(\mu - \frac{\phi}{R-p-\theta_2}\right)$ decreasing in $\phi$. Thus, we have that $\bar{N}$ increasing in $\phi$. □

# D. Proofs of the Results in Online Appendix A

## D.1. Proof of Proposition A.1

To prove Proposition A.1, we give the following Lemma and Corollary.

### D.1.1. Customer Strategy

Lemma D.11 **(Customer Strategy—Tech-Savvy Only)**. *When there are no traditional customers, i.e., $\Lambda_0 = 0$, under the food price $p$ and delivery fee $\theta$, the joining rates of food-delivery and walk-in customers, $\lambda_D$ and $\lambda_W$, are*

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| *When* $p \le R - \frac{c}{\mu-\Lambda_1}$ | | |
| *if* $0 < \theta \le \frac{c-\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| *if* $\frac{c-\phi}{\mu-\Lambda_1} < \theta \le R - p - \frac{\phi}{\mu}$ | $0$ | $\Lambda_1$ |
| *When* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\phi}{\mu}$ | | |
| *if* $0 < \theta \le \max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right)$ | $\Lambda_1$ | $0$ |
| *if* $\max\left(R - p - \frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \le \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$ | $\mu - \frac{\phi}{R-p-\theta}$ | $0$ |
| *if* $\theta > \min\left(\frac{(c-\phi)(R-p)}{c}, R - p - \frac{\phi}{\mu}\right)$ | $0$ | $\mu - \frac{c}{R-p}$ |

*Proof of Lemma D.11.* When $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu} \le R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{\phi}{\mu}$ by Lemma C.4.

1. If $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $\Lambda_1 \le \lambda_W^X = \mu - \frac{c}{R-p}$. From Lemma C.3(1), we have $U_W(\Lambda_1) \ge 0$.

1.1 If $\theta \le \frac{c-\phi}{\mu-\Lambda_1}$, we have $\Lambda_1 \ge \lambda^X(\theta)$ and $U_D(\Lambda_1) \ge U_W(\Lambda_1) \ge 0$ by Lemma C.3(3.1). This means that, if all tech-savvy customers join, they obtain greater utility from using the delivery service. In this case, all tech-savvy customers will join using food delivery service; i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.

1.2 If $\theta > \frac{c-\phi}{\mu-\Lambda_1}$, we have $\Lambda_1 < \lambda^X(\theta)$ by Lemma C.3(3.3). Further, since $\Lambda_1 \le \lambda_W^X$, from Lemma C.3(1) and (3), we have $U_W(\Lambda_1) > U_D(\Lambda_1)$ and $U_W(\Lambda_1) \ge 0$. In this case, all tech-savvy customers will join and walk in themselves, i.e., $\lambda_D = 0$ and $\lambda_W = \Lambda_1$.

2. If $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c}{\mu} \le R - \frac{\phi}{\mu-\Lambda_1}$, we have $0 \le R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c} \le R - p - \frac{\phi}{\mu}$, and $\Lambda_1 > \lambda_W^X = \mu - \frac{c}{R-p} \ge 0$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.

2.1 If $0 < \theta \le R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2) and (2.2), we have $U_D(\Lambda_1) \ge 0 > U_W(\Lambda_1)$. This says that if all tech-savvy customers join, their utility of using the food delivery service is non-negative while the walk-in utility is negative. Thus, all tech-savvy customers will join and use the delivery service, i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.

2.2 If $R - p - \frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2), (2.2), and (3.3), we have $\lambda^X(\theta) \le \lambda_W^X \le \lambda_D^X(\theta) < \Lambda_1$ and $U_D(\Lambda_1) < 0$. This says that when all tech-savvy customers join, both walk-in and food delivery are unattractive. Some tech-savvy customers may balk to avoid negative utility, until the total arrival rate to the system returns to $\lambda_D^X(\theta)$, where $U_W(\lambda_D^X(\theta)) < U_D(\lambda_D^X(\theta)) = 0$ (using Lemma C.3(1) and (2)). Thus, the tech-savvy customers join and use the food delivery service with rate $\lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$ and other customers balk: i.e., $\lambda_D = \lambda_D^X(\theta) = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$. In this case, all customers have zero utility.

2.3 If $\frac{(c-\phi)(R-p)}{c} < \theta \le R - p - \frac{\phi}{\mu}$, we have $\lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta)$ by Lemma C.3(3.3), and $\lambda_D^X(\theta) < \lambda_W^X < \Lambda_1$. Then, all tech-savvy customers joining (either walk in or order food delivery) does not lead to positive utility – some customers will balk, until the total arrival rate to the system drops to $\lambda_W^X$, where $U_D(\lambda_W^X) < U_W(\lambda_W^X) = 0$ by Lemma C.3(1) and (3). This means that tech-savvy customers will walk in themselves until the joining rate reaches $\mu - \frac{c}{R-p}$, and nobody will use food delivery; i.e., $\lambda_D = 0$ and $\lambda_W = \mu - \frac{c}{R-p}$. All customers have zero utility.

3. If $R - \frac{c}{\mu-\Lambda_1} < R - \frac{c}{\mu} < p \le R - \frac{\phi}{\mu-\Lambda_1}$, we have $0 \le R - p - \frac{\phi}{\mu-\Lambda_1} < R - p - \frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, $\lambda_W^X = \mu - \frac{c}{R-p} < \Lambda_1 \le \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$, and $\mu - \frac{c}{R-p} < 0$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.
   3.1 If $0 < \theta \le R - p - \frac{\phi}{\mu-\Lambda_1}$, similar to Case 2.1 above, we have $U_D(\Lambda_1) \ge 0 > U_W(\Lambda_1)$ – if all tech-savvy customers join, their utility of using the food delivery service is non-negative while the walk-in utility is negative. Then, all tech-savvy customers will join and use the delivery service; i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.
   3.2 If $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2 above, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.
4. If $R - \frac{c}{\mu} \le R - \frac{\phi}{\mu-\Lambda_1} < p \le R - \frac{\phi}{\mu}$, we have $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < R - p - \frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$ and $\mu - \frac{c}{R-p} < 0$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.
   4.1 If $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < \theta \le R - p - \frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2 above, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.

When $\Lambda_1 > \frac{c-\phi}{c}\mu$, we have $R - \frac{c}{\mu-\Lambda_1} < R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{c}{\mu} < R - \frac{\phi}{\mu}$ by Lemma C.4.

1. If $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $\Lambda_1 \le \lambda_W^X = \mu - \frac{c}{R-p}$. This case is the same as Case 1 when $\Lambda_1 \le \frac{c-\phi}{c}\mu$.
2. If $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\phi}{\mu-\Lambda_1}$, we have $0 \le R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$ and $\lambda_W^X = \mu - \frac{c}{R-p} \le \Lambda_1 \le \lambda_D^X(0) = \mu - \frac{\phi}{R-p}$. From Lemma C.3(1), we have $U_W(\Lambda_1) < 0$.
   2.1 If $0 < \theta \le R - p - \frac{\phi}{\mu-\Lambda_1} < \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2) and (2.2), we have $U_D(\Lambda_1) \ge 0 > U_W(\Lambda_1)$. Similar to the Case 2.1 when $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have that all tech-savvy customers will join and use the delivery service, i.e., $\lambda_D = \Lambda_1$ and $\lambda_W = 0$.
   2.2 If $R - p - \frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$, from Lemma C.3(2), (2.2), and (3.3), we have $\lambda^X(\theta) \le \lambda_W^X \le \lambda_D^X(\theta) < \Lambda_1$ and $U_D(\Lambda_1) < 0$. Similar to the Case 2.2 when $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have that tech-savvy customers join and use the delivery service with rate $\lambda_D^X(\theta)$ and other customers balk; i.e., $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$, and $\lambda_W = 0$. Here, all customers have zero utility.
   2.3 If $\frac{(c-\phi)(R-p)}{c} < \theta \le R - p - \frac{\phi}{\mu}$, we have $\lambda_D^X(\theta) < \lambda_W^X < \lambda^X(\theta)$ by Lemma C.3(3.3), and $\lambda_D^X(\theta) < \lambda_W^X < \Lambda_1$. Similar to Case 2.2 above, we have $\lambda_D = 0$ and $\lambda_W = \mu - \frac{c}{R-p}$.
3. If $R - \frac{\phi}{\mu-\Lambda_1} < p \le R - \frac{c}{\mu}$, we have $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < \frac{(c-\phi)(R-p)}{c} \le R - p - \frac{\phi}{\mu}$ and $\mu - \frac{c}{R-p} \ge 0$.
   3.1 If $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < \theta \le \frac{(c-\phi)(R-p)}{c}$, similar to Case 2.2, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.
   3.2 If $\frac{(c-\phi)(R-p)}{c} < \theta \le R - p - \frac{\phi}{\mu}$, similar to Case 2.3, we have $\lambda_D = 0$ and $\lambda_W = \mu - \frac{c}{R-p}$.
4. If $R - \frac{\phi}{\mu-\Lambda_1} < R - \frac{c}{\mu} < p \le R - \frac{\phi}{\mu}$, we have $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < R - p - \frac{\phi}{\mu} < \frac{(c-\phi)(R-p)}{c}$ and $\mu - \frac{c}{R-p} < 0$. From Lemma C.3, we have $U_W(\Lambda_1) < 0$.
   4.1 If $R - p - \frac{\phi}{\mu-\Lambda_1} < 0 < \theta \le R - p - \frac{\phi}{\mu}$, similar to Case 2.2, we have $\lambda_D = \mu - \frac{\phi}{R-p-\theta}$ and $\lambda_W = 0$.

To summarize, in equilibrium, the joining rates of food-delivery and walk-in customers, $\lambda_D$ and $\lambda_W$, under the food price $p$ and delivery fee $\theta$, are

1. For $\Lambda_1 \le \frac{c-\phi}{c}\mu$,

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| 1.1 When $p \le R - \frac{c}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le \frac{c-\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| if $\frac{c-\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ | $\Lambda_1$ |
| 1.2 When $R-\frac{c}{\mu-\Lambda_1} < p \le R-\frac{c}{\mu}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| if $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ | $\mu-\frac{c}{R-p}$ |
| 1.3 When $R-\frac{c}{\mu} < p \le R-\frac{\phi}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| if $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |
| 1.4 When $R-\frac{\phi}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |

2. For $\Lambda_1 > \frac{c-\phi}{c}\mu$,

| | $\lambda_D(p,\theta)$ | $\lambda_W(p,\theta)$ |
|---|---|---|
| 2.1 When $p \le R - \frac{c}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le \frac{c-\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| if $\frac{c-\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ | $\Lambda_1$ |
| 2.2 When $R-\frac{c}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu-\Lambda_1}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | $0$ |
| if $R-p-\frac{\phi}{\mu-\Lambda_1} < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ | $\mu-\frac{c}{R-p}$ |
| 2.3 When $R-\frac{\phi}{\mu-\Lambda_1} < p \le R-\frac{c}{\mu}$ | | |
| if $0 < \theta \le \frac{(c-\phi)(R-p)}{c}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |
| if $\frac{(c-\phi)(R-p)}{c} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ | $\mu-\frac{c}{R-p}$ |
| 2.4 When $R-\frac{c}{\mu} < p \le R-\frac{\phi}{\mu}$ | | |
| if $0 < \theta \le R-p-\frac{\phi}{\mu}$ | $\mu-\frac{\phi}{R-p-\theta}$ | $0$ |

This result directly leads to Lemma D.11. □

### D.1.2. Food Delivery Platform Strategy

From Lemma D.11, we obtain the platform's profit.

COROLLARY D.6. *When there are no traditional customers, i.e., $\Lambda_0 = 0$, under the food price $p$ and delivery fee $\theta$, the platform's profit $\pi(p,\theta)$ is*

| | $\pi(p,\theta)$ |
|---|---|
| *When* $p \le R-\frac{c}{\mu-\Lambda_1}$ | |
| *if* $0 < \theta \le \frac{c-\phi}{\mu-\Lambda_1}$ | $\theta\Lambda_1$ |
| *if* $\frac{c-\phi}{\mu-\Lambda_1} < \theta \le R-p-\frac{\phi}{\mu}$ | $0$ |
| *When* $R-\frac{c}{\mu-\Lambda_1} < p \le R-\frac{\phi}{\mu}$ | |
| *if* $0 < \theta \le \max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right)$ | $\theta\Lambda_1$ |
| *if* $\max\left(R-p-\frac{\phi}{\mu-\Lambda_1}, 0\right) < \theta \le \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ | $\theta\left(\mu-\frac{\phi}{R-p-\theta}\right)$ |
| *if* $\theta > \min\left(\frac{(c-\phi)(R-p)}{c}, R-p-\frac{\phi}{\mu}\right)$ | $0$ |

From Corollary D.6, we see that, similar to the case when there is abundant traditional customers (i.e., $\Lambda_0 > \mu$ in Corollary C.3), the platform will set the delivery fee such that either all tech-savvy customers will use the food delivery service, or a fraction of them will use the food delivery service. Different from Corollary C.3, in the first case when platform serves all tech-savvy customers, if the food price is relatively low, i.e., $p \le R - \frac{c}{\mu-\Lambda_1}$, the highest delivery fee decreases from $\frac{(c-\phi)(R-p)}{c}$ to $\frac{c-\phi}{\mu-\Lambda_1}$.

Proposition D.7 **(Food Delivery Platform Strategy—Tech-Savvy Only)**. *When there are no traditional customers, i.e., $\Lambda_0 = 0$, under the restaurant's food price $p$, the platform's best-response delivery fee $\theta^*(p)$, the joining rates of food-delivery and walk-in customers $\lambda_D(p,\theta^*(p))$ and $\lambda_W(p,\theta^*(p))$, are*

1. *For $\Lambda_1 \le \frac{c-\phi}{c}\mu$,*

| | $\theta^*(p)$ | $\lambda_D(p,\theta^*(p))$ | $\lambda_W(p,\theta^*(p))$ |
|---|---|---|---|
| *If* $p \le R - \frac{c}{\mu-\Lambda_1}$ | $\frac{c-\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $R - p - \frac{\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| *If* $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < p \le R - \frac{\phi}{\mu}$ | $R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\mu - \sqrt{\frac{\mu\phi}{R-p}}$ | 0 |

2. *For $\Lambda_1 > \frac{c-\phi}{c}\mu$,*

| | $\theta^*(p)$ | $\lambda_D(p,\theta^*(p))$ | $\lambda_W(p,\theta^*(p))$ |
|---|---|---|---|
| *If* $p \le R - \frac{c}{\mu-\Lambda_1}$ | $\frac{c-\phi}{\mu-\Lambda_1}$ | $\Lambda_1$ | 0 |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c^2}{\mu\phi}$ | $\frac{(c-\phi)(R-p)}{c}$ | $\mu - \frac{c}{R-p}$ | 0 |
| *If* $R - \frac{c^2}{\mu\phi} < p \le R - \frac{\phi}{\mu}$ | $R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\mu - \sqrt{\frac{\mu\phi}{R-p}}$ | 0 |

*Moreover, the food delivery platform's profit under the platform's best response delivery fee $\pi^*(p)$ is a weakly decreasing function of the food price $p$.*

*Proof of Proposition D.7.* When $\Lambda_1 \le \frac{c-\phi}{c}\mu$,

1. If $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $\frac{c-\phi}{\mu-\Lambda_1} \le \frac{(c-\phi)(R-p)}{c} \le R - p - \frac{\phi}{\mu-\Lambda_1}$. From Lemma D.11, tech-savvy customers are willing to pay a delivery fee at most $\frac{c-\phi}{\mu-\Lambda_1}$ for the delivery service, and the demand for the delivery service is $\lambda_D = \Lambda_1$ when $\theta \le \frac{c-\phi}{\mu-\Lambda_1}$. Therefore, the platform will charge the highest food delivery fee $\theta^* = \frac{c-\phi}{\mu-\Lambda_1}$ to maximize its profit, we have $\lambda_D = \Lambda_1$ and $\lambda_W = 0$. The food delivery platform's profit $\pi^*(p) = \frac{c-\phi}{\mu-\Lambda_1}\Lambda_1$ is a constant regarding the food price $p$.
2. If $p > R - \frac{c}{\mu-\Lambda_1}$, the discussion follows Cases 2 and 3 when $\Lambda_1 \le \frac{c-\phi}{c}\mu$ in the proof of Proposition C.3, and we have the food delivery platform's profit under the platform's best response delivery fee $\pi^*(p)$ is a a decreasing function of food price $p$.

When $\Lambda_1 > \frac{c-\phi}{c}\mu$,

1. If $p \le R - \frac{c}{\mu-\Lambda_1}$, we have $\frac{c-\phi}{\mu-\Lambda_1} \le \frac{(c-\phi)(R-p)}{c} \le R - p - \frac{\phi}{\mu-\Lambda_1} < R - p - \frac{\phi}{\mu}$. From Lemma D.11, similar to Case 1 when $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have $\theta^* = \frac{c-\phi}{\mu-\Lambda_1}$, $\lambda_D = \Lambda_1$, and $\lambda_W = 0$. The food delivery platform's profit $\pi^*(p) = \frac{c-\phi}{\mu-\Lambda_1}\Lambda_1$ is a constant regarding the food price $p$.

2. If $p > R - \frac{c}{\mu - \Lambda_1}$, the discussion follows Cases 2 and 3 when $\Lambda_1 > \frac{c-\phi}{c}\mu$ in the proof of Proposition C.3, and we have the food delivery platform's profit under the platform's best response delivery fee $\pi^*(p)$ is a a decreasing function of food price $p$. $\square$

### D.1.3. Restaurant Strategy

From Corollary D.6, we obtain the restaurant's profit.

Corollary D.7. *When there are no traditional customers, i.e.,* $\Lambda_0 = 0$, *under the food price* $p$, *the restaurant's profit* $\Pi(p)$ *is*

1. *For* $\Lambda_1 \le \frac{c-\phi}{c}\mu$,

| | $\Pi(p)$ |
|---|---|
| *If* $p \le R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $p\Lambda_1$ |
| *If* $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < p \le R - \frac{\phi}{\mu}$ | $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ |

2. *For* $\Lambda_1 > \frac{c-\phi}{c}\mu$,

| | $\Pi(p)$ |
|---|---|
| *If* $p \le R - \frac{c}{\mu-\Lambda_1}$ | $p\Lambda_1$ |
| *If* $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c^2}{\mu\phi}$ | $p\left(\mu - \frac{c}{R-p}\right)$ |
| *If* $R - \frac{c^2}{\mu\phi} < p \le R - \frac{\phi}{\mu}$ | $p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$ |

The following proposition characterizes the restaurant's optimal strategy as a Stackelberg leader.

Proposition D.8 **(Restaurant Strategy—Tech-Savvy Only)**. *When there are no traditional customers, i.e.,* $\Lambda_0 = 0$, *there exist threshold values* $\phi_1$, $\phi_2$, $\lambda_2$, *and* $\lambda_3$, *such that, in equilibrium, the restaurant's optimal price* $p^*$, *profit* $\Pi^*$, *and throughput* $\lambda_D^* + \lambda_W^*$ *are*

| | $p^*(\Lambda_1)$ | $\Pi^*(\Lambda_1)$ | $\lambda_D^* + \lambda_W^*$ |
|---|---|---|---|
| *When* $\phi \le \phi_1$ | | | |
| *if* $\Lambda_1 \le \lambda_2$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ | $\Lambda_1$ |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\mu-\chi$ |
| *When* $\phi_1 < \phi \le \phi_2$ | | | |
| *if* $\Lambda_1 \le \lambda_2$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ | $\Lambda_1$ |
| *if* $\lambda_2 < \Lambda_1 \le \lambda_3$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\mu-\chi$ |
| *if* $\lambda_3 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $R - \frac{c}{\mu-\Lambda_1}$ | $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ | $\Lambda_1$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $R - \sqrt{\frac{c}{\mu}R}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $\mu - \sqrt{\frac{c\mu}{R}}$ |
| *When* $\phi > \phi_2$ | | | |
| *if* $\Lambda_1 \le \frac{c-\phi}{c}\mu$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ | $\Lambda_1$ |
| *if* $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $R - \frac{c}{\mu-\Lambda_1}$ | $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ | $\Lambda_1$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $R - \sqrt{\frac{c}{\mu}R}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $\mu - \sqrt{\frac{c\mu}{R}}$ |

*Moreover, we have*

*(i)* $\Pi^*(\Lambda_1)$ *is a weakly increasing function of* $\Lambda_1$ *for* $\forall\phi$.

*(ii)* $p^* \geq p_T^*$ *and* $\Pi^* \geq \Pi_T^*$*, if (i)* $\phi \leq \phi_1$*; or (ii)* $\phi_1 < \phi \leq \phi_2$ *and* $\Lambda_1 \leq \lambda_3$*; or (iii)* $\phi > \phi_2$ *and* $\Lambda_1 \leq (c-\phi)\mu/c$*; and* $p^* = p_T^*$ *and* $\Pi^* = \Pi_T^*$*, if (i)* $\phi_1 < \phi \leq \phi_2$ *and* $\Lambda_1 > \lambda_3$*; or (ii)* $\phi > \phi_2$ *and* $\Lambda_1 > (c-\phi)\mu/c$.

*Proof of Proposition D.8* Recall from Corollary D.7 that (i) For $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, the restaurant's profit $\Pi(p)$ linearly increases with $p$ on $\left(0, R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right]$, so the restaurant's maximum profit $\Pi^*$ is $\max_{p\in\left[R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}, R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$. (ii) For $\Lambda_1 > \frac{c-\phi}{c}\mu$, the restaurant's profit $\Pi(p)$ linearly increases with $p$ on $\left(0, R-\frac{c}{\mu-\Lambda_1}\right]$, so we only need to compare $\max_{p\in\left[R-\frac{c}{\mu-\Lambda_1}, R-\frac{c^2}{\mu\phi}\right]} p\left(\mu-\frac{c}{R-p}\right)$ and $\max_{p\in\left(R-\frac{c^2}{\mu\phi}, R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ to determine the restaurant's maximum profit $\Pi^*$.

1. If $\phi \leq \phi_1$, following the same discussion in the proof of Proposition C.4, we have $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)^2}{2\mu-\chi} \geq \max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\frac{c}{R-p}\right) = \left(\sqrt{R\mu}-\sqrt{c}\right)^2$, and $\arg\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)}{2\mu-\chi} \geq R-\frac{c^2}{\mu\phi}$.
   - If $\Lambda_1 \leq \lambda_2$, the maximum point of $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ is in $\left(0, R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right]$ by the definition of $\lambda_2$. We have $\max_{p\in\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}, R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) = p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}}$. In this case, $\Pi^*(\Lambda_1) = \left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ and $p^*(\Lambda_1) = R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$. Note that $\left(R-\frac{\mu\phi}{(\mu-\lambda)^2}\right)\lambda$ is a unimodal function with the maximum at $\lambda_2$, so $\Pi^*(\Lambda_1)$ increases in $\Lambda_1$ on $(0, \lambda_2]$.
   - If $\lambda_2 < \Lambda_1 \leq \frac{c-\phi}{c}\mu$, the maximum point of $p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ is in $\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}, R-\frac{\phi}{\mu}\right]$ by the definition of $\lambda_2$. In this case, $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.
   - If $\Lambda_1 > \frac{c-\phi}{c}\mu$, $\max_{p\in\left(R-\frac{c}{\mu-\Lambda_1}, R-\frac{c^2}{\mu\phi}\right]} p\left(\mu-\frac{c}{R-p}\right) \leq \max_{p\in\left(R-\frac{c^2}{\mu\phi}, R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) = \max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)}{2\mu-\chi}$. In this case, $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.

   To summarize, the restaurant's maximum profit $\Pi^*$ and the profit-maximizing price $p^*$ are

   | | $\Pi^*(\Lambda_1)$ | $p^*(\Lambda_1)$ |
   |---|---|---|
   | if $0 < \Lambda_1 \leq \lambda_2$ | $\Lambda_1\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ | $R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$ |
   | if $\Lambda_1 > \lambda_2$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ |

   We next compare $p^*(\Lambda_1)$ and $\Pi^*(\Lambda_1)$ to the restaurant's revenue maximizing food price and maximum revenue without food delivery service, $p_T^*(\Lambda_1)$ and $\Pi_T^*(\Lambda_1)$. In a classical unobservable queue without food delivery service (see, e.g., Edelson and Hilderbrand 1975), we have

   $$p_T^*(\Lambda_1) = \begin{cases} R-\frac{c}{\mu-\Lambda_1} & \text{if } \Lambda_1 \leq \mu-\sqrt{\frac{c\mu}{R}} \\ R-\sqrt{cR/\mu} & \text{if } \Lambda_1 > \mu-\sqrt{\frac{c\mu}{R}} \end{cases} \text{ and } \Pi_T^*(\Lambda_1) = \begin{cases} \Lambda_1\left(R-\frac{c}{\mu-\Lambda_1}\right) & \text{if } \Lambda_1 \leq \mu-\sqrt{\frac{c\mu}{R}} \\ \left(\sqrt{R\mu}-\sqrt{c}\right)^2 & \text{if } \Lambda_1 > \mu-\sqrt{\frac{c\mu}{R}} \end{cases}.$$

   Note that given $\phi \leq \phi_1$, we have $\lambda_2 \leq \frac{c-\phi}{c}\mu$ and $\frac{2R(\mu-\chi)^2}{2\mu-\chi} \geq \left(\sqrt{R\mu}-\sqrt{c}\right)^2$.

   For $\lambda_2 \leq \mu-\sqrt{\frac{c\mu}{R}}$, if $0 < \Lambda_1 \leq \lambda_2$, we have $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, which implies $p^*(\Lambda_1) = R-\frac{\mu\phi}{(\mu-\Lambda_1)^2} > p_T^*(\Lambda_1) = R-\frac{c}{\mu-\Lambda_1}$. Moreover, we have $\Pi^*(\Lambda_1) = p^*(\Lambda_1)\Lambda_1 > \Pi_T^*(\Lambda_1) = p_T^*(\Lambda_1)\Lambda_1$. If $\Lambda_1 > \lambda_2$,

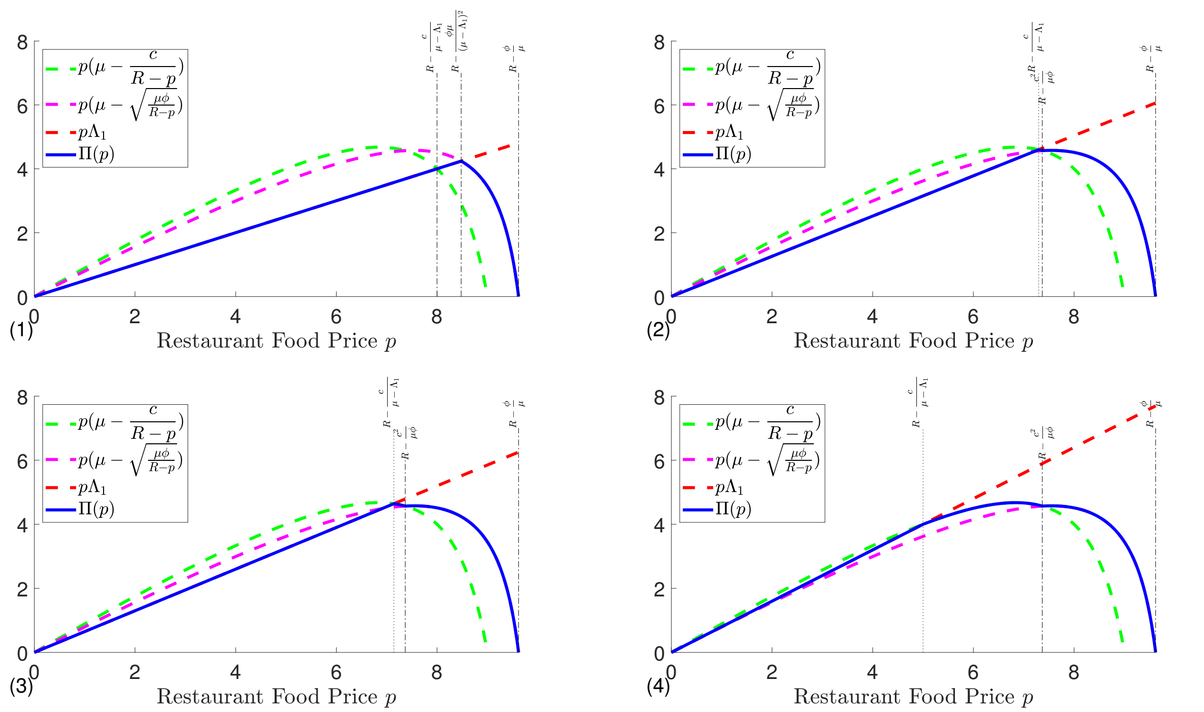


**Figure D.17** **The restaurant's profit** $\Pi$ **as a function of** $p$**, when** $\Lambda_1=0.5, 0.63, 0.65, 0.8$**, for the parameter setting** $R=10$**,** $\mu=c=1$**,** $\Lambda_0=0$**, and** $\phi=0.38$**.**

we have $p^*(\Lambda_1)$ is a constant regarding $\Lambda_1$ while $p_T^*(\Lambda_1)$ decreases with $\Lambda_1$ on $\left(\lambda_2,\mu-\sqrt{\frac{c\mu}{R}}\right]$ and then stays constant, which implies $p^*(\Lambda_1)\ge p_T^*(\Lambda_1)$. From $\phi\le\phi_1$, we have $\frac{2R(\mu-\chi)^2}{2\mu-\chi}\ge\left(\sqrt{R\mu}-\sqrt{c}\right)^2\Leftrightarrow\Pi^*(\Lambda_1)\ge\Pi_T^*(\Lambda_1)$.

Otherwise, for $\lambda_2>\mu-\sqrt{\frac{c\mu}{R}}$, if $0<\Lambda_1\le\mu-\sqrt{\frac{c\mu}{R}}$, we have $\Lambda_1\le\frac{c-\phi}{c}\mu$, which implies $p^*(\Lambda_1)=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}>p_T^*(\Lambda_1)=R-\frac{c}{\mu-\Lambda_1}$. Moreover, we have $\Pi^*(\Lambda_1)=p^*(\Lambda_1)\Lambda_1>\Pi_T^*(\Lambda_1)=p_T^*(\Lambda_1)\Lambda_1$. If $\Lambda_1>\mu-\sqrt{\frac{c\mu}{R}}$, we have $p_T^*(\Lambda_1)=R-\sqrt{\frac{c}{\mu}R}$ is a constant regarding $\Lambda_1$, while $p^*(\Lambda_1)$ decreases with $\Lambda_1$ on $\left(\mu-\sqrt{\frac{c\mu}{R}},\lambda_2\right]$ and then equals to $\frac{2R(\mu-\chi)}{2\mu-\chi}$. We have $R-\sqrt{\frac{c}{\mu}R}<\frac{2R(\mu-\chi)}{2\mu-\chi}$ by Lemma C.7(4), which implies $p^*(\Lambda_1)\ge p_T^*(\Lambda_1)$. Moreover, we have $\Pi^*(\Lambda_1)$ is a nondecreasing function of $\Lambda_1$ while $\Pi_T^*(\Lambda_1)$ is a constant regarding $\Lambda_1$, which implies $\Pi^*(\Lambda_1)\ge\Pi_T^*(\Lambda_1)$.

2. If $\phi_1<\phi\le\phi_2\equiv\frac{\sqrt{c^3(c+8R\mu)}-c^2}{2R\mu}$, we have $\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)^2}{2\mu-\chi}<\max_{p\in\left(0,R-\frac{\phi}{\mu}\right]}p\left(\mu-\frac{c}{R-p}\right)=\left(\sqrt{R\mu}-\sqrt{c}\right)^2$, $\arg\max p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)=\frac{2R(\mu-\chi)}{2\mu-\chi}\ge R-\frac{c^2}{\mu\phi}$, and $\lambda_2\le\frac{c-\phi}{c}\mu$. Figure D.17 illustrates the restaurant's profit as a function of $p$ under different $\Lambda_1$. Note that when $\frac{c-\phi}{c}\mu<\Lambda_1\le\mu-\sqrt{\frac{c\mu}{R}}$, we have $\arg\max p\left(\mu-\frac{c}{R-p}\right)=R-\sqrt{\frac{c}{\mu}R}\le R-\frac{c}{\mu-\Lambda_1}$, which implies $\max_{p\in\left[R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]}p\left(\mu-\frac{c}{R-p}\right)=p\left(\mu-\frac{c}{R-p}\right)\Big|_{p=R-\frac{c}{\mu-\Lambda_1}}=\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1$. So we need to compare $\left(R-\frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ and $\max_{p\in\left(R-\frac{c^2}{\mu\phi},R-\frac{\phi}{\mu}\right]}p\left(\mu-\sqrt{\frac{\mu\phi}{R-p}}\right)$ to determine the restaurant's maximum profit $\Pi^*$.

- If $\Lambda_1\le\lambda_2$, similar to the $\Lambda_1\le\lambda_2$ case discussed in Case 1, as Figure D.17(1) shows, we have $\Pi^*(\Lambda_1)=\left(R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ and $p^*(\Lambda_1)=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}$. We have $\Pi^*(\Lambda_1)$ increases in $\Lambda_1$ on

$(0, \lambda_2]$.

- If $\lambda_2 < \Lambda_1 \leq \frac{c-\phi}{c}\mu$, similar to the $\lambda_2 < \Lambda_1 \leq \frac{c-\phi}{c}\mu$ discussed in $\phi \leq \phi_1$ case, we have $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.
- If $\frac{c-\phi}{c}\mu < \Lambda_1 \leq \lambda_3 \leq \mu - \sqrt{\frac{c\mu}{R}}$, we have $\arg\max p\left(\mu - \frac{c}{R-p}\right) = R - \sqrt{\frac{c}{\mu}R} \leq R - \frac{c}{\mu-\Lambda_1}$ and $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1 \leq \frac{2R(\mu-\chi)^2}{2\mu-\chi}$, as illustrated in Figure D.17(2). In this case, $\Pi^*(\Lambda_1) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$ and $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.
- If $\lambda_3 < \Lambda_1 \leq \mu - \sqrt{\frac{c\mu}{R}}$, we have $\arg\max p\left(\mu - \frac{c}{R-p}\right) = R - \sqrt{\frac{c}{\mu}R} \leq R - \frac{c}{\mu-\Lambda_1}$ and $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1 > \frac{2R(\mu-\chi)^2}{2\mu-\chi}$, as illustrated in Figure D.17(3). In this case, $\Pi^*(\Lambda_1) = \left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ and $p^*(\Lambda_1) = R - \frac{c}{\mu-\Lambda_1}$. Note that $\left(R - \frac{c}{\mu-\lambda}\right)\lambda$ is a unimodal function with the maximum at $\mu - \sqrt{\frac{c\mu}{R}}$, so $\Pi^*(\Lambda_1)$ increases in $\Lambda_1$ on $\left(\lambda_3, \mu - \sqrt{\frac{c\mu}{R}}\right]$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, we have $R - \frac{c}{\mu-\Lambda_1} \leq R - \sqrt{\frac{c}{\mu}R} < R - \frac{c^2}{\mu\phi}$, which implies $\max_{p\in\left(R-\frac{c}{\mu-\Lambda_1}, R-\frac{c^2}{\mu\phi}\right]} p\left(\mu - \frac{c}{R-p}\right) = \max_{p\in\left(0, R-\frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) > \max_{p\in\left(0, R-\frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)^2}{2\mu-\chi}$, as illustrated in Figure D.17(4). In this case, $\Pi^* = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$ and $p^* = R - \sqrt{\frac{c}{\mu}R}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.

To summarize, the restaurant's maximum profit $\Pi^*$ and the profit-maximizing price $p^*$ are

| | $\Pi^*(\Lambda_1)$ | $p^*(\Lambda_1)$ |
|---|---|---|
| if $0 < \Lambda_1 \leq \lambda_2$ | $\Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ |
| if $\lambda_2 < \Lambda_1 \leq \lambda_3$ | $\frac{2R(\mu-\chi)^2}{2\mu-\chi}$ | $\frac{2R(\mu-\chi)}{2\mu-\chi}$ |
| if $\lambda_3 < \Lambda_1 \leq \mu - \sqrt{\frac{c\mu}{R}}$ | $\Lambda_1\left(R - \frac{c}{\mu-\Lambda_1}\right)$ | $R - \frac{c}{\mu-\Lambda_1}$ |
| if $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $R - \sqrt{\frac{c}{\mu}R}$ |

We next compare $p^*(\Lambda_1)$ and $\Pi^*(\Lambda_1)$ to the restaurant's revenue maximizing food price and maximum revenue without food delivery service, $p_T^*(\Lambda_1)$ and $\Pi_T^*(\Lambda_1)$, respectively. Note that given $\phi \leq \phi_2$, we have $\lambda_2 \leq \frac{c-\phi}{c}\mu \leq \lambda_3 < \mu - \sqrt{\frac{c\mu}{R}}$ and $\frac{2R(\mu-\chi)^2}{2\mu-\chi} < \left(\sqrt{R\mu} - \sqrt{c}\right)^2$. If $\Lambda_1 \leq \lambda_2$, similar to the $\lambda_2 \leq \mu - \sqrt{\frac{c\mu}{R}}$ case in Case 1, we have $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, which implies $p^*(\Lambda_1) = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} \geq p_T^*(\Lambda_1) = R - \frac{c}{\mu-\Lambda_1}$ and $\Pi^*(\Lambda_1) = p^*(\Lambda_1)\Lambda_1 \geq \Pi_T^*(\Lambda_1) = p_T^*(\Lambda_1)\Lambda_1$. If $\lambda_2 < \Lambda_1 \leq \lambda_3$, we have $p^*(\Lambda_1)$ is a constant regarding $\Lambda_1$, while $p_T^*(\Lambda_1)$ decreases with $\Lambda_1$, which implies $p^*(\Lambda_1) \geq p_T^*(\Lambda_1)$. Moreover, by the definition of $\lambda_3$, we have $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1 \leq \frac{2R(\mu-\chi)^2}{2\mu-\chi}$, which implies $\Pi^*(\Lambda_1) \geq \Pi_T^*(\Lambda_1)$. If $\Lambda_1 > \lambda_3$, clearly we have $p^*(\Lambda_1) = p_T^*(\Lambda_1)$ and $\Pi^*(\Lambda_1) = \Pi_T^*(\Lambda_1)$.

3. If $\phi > \phi_2$, we have $\max_{p\in\left(0, R-\frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)^2}{2\mu-\chi} < \max_{p\in\left(0, R-\frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$, $\arg\max p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)}{2\mu-\chi} < R - \frac{c^2}{\mu\phi}$, and $\frac{c-\phi}{c}\mu < \lambda_2$. Figure D.18 illustrates the restaurant's profit as a function of $p$ under different $\Lambda_1$.

- If $\Lambda_1 \leq \frac{c-\phi}{c}\mu$, similar to the $\Lambda_1 \leq \lambda_2$ case discussed in Case 1, as Figure D.18(1) shows, we have $\Pi^*(\Lambda_1) = \left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1$ and $p^*(\Lambda_1) = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$. Note that $\Pi^*(\Lambda_1)$ increases with $\Lambda_1$ on $\left(0, \frac{c-\phi}{c}\mu\right]$.

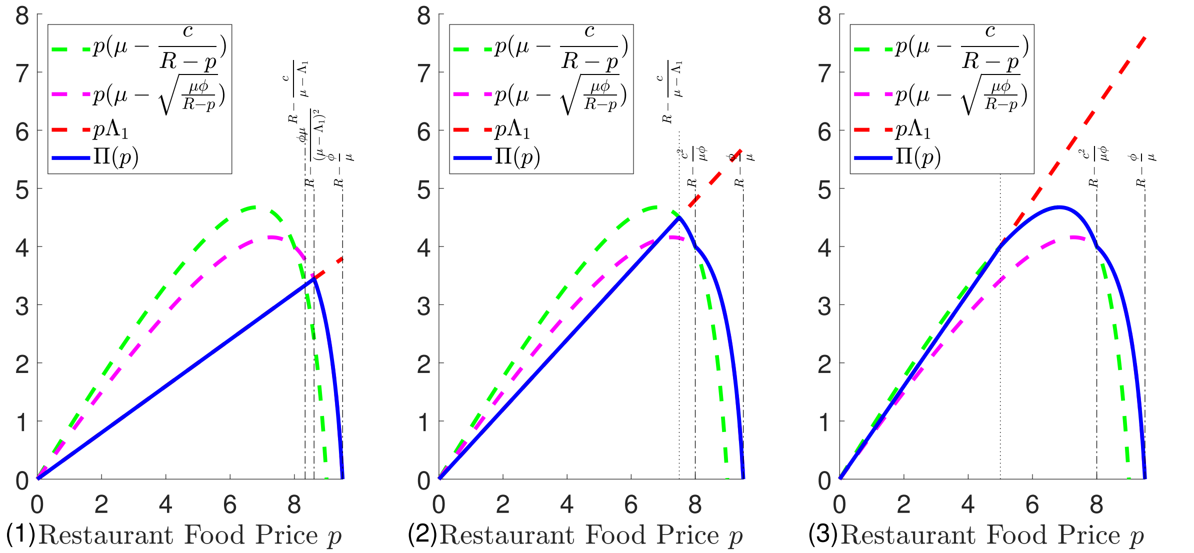


**Figure D.18** **The restaurant's profit $\Pi$ as a function of $p$, when $\Lambda_1 = 0.4,\ 0.6,\ 0.8$, for the parameter setting $R = 10$, $\Lambda_0 = 0$, $\mu = c = 1$, and $\phi = 0.5$.**

- If $\frac{c-\phi}{c}\mu < \Lambda \le \mu - \sqrt{\frac{c\mu}{R}}$, we have $\arg\max p\left(\mu - \frac{c}{R-p}\right) = R - \sqrt{\frac{c}{\mu}R} < R - \frac{c}{\mu-\Lambda_1}$ and $\arg\max p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right) = \frac{2R(\mu-\chi)}{2\mu-\chi} < R - \frac{c^2}{\mu\phi}$, as illustrated in Figure D.18(2). In this case, $\Pi^*(\Lambda_1) = \left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ and $p^*(\Lambda_1) = R - \frac{c}{\mu-\Lambda_1}$. Note that $\Pi^*(\Lambda_1)$ increases with $\Lambda_1$ on $\left(\frac{c-\phi}{c}\mu, \mu - \sqrt{\frac{c\mu}{R}}\right]$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, we have $R - \frac{c}{\mu-\Lambda_1} \le R - \sqrt{\frac{c}{\mu}R} < R - \frac{c^2}{\mu\phi}$, which implies $\max_{p\in\left(R-\frac{c}{\mu-\Lambda_1}, R-\frac{c^2}{\mu\phi}\right]} p\left(\mu - \frac{c}{R-p}\right) = \max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu - \frac{c}{R-p}\right) > \max_{p\in\left(0,R-\frac{\phi}{\mu}\right]} p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)$, as illustrated in Figure D.18(3). In this case, $\Pi^* = \left(\sqrt{R\mu} - \sqrt{c}\right)^2$ and $p^* = R - \sqrt{\frac{c}{\mu}R}$. Note that $\Pi^*(\Lambda_1)$ here is a constant regarding $\Lambda_1$.

To summarize, the restaurant's maximum profit $\Pi^*$ and the profit-maximizing price $p^*$ are

| | $\Pi^*(\Lambda_1)$ | $p^*(\Lambda_1)$ |
|---|---|---|
| if $0 < \Lambda_1 \le \frac{c-\phi}{c}\mu$ | $\Lambda_1\left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)$ | $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ |
| if $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $\Lambda_1\left(R - \frac{c}{\mu-\Lambda_1}\right)$ | $R - \frac{c}{\mu-\Lambda_1}$ |
| if $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $\left(\sqrt{R\mu} - \sqrt{c}\right)^2$ | $R - \sqrt{\frac{c}{\mu}R}$ |

We next compare $p^*(\Lambda_1)$ and $\Pi^*(\Lambda_1)$ to the restaurant's revenue maximizing food price and maximum revenue without food delivery service, $p_T^*(\Lambda_1)$ and $\Pi_T^*(\Lambda_1)$. If $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have $p^*(\Lambda_1) \ge p_T^*(\Lambda_1)$ and $\Pi^*(\Lambda_1) = \Lambda_1 p^*(\Lambda_1) \ge \Pi_T^*(\Lambda_1) = \Lambda_1 p_T^*(\Lambda_1)$. If $\Lambda_1 > \frac{c-\phi}{c}\mu$, clearly we have $p^*(\Lambda_1) = p_T^*(\Lambda_1)$ and $\Pi^*(\Lambda_1) = \Pi_T^*(\Lambda_1)$. $\square$

Given the restaurant's optimal food price $p^*(\Lambda_1)$ in Proposition D.8, we can use Proposition D.7 to obtain the platform's optimal delivery fee and corresponding profit in equilibrium of the Stackelberg game.

COROLLARY D.8 **(Food Delivery Platform's Profit—Tech-Savvy Only)**. *When there are no traditional customers, i.e.,* $\Lambda_0 = 0$*, there exist threshold values* $\phi_1$*,* $\phi_2$*,* $\lambda_2$*, and* $\lambda_3$*, such that, in equilibrium, the food delivery platform's delivery fee* $\theta^*$*, equilibrium profit* $\pi^*$*, the joining rates of food-delivery and walk-in customers* $\lambda_D^*$ *and* $\lambda_W^*$ *under the restaurant's equilibrium price* $p^*$ *are*

| | $\theta^*(p^*(\Lambda_1))$ | $\pi^*(p^*(\Lambda_1))$ | $\lambda_D^*$ | $\lambda_W^*$ |
|---|---|---|---|---|
| *When* $\phi \le \phi_1$ | | | | |
| *if* $\Lambda_1 \le \lambda_2$ | $\frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$ | $\frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}$ | $\Lambda_1$ | 0 |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$ | $\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ | $\mu-\chi$ | 0 |
| *When* $\phi_1 < \phi \le \phi_2$ | | | | |
| *if* $\Lambda_1 \le \lambda_2$ | $\frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$ | $\frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}$ | $\Lambda_1$ | 0 |
| *if* $\lambda_2 < \Lambda_1 \le \lambda_3$ | $\frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$ | $\frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ | $\mu-\chi$ | 0 |
| *if* $\lambda_3 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $\frac{c-\phi}{\mu-\Lambda_1}$ | $\left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ | $\Lambda_1$ | 0 |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $(c-\phi)\left(\sqrt{\frac{R\mu}{c}}-1\right)$ | $\mu-\sqrt{\frac{c\mu}{R}}$ | 0 |
| *When* $\phi > \phi_2$ | | | | |
| *if* $\Lambda_1 \le \frac{c-\phi}{c}\mu$ | $\frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$ | $\frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}$ | $\Lambda_1$ | 0 |
| *if* $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $\frac{c-\phi}{\mu-\Lambda_1}$ | $\left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ | $\Lambda_1$ | 0 |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $(c-\phi)\sqrt{\frac{R}{c\mu}}$ | $(c-\phi)\left(\sqrt{\frac{R\mu}{c}}-1\right)$ | $\mu-\sqrt{\frac{c\mu}{R}}$ | 0 |

*Moreover, we have* $\pi^*(p^*)$ *is a weakly increasing function of* $\Lambda_1$ *for* $\forall\phi$*.*

*Proof of Corollary D.8* (i) When $\phi \le \phi_1$, similar to the $\phi \le \phi_1$ case in Proposition D.8, we have $\lambda_2 \le \frac{c-\phi}{c}\mu$.

- If $\Lambda_1 \le \lambda_2 \le \frac{c-\phi}{c}\mu$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} > R - \frac{c}{\mu-\Lambda_1}$ by Proposition D.8. From Proposition D.7(1), the platform's profit $\pi$ and profit-maximizing delivery fee $\theta^*$ are $\pi(p^*) = \Lambda_1\left(R - p - \frac{\phi}{\mu-\Lambda_1}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}} = \frac{\phi\Lambda_1^2}{(\mu-\Lambda_1)^2}$ and $\theta^*(p^*) = \frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$. Note that $\pi(p^*)$ increases with $\Lambda_1$ on $\left(0, \frac{c-\phi}{c}\mu\right]$.
- If $\Lambda_1 > \lambda_2$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} > R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ by Proposition D.8. From Proposition D.7(1), the platform's profit $\pi$ and profit-maximizing delivery fee $\theta^*$ are $\pi(p^*) = \left(R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}\right)\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\Big|_{p=\frac{2R(\mu-\chi)}{2\mu-\chi}} = \frac{\phi(\mu-\chi)^2}{\chi^2} = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ and $\theta^*(p^*) = \frac{\phi(\mu-\chi)}{\chi^2} = \frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$. Note that here $\pi(p^*)$ is a constant regarding $\Lambda_1$.

(ii) When $\phi_1 < \phi \le \phi_2$, we have

- If $\Lambda_1 \le \lambda_2$, similar to Case (i), $\pi(p^*) = \Lambda_1\left(R - p - \frac{\phi}{\mu-\Lambda_1}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}} = \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2}$ and $\theta^*(p^*) = \frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$. Note that $\pi(p^*)$ increases with $\Lambda_1$ on $(0, \lambda_2]$.
- If $\lambda_2 < \Lambda_1 \le \frac{c-\phi}{c}\mu$, similar to Case (i), $\pi(p^*) = p\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\Big|_{p=\frac{2R(\mu-\chi)}{2\mu-\chi}} = \frac{\phi(\mu-\chi)^2}{\chi^2} = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ and $\theta^*(p^*) = \frac{\phi(\mu-\chi)}{\chi^2} = \frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$. Note that here $\pi(p^*)$ is a constant regarding $\Lambda_1$.
- If $\frac{c-\phi}{c}\mu < \Lambda_1 \le \lambda_3$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = \frac{2R(\mu-\chi)}{2\mu-\chi} \ge R - \frac{c^2}{\mu\phi}$ by

Proposition D.8. From Proposition D.7(2), $\pi(p^*) = \left(R - p - \frac{\sqrt{\mu\phi(R-p)}}{\mu}\right)\left(\mu - \sqrt{\frac{\mu\phi}{R-p}}\right)\Big|_{p=\frac{2R(\mu-\chi)}{2\mu-\chi}} = \frac{\phi(\mu-\chi)^2}{\chi^2} = \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)}$ and $\theta^*(p^*) = \frac{\phi(\mu-\chi)}{\chi^2} = \frac{R\chi(\mu-\chi)}{\mu(2\mu-\chi)}$. Note that here $\pi(p^*)$ is a constant regarding $\Lambda_1$.

- If $\lambda_3 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \frac{c}{\mu-\Lambda_1}$ by Proposition D.8. From Proposition D.7(2), the platform's profit $\pi$ is $\pi(p^*) = \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1\Big|_{p=R-\frac{c}{\mu-\Lambda_1}} = \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ and $\theta^*(p^*) = \frac{c-\phi}{\mu-\Lambda_1}$. Note that $\pi(p^*)$ increases with $\Lambda_1$ on $\left(\lambda_3, \mu - \sqrt{\frac{c\mu}{R}}\right]$.

- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, the restaurant's profit-maximizing price is $p^*(\Lambda_1) = R - \sqrt{\frac{c}{\mu}R}$ and we have $R - \frac{c}{\mu-\Lambda_1} \le R - \sqrt{\frac{c}{\mu}R} < R - \frac{c^2}{\mu\phi}$ by Proposition D.8. From Proposition D.7(2), $\pi(p^*) = \left(\frac{(c-\phi)(R-p)}{c}\right)\left(\mu - \frac{c}{R-p}\right)\Big|_{p=R-\sqrt{\frac{c}{\mu}R}} = (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right)$ and $\theta^*(p^*) = \frac{(c-\phi)(R-p)}{c}$. Note that here $\pi(p^*)$ is a constant regarding $\Lambda_1$.

(iii) When $\phi > \phi_2$, following the same discussion as in the $\phi \le \phi_1$ and $\phi_1 < \phi \le \phi_2$ cases, we have

- If $\Lambda_1 \le \frac{c-\phi}{c}\mu$, the platform's profit $\pi$ is $\pi(p^*) = \Lambda_1\left(R - p - \frac{\phi}{\mu-\Lambda_1}\right)\Big|_{p=R-\frac{\mu\phi}{(\mu-\Lambda_1)^2}} = \frac{\phi\Lambda_1^2}{(\mu-\Lambda_1)^2}$ and $\theta^*(p^*) = \frac{\phi\Lambda_1}{(\mu-\Lambda_1)^2}$. Note that $\pi(p^*)$ increases with $\Lambda_1$ on $\left(0, \frac{c-\phi}{c}\mu\right]$.
- If $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, the platform's profit $\pi$ is $\pi(p^*) = \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1\Big|_{p=R-\frac{c}{\mu-\Lambda_1}} = \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ and $\theta^*(p^*) = \frac{c-\phi}{\mu-\Lambda_1}$. Note that $\pi(p^*)$ increases with $\Lambda_1$ on $\left(\frac{c-\phi}{c}\mu, \mu - \sqrt{\frac{c\mu}{R}}\right]$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, the platform's profit $\pi$ is $\pi(p^*) = (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right)$ and $\theta^*(p^*) = \frac{(c-\phi)(R-p)}{c}$.

Note that here $\pi(p^*)$ is a constant regarding $\Lambda_1$. □

#### D.1.4. Social Welfare

We next investigate social welfare, which is defined as the sum of the restaurant's equilibrium profit in Proposition D.8 and the platform's equilibrium profit in Corollary D.8 under the restaurant's optimal food price $p^*(\Lambda_1)$ in Proposition D.8 and the food delivery platform's best-response delivery fee $\theta^*(p^*)$ in Corollary D.8. Similar to Corollary C.4, customers have zero utility in equilibrium; otherwise, either the restaurant or the food delivery platform could raise the price without changing the throughput, which would lead to a higher profit.

COROLLARY D.9 **(Social Welfare—Tech-Savvy Only)**. *When there are no traditional customers, i.e., $\Lambda_0 = 0$, social welfare under the restaurant's optimal food price $p^*$ in Proposition D.8 and the food delivery platform's best response fee $\theta(p^*)$ in Corollary D.8 is*

| | $S^*(\Lambda_1)$ |
|---|---|
| *When* $\phi \le \phi_1$ | |
| *if* $\Lambda_1 \le \lambda_2$ | $R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$ |
| *if* $\Lambda_1 > \lambda_2$ | $\frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$ |
| *When* $\phi_1 < \phi \le \phi_2$ | |
| *if* $\Lambda_1 \le \lambda_2$ | $R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$ |
| *if* $\lambda_2 < \Lambda_1 \le \lambda_3$ | $\frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$ |
| *if* $\lambda_3 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $\phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$ |
| *When* $\phi > \phi_2$ | |
| *if* $\Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$ | $R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$ |
| *if* $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$ | $\phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$ |

*Moreover, we have* $S^*(\Lambda_1)$ *is a weakly increasing function of* $\Lambda_1$ *for* $\forall\phi$.

*Proof of Corollary D.9.* From Proposition D.8 and Corollary D.8, we have

(i) When $\phi \le \phi_1$,

- If $\Lambda_1 \le \lambda_2$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1 + \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2} = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\Lambda_1 > \lambda_2$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \frac{2R(\mu-\chi)^2}{2\mu-\chi} + \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)} = \frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$.

(ii) When $\phi_1 < \phi \le \phi_2$,

- If $\Lambda_1 \le \lambda_2$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1 + \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2} = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\lambda_2 < \Lambda_1 \le \lambda_3$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \frac{2R(\mu-\chi)^2}{2\mu-\chi} + \frac{R\chi(\mu-\chi)^2}{\mu(2\mu-\chi)} = \frac{R(2\mu+\chi)(\mu-\chi)^2}{\mu(2\mu-\chi)}$.
- If $\lambda_3 < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1 + \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1 = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2 + (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right) = \phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$.

(iii) When $\phi > \phi_2$,

- If $\Lambda_1 \le \frac{c-\phi}{c}\mu$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}\right)\Lambda_1 + \frac{\phi(\Lambda_1)^2}{(\mu-\Lambda_1)^2} = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\frac{c-\phi}{c}\mu < \Lambda_1 \le \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1 + \left(\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1 = R\Lambda_1 - \frac{\phi\Lambda_1}{\mu-\Lambda_1}$.
- If $\Lambda_1 > \mu - \sqrt{\frac{c\mu}{R}}$, we have $S(\Lambda_1) = \Pi^*(\Lambda_1) + \pi^*(p^*) = \left(\sqrt{R\mu} - \sqrt{c}\right)^2 + (c-\phi)\left(\sqrt{\frac{R\mu}{c}} - 1\right) = \phi + R\mu - (c+\phi)\sqrt{\frac{R\mu}{c}}$. □

We have Proposition A.1 from Proposition D.8, Corollary D.8 and Corollary D.9. □

### D.2. Proof of of Lemma A.1

Similar to Lemma 1, the optimal monopoly food price $p^o$ and delivery fee $\theta^o$ maximize not only the aggregated profit but also social welfare; and the centralized owner's goal of maximizing the aggregated profit aligns with a social planner's goal of maximizing social welfare.

Similar to the proof of Lemma 1, we first derive the socially optimal joining rates of food-delivery and walk-in customers.

Under the condition of no traditional customers, i.e., $\Lambda_0 = 0$, due to the lower waiting cost by using delivery service, it is socially optimal for tech-savvy customers to join using food delivery service. Then, the socially optimal behavior of tech-savvy customers is identical to unobservable queues in Chap 3, Hassin and Haviv (2003). We give the following proposition without proof.

PROPOSITION D.9 **(Social Optimization—Tech-Savvy Only)**. *When there are no traditional customers, i.e.,* $\Lambda_0 = 0$*, the maximum social welfare and socially optimal joining rates of food-delivery and walk-in customers* $\lambda_D$ *and* $\lambda_W$ *are*

| | $S^o$ | $\lambda_D^o$ | $\lambda_W^o$ |
|---|---|---|---|
| *If* $0 \le \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\Lambda_1\left(R - \frac{\phi}{\mu-\Lambda_1}\right)$ | $\Lambda_1$ | $0$ |
| *If* $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\left(\sqrt{R\mu} - \sqrt{\phi}\right)^2$ | $\mu - \sqrt{\frac{\phi\mu}{R}}$ | $0$ |

*Moreover, we have:*

*(i) The optimal social welfare* $S^o$ *is a weakly increasing function of* $\Lambda_1$.

*(ii) The socially optimal throughput* $\lambda_D^o + \lambda_W^o$ *is a weakly increasing function of* $\Lambda_1$.

Next we study how to achieve the socially optimal joining rates of food-delivery and walk-in customers, $\lambda_D^o$ and $\lambda_W^o$, characterized in Proposition D.9. Let $p^o$ and $\theta^o$ denote the optimal monopoly food price and delivery fee that induce the socially optimal joining rates $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9. We expect that the solution of $p + \theta^*(p) = p_m$, where $\theta^*(p)$ is the platform's best response delivery fee from Proposition D.7 and

$$p_m = \begin{cases} R - \frac{\phi}{\mu-\Lambda_1} & \text{if } \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}} \\ R - \sqrt{\frac{\phi R}{\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \end{cases}.$$

is the social optimal price in an unobservable M/M/1 with waiting cost $\phi$ (see, e.g., Hassin and Haviv (2003)), is the socially optimal food price $p^o$, and the platform's best response to $p^o$ is $\theta^o$; i.e., $\theta^*(p^o) = \theta^o$. We next verify this conjecture.

From Proposition D.7, we have the sum of food price $p$ and platform's best response delivery fee $\theta^*(p)$:

1. For $\Lambda_1 \le \frac{c-\phi}{c}\mu$,

| | $p+\theta^*(p)$ | |
|---|---|---|
| If $p \le R - \frac{c}{\mu-\Lambda_1}$ | $p + \frac{c-\phi}{\mu-\Lambda_1}$ | $\in \left[\frac{c-\phi}{\mu-\Lambda_1}, R - \frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $R - \frac{\phi}{\mu-\Lambda_1}$ | $\in \left(R - \frac{\phi}{\mu-\Lambda_1}, R - \frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < p \le R - \frac{\phi}{\mu}$ | $R - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\in \left(R - \frac{\phi}{\mu-\Lambda_1}, R - \frac{\phi}{\mu}\right]$ |

2. For $\Lambda_1 > \frac{c-\phi}{c}\mu$,

| | $p+\theta^*(p)$ | |
|---|---|---|
| If $p \le R - \frac{c}{\mu-\Lambda_1}$ | $p + \frac{c-\phi}{\mu-\Lambda_1}$ | $\in \left[\frac{c-\phi}{\mu-\Lambda_1}, R - \frac{\phi}{\mu-\Lambda_1}\right]$ |
| If $R - \frac{c}{\mu-\Lambda_1} < p \le R - \frac{c^2}{\mu\phi}$ | $R - \frac{\phi}{c}(R-p)$ | $\in \left(R - \frac{\phi}{\mu-\Lambda_1}, R - \frac{c}{\mu}\right]$ |
| If $R - \frac{c^2}{\mu\phi} < p \le R - \frac{\phi}{\mu}$ | $R - \frac{\sqrt{\mu\phi(R-p)}}{\mu}$ | $\in \left(R - \frac{c}{\mu}, R - \frac{\phi}{\mu}\right]$ |

When $\phi \le \frac{c^2}{R\mu}$, we have $\mu - \sqrt{\frac{\phi\mu}{R}} \le \frac{c-\phi}{c}\mu$, $R - \sqrt{\frac{\phi R}{\mu}} \ge R - \frac{c}{\mu}$, and $R - \frac{c^2}{\mu\phi} < 0$.

- If $\Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$, the solution of $p + \theta^*(p) = p_m \Leftrightarrow p + \frac{c-\phi}{\mu-\Lambda_1} = R - \frac{\phi}{\mu-\Lambda_1}$ is $p^o = R - \frac{c}{\mu-\Lambda_1}$. Also, by Proposition D.7, we have $\theta^o = \theta^*(p^o) = \frac{(c-\phi)(R-p^o)}{c} = \frac{c-\phi}{\mu-\Lambda_1}$. By Lemma D.11, we have $\lambda_D = \Lambda_1$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9.
- If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$, we have $R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$, $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < 0$, and $R - \frac{c}{\mu-\Lambda_1} < 0$. The solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\sqrt{\mu\phi(R-p)}}{\mu} = R - \sqrt{\frac{\phi R}{\mu}}$ is $p^o = 0$. By Proposition D.7, the food delivery platform's best response is $\theta^o = R - p^o - \frac{\sqrt{\mu\phi(R-p^o)}}{\mu} = R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$. By Lemma D.11, we have $\lambda_D = \mu - \frac{\phi}{R-p^o-\theta^o} = \mu - \sqrt{\frac{\mu\phi}{R}}$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9.

When $\phi > \frac{c^2}{R\mu}$, we have $\frac{c-\phi}{c}\mu < \mu - \sqrt{\frac{\phi\mu}{R}}$, and $R - \sqrt{\frac{\phi R}{\mu}} < R - \frac{c}{\mu}$.

- If $\Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}}$, the solution of $p + \theta^*(p) = p_m \Leftrightarrow p + \frac{c-\phi}{\mu-\Lambda_1} = R - \frac{\phi}{\mu-\Lambda_1}$ is $p^o = R - \frac{c}{\mu-\Lambda_1}$. Also, by Proposition D.7, we have $\theta^o = \theta^*(p^o) = \frac{(c-\phi)(R-p^o)}{c} = \frac{c-\phi}{\mu-\Lambda_1}$. By Lemma D.11, we have $\lambda_D = \Lambda_1$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9.
- If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$, we have $R - \sqrt{\frac{\phi R}{\mu}} > R - \frac{\phi}{\mu-\Lambda_1}$. The solution of $p + \theta^*(p) = p_m \Leftrightarrow R - \frac{\phi}{c}(R-p) = R - \sqrt{\frac{\phi R}{\mu}}$ is $p^o = R - c\sqrt{\frac{R}{\phi\mu}} < R - \frac{c^2}{\mu\phi}$. By Proposition D.7(2), the food delivery platform's best response is $\theta^o = \frac{(c-\phi)(R-p^o)}{c} = (c-\phi)\sqrt{\frac{R}{\phi\mu}}$. By Lemma D.11, we have $\lambda_D = \mu - \frac{\phi}{R-p^o-\theta^o} = \mu - \sqrt{\frac{\mu\phi}{R}}$ and $\lambda_W = 0$, which are identical to $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9.

To summarize, the centralized owner of the food service chain can set the food price as

$$p^o = \begin{cases} R - \frac{c}{\mu-\Lambda_1} & \text{if } \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}} \\ 0 & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi \le \frac{c^2}{R\mu} \\ R - c\sqrt{\frac{R}{\phi\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi > \frac{c^2}{R\mu} \end{cases} \tag{D.13}$$

to induce the socially optimal joining rates $\lambda_D^o$ and $\lambda_W^o$ in Proposition D.9 and extract all the surpluses from customers; and the delivery platform's best response is

$$\theta^o = \begin{cases} \frac{c-\phi}{\mu-\Lambda_1} & \text{if } \Lambda_1 \le \mu - \sqrt{\frac{\phi\mu}{R}} \\ R - \sqrt{\frac{\phi R}{\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi \le \frac{c^2}{R\mu} \\ (c-\phi)\sqrt{\frac{R}{\phi\mu}} & \text{if } \Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}} \text{ and } \phi > \frac{c^2}{R\mu} \end{cases}. \tag{D.14}$$

From the expression of $p^o$ and $\theta^o$ in (D.13) and (D.14), it is easy to verify that $p^o$ and $p^o + \theta^o$ are weakly decreasing functions of $\Lambda_1$, and that $\theta^o$ is a weakly increasing function of $\Lambda_1$.

The restaurant's corresponding profit is

| | $\Pi^o(\Lambda_1) = p^o(\lambda_D^o + \lambda_W^o)$ |
|---|---|
| If $0 < \Lambda_1 \leq \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ and $\phi \leq \frac{c^2}{R\mu}$ | $0$ |
| If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ and $\phi > \frac{c^2}{R\mu}$ | $\left(R - c\sqrt{\frac{R}{\phi\mu}}\right)\left(\mu - \sqrt{\frac{\phi\mu}{R}}\right)$ |

From proof of Lemma 1, we have $\left(R - \frac{c}{\mu-\Lambda_1}\right)\Lambda_1$ is a decreasing function of $\Lambda_1$, 0 and $\left(R - c\sqrt{\frac{R}{\phi\mu}}\right)\left(\mu - \sqrt{\frac{\phi\mu}{R}}\right)$ are constants regarding $\Lambda_1$. Thus, $\Pi^o(\Lambda_1)$ is a weakly decreasing function of $\Lambda_1$.

The platform's corresponding profit is

| | $\pi^o(p^o(\Lambda_1)) = \theta^o\lambda_D^o$ |
|---|---|
| If $0 < \Lambda_1 \leq \mu - \sqrt{\frac{\phi\mu}{R}}$ | $\frac{c-\phi}{\mu-\Lambda_1}\Lambda_1$ |
| If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ and $\phi \leq \frac{c^2}{R\mu}$ | $\left(R - \sqrt{\frac{\phi R}{\mu}}\right)\left(\mu - \sqrt{\frac{\phi\mu}{R}}\right)$ |
| If $\Lambda_1 > \mu - \sqrt{\frac{\phi\mu}{R}}$ and $\phi > \frac{c^2}{R\mu}$ | $(c-\phi)\sqrt{\frac{R}{\phi\mu}}\left(\mu - \sqrt{\frac{\phi\mu}{R}}\right)$ |

which is clearly a weakly increasing function of $\Lambda_1$. $\square$

## D.3. Proof of Proposition A.2

We discuss the one-way and two-way RS contracts separately.

1. We first discuss the one-way RS contract with a price ceiling. In a decentralized system under the platform's best response delivery fee $\theta^*(p)$, social welfare can be derived as

1.1 For $\Lambda_1 \leq \frac{c-\phi}{c}\mu$,

| | $\Pi(p) + \pi^*(p)$ |
|---|---|
| If $p \leq R - \frac{c}{\mu-\Lambda_1}$ | $\left(p + \frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{\mu\phi}{(\mu-\Lambda_1)^2}$ | $\left(R - \frac{\phi}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $R - \frac{\mu\phi}{(\mu-\Lambda_1)^2} < p \leq R - \frac{\phi}{\mu}$ | $R\mu + \phi - \sqrt{\mu\phi}\left(\sqrt{R-p} + \frac{R}{\sqrt{R-p}}\right)$ |

1.2 For $\Lambda_1 > \frac{c-\phi}{c}\mu$,

| | $\Pi(p) + \pi^*(p)$ |
|---|---|
| If $p \leq R - \frac{c}{\mu-\Lambda_1}$ | $\left(p + \frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ |
| If $R - \frac{c}{\mu-\Lambda_1} < p \leq R - \frac{c^2}{\mu\phi}$ | $R\mu + \phi - \left(\frac{Rc}{R-p} + \frac{\phi\mu(R-p)}{c}\right)$ |
| If $R - \frac{c^2}{\mu\phi} < p \leq R - \frac{\phi}{\mu}$ | $R\mu + \phi - \sqrt{\mu\phi}\left(\sqrt{R-p} + \frac{R}{\sqrt{R-p}}\right)$ |

We have the following results.

- $\left(p + \frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ increases in the food price $p$, because $\frac{\partial\left(\left(p+\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1\right)}{\partial p} = \Lambda_1 > 0$.
- $\frac{\partial\left(R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)\right)}{\partial p} = \frac{\mu\phi}{c} - \frac{Rc}{(R-p)^2}$, which is zero when $p = R - c\sqrt{\frac{R}{\mu\phi}}$; $R - c\sqrt{\frac{R}{\mu\phi}} \leq R - \frac{c}{\mu-\Lambda_1} \Leftrightarrow \Lambda_1 \leq \mu - \sqrt{\frac{\mu\phi}{R}}$; and $\frac{\partial^2\left(R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)\right)}{\partial p^2} = \frac{-2Rc}{(R-p)^3} < 0$.

- $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases in the food price $p$, because $\frac{\partial\left(R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)\right)}{\partial p}=-\frac{p\sqrt{\mu\phi}}{2(R-p)^{\frac{3}{2}}}<0$.

Thus, we have:

- When $\Lambda_1\leq\mu-\sqrt{\frac{\mu\phi}{R}}$, we have
  — $\left(p+\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ increases in $p$ for $p\in\left[0,R-\frac{c}{\mu-\Lambda_1}\right]$.
  — $R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ decreases in $p$ for $p\in\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]$.
  — $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases in $p$ for $p\in\left(R-\frac{c^2}{\mu\phi},R-\frac{\phi}{\mu}\right]$.

  Thus, $\Pi(p)+\pi^*(p)$ is a unimodal function of $p$ with the maximum at $p^o=R-\frac{c}{\mu-\Lambda_1}$.
- When $\Lambda_1>\mu-\sqrt{\frac{\mu\phi}{R}}$, we have
  — $\left(p+\frac{c-\phi}{\mu-\Lambda_1}\right)\Lambda_1$ increases in $p$ for $p\in\left[0,R-\frac{c}{\mu-\Lambda_1}\right]$.
  — $R\mu+\phi-\left(\frac{Rc}{R-p}+\frac{\phi\mu(R-p)}{c}\right)$ is a unimodal function of $p$ with the maximum at $p=R-c\sqrt{\frac{R}{\mu\phi}}$ for $p\in\left(R-\frac{c}{\mu-\Lambda_1},R-\frac{c^2}{\mu\phi}\right]$.
  — $R\mu+\phi-\sqrt{\mu\phi}\left(\sqrt{R-p}+\frac{R}{\sqrt{R-p}}\right)$ decreases in $p$ for $p\in\left(R-\frac{c^2}{\mu\phi},R-\frac{\phi}{\mu}\right]$.

  Thus, $\Pi(p)+\pi^*(p)$ is a unimodal function of $p$ with the maximum at $p^o=R-c\sqrt{\frac{R}{\mu\phi}}$.

When the platform shares $\gamma_1$ fraction of its profit with the restaurant, the restaurant's profit is $\Pi(p)+\gamma_1\pi^*(p)=(\Pi(p)+\pi^*(p))-(1-\gamma_1)\pi^*(p)$. We have shown above that $\Pi(p)+\pi^*(p)$ is increasing in $p$ for $p\in[0,p^o]$, and $\pi^*(p)$ is a weakly decreasing function of $p$ by Proposition D.7. Thus, the restaurant's profit $\Pi(p)+\gamma_1\pi^*(p)$ increases for $p\in[0,p^o]$, which means that the restaurant will set the food price $p=p^o$ to maximize its profit. By Lemma A.1, given the food price $p^o$, the platform's best-response delivery fee is $\theta^o$. Hence, the one-way revenue-sharing contract with a price ceiling proposed in Proposition A.2 successfully induces the restaurant and the platform to behave in the socially optimal manner for $\forall\gamma_1\in[0,1]$.

For the decentralized system, we have the restaurant's profit $\Pi^*(\Lambda_1)$ from Proposition D.8, the platform's profit $\pi^*(p^*(\Lambda_1))$ from Corollary D.8, and social welfare $S^*(\Lambda_1)$ from Corollary D.9 in equilibrium. Note that $\Pi^*(\Lambda_1)$ and $\pi^*(p^*(\Lambda_1))$ are the minimum profits the restaurant and the delivery platform target. In the centralized system, under the optimal monopoly food price $p^o$ and delivery fee $\theta^o$, we have the restaurant's profit $\Pi^o(\Lambda_1)$, the platform's profit $\pi^o(\Lambda_1)$, and social welfare $S^o(\Lambda_1)$ from Lemma A.1. The range of sharing fraction

$$\gamma_1\in\left[\frac{\Pi^*(\Lambda_1)-\Pi^o(\Lambda_1)}{\pi^o(p^o(\Lambda_1))},1-\frac{\pi^*(p^*(\Lambda_1))}{\pi^o(p^o(\Lambda_1))}\right] \tag{D.15}$$

makes sure that the restaurant and the platform can reach a win-win situation for the restaurant and the platform.

2. The two-way revenue sharing contract turns the restaurant's profit function into an affine transformation of the aggregated profit of the whole service system. Thus, the restaurant will

set the food price as the socially optimal one $p^o$. Then, from Lemma A.1, the platform's best response is to set the delivery fee as $\theta^o$. This contract coordinates the whole system in the socially optimal fashion. From a discussion similar to the one-way RS contract with a price ceiling, the range of sharing fraction

$$\gamma_2 \in \left[\frac{\Pi^*\left(\Lambda_1\right)}{S^o\left(\Lambda_1\right)}, 1 - \frac{\pi^*\left(p^*\left(\Lambda_1\right)\right)}{S^o\left(\Lambda_1\right)}\right] \tag{D.16}$$

makes sure that the restaurant and the platform can reach a win-win. $\square$