Supplement to "Intertemporal Segmentation via Flexible-Duration Group Buying"

A. Comparison with Group-Buying Literature

Through careful comparison with all the existing group-buying literature, we find that the models in most of the group-buying papers are not readily extendable to the case in which the deadline T goes to infinity. Besides, even though the models in a few papers allow such an extension, they do not study exactly what is studied in our paper. We will explain in detail below.

First and foremost, Anand and Aron (2003), Chen and Zhang (2015) and Marinesi et al. (2018) regard group buying as a mechanism of offering a contingent quantity discount, and show that the profitability of group buying results from its better response to demand uncertainty. Chen and Roma (2011) and Jing and Xie (2011) also view group buying as a way to offer a quantity discount while considering other scenarios. These papers do not take into account customers' dynamic sign-up behavior, since they do not assume that customers arrive sequentially. In addition, these papers do not explicitly consider a deadline T, although there is a deadline for group buying in the practical business examples they discuss. Consequently, the models in these papers cannot be extended to accommodate an infinite deadline of T, and moreover, these papers do not contain the main insights of our paper.

Second, Hu et al. (2013) and Hu et al. (2015) consider customers' sequential arrival and the resulting sequential sign-up process. In particular, they develop a two-person and two-period model where one customer arrives in each period. The group-buying (or crowd-funding) deal succeeds if and only if both customers sign up for the deal. While we find that the models in these two papers can be extended to accommodate an infinite deadline (i.e., an N-person and N-period model where N goes to infinity), they differ significantly from what is studied in our paper. Notice that the minimum number of sign-ups required in these two papers would also be N, and thus the success rate of a group-buying deal is still less than 1 as in the case of a finite N, which means there is a *positive* probability that group buying may not reach the target. Besides, the high-end customers may have an incentive to sign up earlier or pay a higher price compared with the low-end customers due to lack of certainty of success of a group-buying deal, a result of the all-or-nothing nature of fixed-duration group buying. However, recall that an important characteristic of flexible-duration group buying is that the success rate is always 1, which means the group-buying campaign is guaranteed to succeed, even though the waiting times are random and can be long. Moreover, in flexible-duration group buying, the different sign-up incentives for high- and low-end customers are not driven by the success uncertainty of a group-buying deal, as in fixed-duration group buying. Rather, they are driven by waiting costs. Therefore, the models in our paper cannot be regarded as a special case of the models studied in these two papers. (In the main body of the paper, we discuss in detail the differences in what drives sign-up behavior in flexible- and fixed-duration group buying.)

Finally, Surasvadi et al. (2017) and Liu and Tunca (2019) assume that customers arrive according to a Poisson process, and thus, customers sign up for the group-buying deal sequentially and dynamically. Although the models in these two papers can be extended to accommodate an infinite deadline T and the corresponding success rate would be 1, the insights differ significantly from those in our paper. In particular, Surasvadi et al. (2017) propose a contingent markdown mechanism that resembles group buying, in which the discount price will be offered not only to the sign-ups but also to all subsequent buyers if the number of sign-ups reaches a pre-specified threshold, or at the end of the selling season, whichever comes first. In fact, there are important distinctions between the work of Surasvadi et al. (2017) and ours even if the deadline T goes to infinity. First, the reason for signing up differs. In Surasvadi et al. (2017), customers sign up and wait for the discount price, and they also have the option of purchasing the same product at the (higher) regular price and taking immediate possession. On the contrary, in our paper, customers sign up and wait for a different product (that can be either horizontally or vertically differentiated from the regular product), which is unavailable in other channels. Second, customers' sign-up incentives are different. In Surasvadi et al. (2017), the low-end customers can never afford to purchase the product at the regular price and thus have to sign up and wait for the discount price, so their purchasing behavior is independent of the high-end customers' sign-up behavior. By contrast, in our paper, the low-end customers can afford the immediate purchase option and can choose whether to sign up; hence, the low-end customers' sign-up behavior is influenced by that of the high-end customers and vice versa. Third, Surasvadi et al. (2017) essentially study intertemporal price discrimination between customers with different valuations, while our work focuses on intertemporal customer segmentation (we have discussed in detail how intertemporal customer segmentation differs from intertemporal price discrimination; see the second paragraph on page 9). In particular, in intertemporal price discrimination, the firm proactively discriminates between different segments of customers by offering time-varying prices. By contrast, in intertemporal customer segmentation as studied in our paper, the price of the group-buying product remains unchanged; hence, the firm passively discriminates between customers based on their different expected waiting times (and costs). Fourth, Surasvadi et al. (2017) consider "a single instance interaction between the seller and the consumers" and assume that once the deal is on, all subsequent consumers can purchase at the discount price; while in our paper, once a group-buying deal concludes, another new group-buying deal begins immediately. Fifth, Surasvadi et al. (2017) assume a limited inventory of the product, and hence, the group-buying state is composed of the number of sign-ups and the on-hand inventory level; while in our paper, there is no inventory limitation for the group-buying product since the firm produces the group-buying product in a batch of size N immediately after the deal is on, and thus, the state of the group-buying deal is only related to the number of sign-ups.

On the other hand, Liu and Tunca (2019) characterize the stochastic dynamic equilibrium behavior of consumers' pledging in a group buying with *two* threshold levels by a recursive differential and difference equation system. They focus on empirically estimating consumer arrival rates and utility distributions utilizing data from group-buying events hosted by an online retailer, and providing empirical evidence for consumer network effects in group buying. By contrast, in our paper, we consider only one threshold level and focus on the potential implications of intertemporal customer segmentation. While intertemporal customer segmentation can exist in the context studied in their paper, the authors do not formally characterize this behavior, and more importantly, the underlying driving force differs significantly. (In the main body of the paper, we discuss in detail the differences between the driving forces of sign-up behavior in flexible- and fixed-duration group buying.) In particular, Liu and Tunca (2019) do not consider time discounting. Hence intertemporal customer segmentation, even if it exists in their model, is driven by sign-up risks under the finite horizon assumption, and would not exist any more when the horizon is taken to infinity.

B. Premium Group-Buying Product

Proof of Lemma A.1. We prove the lemma by induction. For any state $n\,(1 \leqslant n \leqslant N)$, there are three possible cases depending on which segment signs up: high-end segment only (Case 1); low-end segment only (Case 2); and both segments (Case 3). Suppose a high-end customer signs up at state n (Case 1), implying that the following IR and IC constraints must be satisfied:

$$IR_H : \theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n) \geqslant 0,$$

$$IC_H : \theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n) \geqslant H - r^{\mathrm{G}}.$$

Regardless of what the prices p^{G} and r^{G} are, since $w^{\mathrm{G}}(n)$ is monotonously increasing in n, and the surpluses of purchasing the regular product remain the same for the high-end customers (i.e., $H - r^{\mathrm{G}}$), $\theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n-1) > H - r^{\mathrm{G}}$ always holds. Thus, a high-end customer who arrives at state $n-1$ would also sign up, because the following constraints can always be satisfied:

$$IR_H : \theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n-1) > 0,$$

$$IC_H : \theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n-1) > H - r^{\mathrm{G}}.$$

This analysis ensures that once a high-end customer signs up at the specific state n, all the following high-end customers will also join the group buying. On the other hand, if no high-end customer signs up at the specific state n, it is implied that $\theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n) < \max\{H - r^{\mathrm{G}}, 0\}$. This cannot guarantee that the next arriving high-end customer will also be unwilling to sign up. Cases 2 and 3 can be proved analogously. □

Proof of Proposition A.1. The result follows directly from comparing the profit functions of the volume, margin, and product-line strategies. Below, we demonstrate how we solve for the equilibrium in the product-line strategy. In order to sell the premium product to high-end customers and the regular product to low-end ones, the firm sets the prices while satisfying the following IR and IC constraints:

$$\begin{cases} IR_H : \theta H - p^{\mathrm{P}} \geqslant 0, \\ IC_H : \theta H - p^{\mathrm{P}} \geqslant H - r^{\mathrm{P}}, \\ IR_L : L - r^{\mathrm{P}} \geqslant 0, \\ IC_L : L - r^{\mathrm{P}} \geqslant \theta L - p^{\mathrm{P}}. \end{cases}$$

In equilibrium, IC_H and IR_L are binding, and hence, $r^{\mathrm{P}} = L$ and $p^{\mathrm{P}} = (\theta - 1)H + L$. The total inventory holding cost when the firm sells N premium products is

$$Nh\frac{1}{\gamma\lambda} + (N-1)h\frac{1}{\gamma\lambda} + \cdots + h\frac{1}{\gamma\lambda} = \frac{hN(N+1)}{2\gamma\lambda}.$$

Thus, the firm's long-run average profit is

$$\pi^{\mathrm{P}} = \frac{Np^{\mathrm{P}} + N\frac{1-\gamma}{\gamma}r^{\mathrm{P}} - \frac{hN(N+1)}{2\gamma\lambda}}{\frac{N}{\gamma\lambda}} = (\theta-1)H\gamma\lambda + L\lambda - \frac{(N+1)h}{2}.$$

Moreover, the threshold $\bar{h}_1$ for the inventory holding cost h is defined as

$$\bar{h}_1 \equiv \min\left\{\frac{2(\theta-1)H\gamma\lambda}{N+1}, \frac{2(\theta-2)H\gamma\lambda + 2(1-\gamma)L\lambda}{N+1}\right\}. \quad \square$$

Proof of Corollary 3. The results follow directly from comparing total sales volumes of different strategies in equilibrium. □

Proof of Corollary 4. (i) Both $\bar{N}_1$ and $\bar{N}_2$ increase in λ.

(ii) $\bar{x}_1^{\mathrm{G}}$ decreases in λ.

(iii) p^{G} and π^{G} always increase in λ regardless of the customer segmentation.

(iv) $S_H^{\mathrm{G}}(n)$ and $S_L^{\mathrm{G}}(n)$ always (weakly) decrease in λ regardless of the segmentation. □

C. Contingent Pricing

Proof of Proposition B.1. In similar fashion to the proof of Propositions 1 and 2, there are five possible scenarios in total for customer segmentation.[1] Define the following two cases for ease of analysis. In Case I, $\bar{x}^{\mathrm{C}} = 0$, $1 \leqslant \bar{x}^{\mathrm{C}} < N$, and $\bar{x}^{\mathrm{C}} = N$ represent scenarios $\{H\}$, $\{H; H+L\}$, and $\{H+L\}$, respectively. In Case II, $\bar{x}^{\mathrm{C}} = 0$ and $1 \leqslant \bar{x}^{\mathrm{C}} < N$ represent scenarios $\{L\}$ and $\{L; L+H\}$, respectively. In equilibrium, the prices p_1^{C}, p_2^{C}, and r^{C} should satisfy the IR and IC constraints.

For Case I, the IR and IC constraints are

$$\begin{cases} IR_{H1}: \theta H - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IC_{H1}: \theta H - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant H - r^{\mathrm{C}} & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IR_{H2}: \theta H - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IC_{H2}: \theta H - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant H - r^{\mathrm{C}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IR_{L1}: L - r^{\mathrm{C}} \geqslant 0 & \bar{x}^{\mathrm{C}} < n \leqslant N \\ IC_{L1}: L - r^{\mathrm{C}} \geqslant \theta L - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IR_{L2}: \theta L - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IC_{L2}: \theta L - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant L - r^{\mathrm{C}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{C}} \leqslant N$ and the expected waiting time $w^{\mathrm{C}}(n)$ is

$$w^{\mathrm{C}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ \frac{\bar{x}^{\mathrm{C}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{C}} - 1}{\gamma\lambda} & \bar{x}^{\mathrm{C}} < n \leqslant N. \end{cases}$$

Since IR_{L1} is binding at any state n in equilibrium, we have $r^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) = L$. Since IC_{H1} is binding at state $n = N$, we have $p_1^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) = (\theta - 1)H + L - c \cdot w^{\mathrm{C}}(N)$. Besides, the constraint IC_{H2} is stronger than IC_{L2} because $(\theta - 1)(H - L) > 0$ always holds. In order to satisfy IC_{H2} at state $n = \bar{x}^{\mathrm{C}}$, $p_2^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) = \theta L - c \cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$. So far, we have written all the prices as the functions of $\bar{x}^{\mathrm{C}}$, so the firm's long-run average profit π^{C} can be written as

$$\begin{aligned} \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) &= \left[p_1^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot \gamma\lambda + r^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot (1-\gamma)\lambda\right] \cdot P(n > \bar{x}^{\mathrm{C}}) + \left[p_2^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot \lambda\right] \cdot P(n \leqslant \bar{x}^{\mathrm{C}}) \\ &= \frac{\left[p_1^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot \gamma\lambda + r^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot (1-\gamma)\lambda\right] \cdot (N - \bar{x}^{\mathrm{C}}) + \left[p_2^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \cdot \lambda\right] \cdot \gamma\bar{x}^{\mathrm{C}}}{N - (1-\gamma)\bar{x}^{\mathrm{C}}}. \end{aligned}$$

Plugging $p_1^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$, $p_2^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$, and $r^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ into the firm's long-run average profit π^{C}, we have

$$\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) = \frac{\left[(\theta-1)H\gamma + L\right]\lambda(N - \bar{x}^{\mathrm{C}}) + \theta L\gamma\lambda\bar{x}^{\mathrm{C}} - c\gamma(N-1)\bar{x}^{\mathrm{C}} - c(N - \bar{x}^{\mathrm{C}} - 1)(N - \bar{x}^{\mathrm{C}})}{N - (1-\gamma)\bar{x}^{\mathrm{C}}}.$$

Take the second-order derivative of $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ w.r.t. $\bar{x}^{\mathrm{C}}$. Denote $\bar{N} \equiv N/(1-\gamma) > N$. We find that on the one hand, if $N > \bar{N}_1^{\mathrm{C}}$, when $\bar{x}^{\mathrm{C}} < \bar{N}$, $\frac{\partial^2 \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})}{\partial \bar{x}^{\mathrm{C}2}} < 0$ always holds, implying that $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is

[1] Note that Lemma A.1 is also applicable under contingent pricing. Particularly for the state $1 \leqslant n < \bar{x}^{\mathrm{C}}$ and $\bar{x}^{\mathrm{C}} < n \leqslant N$, this proof is similar to that of Lemma A.1. Moreover, for the tipping state $\bar{x}^{\mathrm{C}}$, regardless of the values of p_1^{C} and p_2^{C}, since the low-end customers begin to join the group buying, i.e., $\theta L - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \geqslant 0$, the high-end customers are also willing to join the group buying. That is because $\theta H - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \geqslant H - L$ can always be satisfied when $\theta > 1$.

concave in $\bar{x}^{\mathrm{C}}$; when $\bar{x}^{\mathrm{C}} > \bar{N}$, $\frac{\partial^2 \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})}{\partial \bar{x}^{C2}} > 0$ always holds, implying that $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is convex in $\bar{x}^{\mathrm{C}}$. Besides, $\lim_{\bar{x}^{\mathrm{C}} \to \bar{N}^-} = -\infty$, $\lim_{\bar{x}^{\mathrm{C}} \to \bar{N}^+} = +\infty$. In this case, the optimal $\bar{x}^{\mathrm{C}}$ within the constraint $0 \leqslant \bar{x}^{\mathrm{C}} \leqslant N$, in which $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is concave in $\bar{x}^{\mathrm{C}}$, is determined by $\bar{x}^{\mathrm{C}} = \bar{x}_1^{\mathrm{C}}$, where $\bar{x}_1^{\mathrm{C}} \equiv N/(1-\gamma) - \sqrt{cN\gamma\,[cN-(1-\gamma)(\theta-1)(L-H\gamma)\lambda]}/[c(1-\gamma)]$ is the unique reasonable solution to the first-order condition $\frac{\partial \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})}{\partial \bar{x}^{\mathrm{C}}} = 0$.[2] It is easy to verify that $\bar{x}_1^{\mathrm{C}} < N$ if and only if $N > \bar{N}_2^{\mathrm{C}}$, and $\bar{x}_1^{\mathrm{C}} > 0$ if and only if $N > \bar{N}_3^{\mathrm{C}}$. Note that $\bar{N}_2^{\mathrm{C}} > \bar{N}_1^{\mathrm{C}} > 0$ if and only if $H/L < 1/\gamma$, and $\bar{N}_3^{\mathrm{C}} > 0 > \bar{N}_1^{\mathrm{C}}$ if and only if $H/L > 1/\gamma$.

On the other hand, if $N < \bar{N}_1^{\mathrm{C}}$, when $\bar{x}^{\mathrm{C}} < \bar{N}$, $\frac{\partial^2 \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})}{\partial \bar{x}^{C2}} > 0$ always holds, implying that $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is convex in $\bar{x}^{\mathrm{C}}$; when $\bar{x}^{\mathrm{C}} > \bar{N}$, $\frac{\partial^2 \pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})}{\partial \bar{x}^{C2}} < 0$ always holds, implying that $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is concave in $\bar{x}^{\mathrm{C}}$. Besides, $\lim_{\bar{x}^{\mathrm{C}} \to \bar{N}^-} = +\infty$, $\lim_{\bar{x}^{\mathrm{C}} \to \bar{N}^+} = -\infty$. In this case, the optimal $\bar{x}^{\mathrm{C}}$ within the constraint $0 \leqslant \bar{x}^{\mathrm{C}} \leqslant N$, in which $\pi^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$ is convex in $\bar{x}^{\mathrm{C}}$, is determined by the boundary, i.e., either 0 or N.

Therefore, combining the analyses above for Case I, the firm sets $\bar{x}^{\mathrm{C}} = \bar{x}_1^{\mathrm{C}}$ when $N > \max\{\bar{N}_2^{\mathrm{C}}, \bar{N}_3^{\mathrm{C}}\}$; otherwise, the firm sets $\bar{x}^{\mathrm{C}} = N$ ($\bar{x}^{\mathrm{C}} = 0$) when $0 < N \leqslant \max\{\bar{N}_2^{\mathrm{C}}, 0\}$ ($0 < N \leqslant \max\{\bar{N}_3^{\mathrm{C}}, 0\}$).

For Case II, the IR and IC constraints are

$$\begin{cases} IR_{L1}: \theta L - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IC_{L1}: \theta L - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant L - r^{\mathrm{C}} & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IR_{L2}: \theta L - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IC_{L2}: \theta L - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant L - r^{\mathrm{C}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IR_{H1}: H - r^{\mathrm{C}} \geqslant 0 & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IC_{H1}: H - r^{\mathrm{C}} \geqslant \theta H - p_1^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) & \bar{x}^{\mathrm{C}} < n \leqslant N, \\ IR_{H2}: \theta H - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ IC_{H2}: \theta H - p_2^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(n) \geqslant H - r^{\mathrm{C}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{C}} < N$ and the expected waiting time $w^{\mathrm{C}}(n)$ is

$$w^{\mathrm{C}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{C}}, \\ \frac{\bar{x}^{\mathrm{C}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{C}}-1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{C}} < n \leqslant N. \end{cases}$$

The constraint IC_{L1} requires $p_1^{\mathrm{C}} \leqslant (\theta-1)L + r^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(N)$. The constraint IC_{H1} requires $p_1^{\mathrm{C}} \geqslant (\theta-1)H + r^{\mathrm{C}} - c \cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$. Since $w^{\mathrm{C}}(\bar{x}^{\mathrm{C}}) \leqslant w^{\mathrm{C}}(N)$, the constraints IC_{L1} and IC_{H1} can never be satisfied at the same time when $\theta > 1$. Hence, Case II cannot form the equilibrium.

Comparing the results above, we define three thresholds for the batch size N:

$$\bar{N}_1^{\mathrm{C}} \equiv \frac{(\theta-1)(L-H\gamma)(1-\gamma)\lambda}{c},$$
$$\bar{N}_2^{\mathrm{C}} \equiv \frac{(\theta-1)(L-H\gamma)\lambda}{c},$$
$$\bar{N}_3^{\mathrm{C}} \equiv \frac{(\theta-1)(H\gamma-L)\gamma\lambda}{c}.$$

[2] The other solution is ruled out because it is always greater than N and is not within the constraint $0 \leqslant \bar{x}^{\mathrm{C}} \leqslant N$.

For a given L, $\bar{N}_1^{\mathrm{C}}$ and $\bar{N}_2^{\mathrm{C}}$ decrease in H/L, while $\bar{N}_3^{\mathrm{C}}$ increases in H/L. Besides, $\bar{N}_2^{\mathrm{C}} < \bar{N}_1$ and $\bar{N}_3^{\mathrm{C}} < \bar{N}_2$ always hold. Thus, the REE when offering group buying under contingent pricing is

(i) when $H/L \leqslant 1/\gamma$,

(1) if $N \leqslant \bar{N}_2^{\mathrm{C}}$, $\bar{x}^{\mathrm{C}} = N$, $r^{\mathrm{C}} = L$, $p_1^{\mathrm{C}} = (\theta-1)H + L - cN/\lambda + c/(\gamma\lambda)$, $p_2^{\mathrm{C}} = \theta L - c(N-1)/\lambda$, and $\pi^{\mathrm{C}} = \theta L\lambda - c(N-1)$;

(2) if $N > \bar{N}_2^{\mathrm{C}}$, $\bar{x}^{\mathrm{C}} = \bar{x}_1^{\mathrm{C}}$, $r^{\mathrm{C}} = L$, $p_1^{\mathrm{C}} = (\theta-1)H + L - c\cdot w^{\mathrm{C}}(N)$, $p_2^{\mathrm{C}} = \theta L - c\cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$, and $\pi^{\mathrm{C}} = \pi_1^{\mathrm{C}}$;

(ii) when $H/L > 1/\gamma$,

(1) if $N \leqslant \bar{N}_3^{\mathrm{C}}$, $\bar{x}^{\mathrm{C}} = 0$, $r^{\mathrm{C}} = L$, $p_1^{\mathrm{C}} = (\theta-1)H + L - c(N-1)/(\gamma\lambda)$, $p_2^{\mathrm{C}} = \theta L$, and $\pi^{\mathrm{C}} = (\theta-1)H\gamma\lambda + L\lambda - c(N-1)$;

(2) if $N > \bar{N}_3^{\mathrm{C}}$, $\bar{x}^{\mathrm{C}} = \bar{x}_1^{\mathrm{C}}$, $r^{\mathrm{C}} = L$, $p_1^{\mathrm{C}} = (\theta-1)H + L - c\cdot w^{\mathrm{C}}(N)$, $p_2^{\mathrm{C}} = \theta L - c\cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{C}})$, and $\pi^{\mathrm{C}} = \pi_1^{\mathrm{C}}$;

where

$$\pi_1^{\mathrm{C}} \equiv \frac{[(\theta-1)H\gamma + L]\,\lambda(N-\bar{x}_1^{\mathrm{C}}) + \theta L\gamma\lambda\bar{x}_1^{\mathrm{C}} - c\gamma(N-1)\bar{x}_1^{\mathrm{C}} - c(N-\bar{x}_1^{\mathrm{C}}-1)(N-\bar{x}_1^{\mathrm{C}})}{N-(1-\gamma)\bar{x}_1^{\mathrm{C}}}. \quad \square$$

Proof of Proposition 3. We can prove the proposition by comparing the equilibrium prices p_1^{C} and p_2^{C} in Proposition B.1. On the one hand, when $N > \max\{\bar{N}_2^{\mathrm{C}}, \bar{N}_3^{\mathrm{C}}\}$, we can prove that $p_1^{\mathrm{C}} > p^{\mathrm{G}} > p_2^{\mathrm{C}}$ if and only if $N < \bar{N}_8^{\mathrm{C}}$, and $p_1^{\mathrm{C}} < p^{\mathrm{G}} < p_2^{\mathrm{C}}$ if and only if $N > \bar{N}_8^{\mathrm{C}}$, where $\bar{N}_8^{\mathrm{C}}$ is the unique reasonable solution to the equation $p_1^{\mathrm{C}} = p_2^{\mathrm{C}}$. Note that $\bar{N}_8^{\mathrm{C}} > \max\{\bar{N}_1, \bar{N}_2\} > \max\{\bar{N}_2^{\mathrm{C}}, \bar{N}_3^{\mathrm{C}}\}$ always holds. On the other hand, when $N \leqslant \max\{\bar{N}_2^{\mathrm{C}}, \bar{N}_3^{\mathrm{C}}\}$, we can easily prove that $p_1^{\mathrm{C}} \geqslant p^{\mathrm{G}} \geqslant p_2^{\mathrm{C}}$ always holds. Therefore, the corollary can be proved. $\square$

Proof of Proposition B.2. By comparing the equilibrium profits of uniform pricing (see Proposition 2) and contingent pricing (see Proposition B.1) for each customer segmentation, we can prove that $\pi^{\mathrm{C}} > \pi^{\mathrm{G}}$ always holds. In addition, by comparing the values of thresholds in Theorems 2 and B.1, the second part of the corollary can be proved. In detail, the inequations $\bar{N}_5^{\mathrm{C}} > \bar{N}_4$, $\bar{N}_6^{\mathrm{C}} = \bar{N}_5$, $\bar{N}_7^{\mathrm{C}} > \bar{N}_5$, and $\bar{N}_7^{\mathrm{C}} > \bar{N}_6$ always hold. $\square$

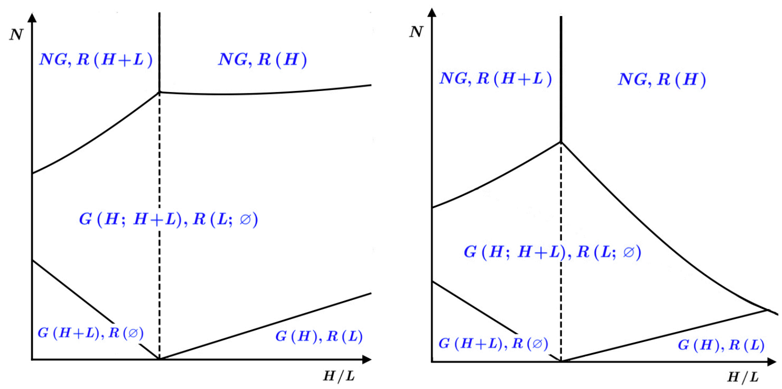
Proof of Theorem B.1. The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the batch size N:

$$\begin{aligned}
\bar{N}^{\mathrm{C}} &\equiv \begin{cases} \bar{N}_5^{\mathrm{C}} & H/L \leqslant 1/\gamma, \\ \bar{N}_7^{\mathrm{C}} & H/L > 1/\gamma\,, \bar{N}_6^{\mathrm{C}} > \bar{N}_3^{\mathrm{C}}, \\ \max\{\bar{N}_6^{\mathrm{C}}, \bar{N}_7^{\mathrm{C}}\} & H/L > 1/\gamma\,, \bar{N}_6^{\mathrm{C}} \leqslant \bar{N}_3^{\mathrm{C}}, \end{cases} \\
\bar{N}_4^{\mathrm{C}} &\equiv \frac{(\theta-1)L\lambda}{c} + 1, \\
\bar{N}_6^{\mathrm{C}} &\equiv \frac{[L + (\theta-2)H\gamma]\,\lambda}{c} + 1,
\end{aligned}$$

where $\bar{N}_5^{\mathrm{C}}$ and $\bar{N}_7^{\mathrm{C}}$ are the unique positive solution to the equations $\pi^{\mathrm{C}} = \pi^{\mathrm{V}}$ and $\pi^{\mathrm{C}} = \pi^{\mathrm{M}}$, respectively. By comparison, $\bar{N}_4^{\mathrm{C}} = \bar{N}_3 > \bar{N}_2^{\mathrm{C}}$ and $\bar{N}_5^{\mathrm{C}} > \bar{N}_2^{\mathrm{C}}$ always hold. $\bar{N}_4^{\mathrm{C}}$ increases in L. For a given L, $\bar{N}_5^{\mathrm{C}}$ increases in H/L, $\bar{N}_6^{\mathrm{C}}$ increases in H/L when $\theta > 2$ while decreasing in H/L when $1 < \theta \leqslant 2$, $\bar{N}_7^{\mathrm{C}}$ increases in H/L when $\theta > \bar{\theta}^{\mathrm{C}}$ while decreasing in H/L when $1 < \theta \leqslant \bar{\theta}^{\mathrm{C}}$, where $\bar{\theta}^{\mathrm{C}} > 2$. Besides, when $\theta > \bar{\theta}^{\mathrm{C}}$, we have $\bar{N}_6^{\mathrm{C}} > \bar{N}_3^{\mathrm{C}}$ and $\bar{N}_7^{\mathrm{C}} > \bar{N}_3^{\mathrm{C}}$.

Figure C.1 illustrates the regions in the parameter space where it is optimal to offer group buying, and the resulting market segmentation.

Figure C.1 Optimal Group-Buying Strategy and Customer Segmentation under Contingent Pricing



(a) $\theta > \bar{\theta}^{\mathrm{C}}$ (b) $1 < \theta \leqslant \bar{\theta}^{\mathrm{C}}$

□

D. Unobservable Group Buying

Proof of Proposition C.1. When the pledge-to-go state n is unobservable, there are only three possible scenarios: only high-end customers sign up (referred to as scenario $\{H\}$); only low-end customers sign up (referred to as scenario $\{L\}$); both high- and low-end customers sign up (referred to as scenario $\{H+L\}$). Note that market partitions $\{H; H+L\}$ and $\{L; L+H\}$ are not feasible here because customers cannot infer the expected waiting time from different states when n is unobservable.

For scenario $\{H\}$, the IR and IC constraints are

$$\begin{cases} IR_L : L - r^{\mathrm{U}} \geqslant 0, \\ IC_L : L - r^{\mathrm{U}} \geqslant \theta L - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}}, \\ IR_H : \theta H - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant 0, \\ IC_H : \theta H - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant H - r^{\mathrm{U}}, \end{cases}$$

where the expected waiting time w^{U} is

$$w^{\mathrm{U}} = \sum_{n=1}^{N} \frac{n-1}{\gamma\lambda} \cdot P(n) = \frac{N-1}{2\gamma\lambda}.$$

In equilibrium, IR_L and IC_H are binding. Thus, $r^{\mathrm{U}} = L$, and $p^{\mathrm{U}} = (\theta-1)H + L - c(N-1)/(2\gamma\lambda)$, $\pi^{\mathrm{U}} = (\theta-1)H\gamma\lambda + L\lambda - c(N-1)/2$.

For scenario $\{L\}$, the IR and IC constraints are

$$\begin{cases} IR_L : \theta L - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant 0, \\ IC_L : \theta L - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant L - r^{\mathrm{U}}, \\ IR_H : H - r^{\mathrm{U}} \geqslant 0, \\ IC_H : H - r^{\mathrm{U}} \geqslant \theta H - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}}, \end{cases}$$

where the expected waiting time w^{U} is

$$w^{\mathrm{U}} = \sum_{n=1}^{N} \frac{n-1}{(1-\gamma)\lambda} \cdot P(n) = \frac{N-1}{2(1-\gamma)\lambda}.$$

The constraints IC_L and IC_H cannot be satisfied at the same time, because IC_L requires $p^{\mathrm{U}} \leqslant (\theta-1)L + r^{\mathrm{U}} - c \cdot w^{\mathrm{U}}$, and IC_H requires $p^{\mathrm{U}} \geqslant (\theta-1)H + r^{\mathrm{U}} - c \cdot w^{\mathrm{U}}$, which are contradictory. Hence, the second scenario cannot become the equilibrium.

For scenario $\{H+L\}$, the IR and IC constraints are

$$\begin{cases} IR_L : \theta L - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant 0, \\ IC_L : \theta L - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant L - r^{\mathrm{U}}, \\ IR_H : \theta H - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant 0, \\ IC_H : \theta H - p^{\mathrm{U}} - c \cdot w^{\mathrm{U}} \geqslant H - r^{\mathrm{U}}, \end{cases}$$

where the expected waiting time w^{U} is

$$w^{\mathrm{U}} = \sum_{n=1}^{N} \frac{n-1}{\lambda} \cdot P(n) = \frac{N-1}{2\lambda}.$$

In equilibrium, $r^{\mathrm{U}} \in [L, +\infty)$, $p^{\mathrm{U}} = \theta L - c(N-1)/(2\lambda)$, and $\pi^{\mathrm{U}} = \theta L\lambda - c(N-1)/2$.[3]

Comparing the results above, the REE when offering the unobservable group buying is

(i) when $H/L \leqslant 1/\gamma$, $r^{\mathrm{U}} = \theta L$, $p^{\mathrm{U}} = \theta L - c(N-1)/(2\lambda)$, and $\pi^{\mathrm{U}} = \theta L\lambda - c(N-1)/2$;

(ii) when $H/L > 1/\gamma$, $r^{\mathrm{U}} = L$, $p^{\mathrm{U}} = (\theta-1)H + L - c(N-1)/(2\gamma\lambda)$, and $\pi^{\mathrm{U}} = (\theta-1)H\gamma\lambda + L\lambda - c(N-1)/2$. $\square$

[3] Since no customer purchases the regular product in this scenario, the value of r^{U} has no impact on the equilibrium profit and customer segmentation as long as $r^{\mathrm{U}} \geqslant L$.

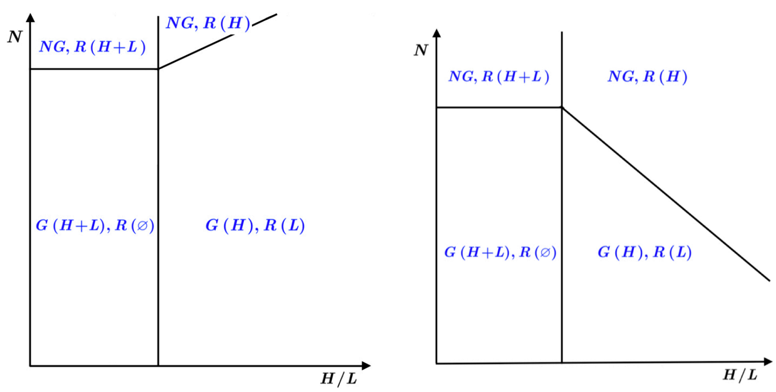
Proof of Theorem C.1. The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the batch size N:

$$\bar{N}^{\mathrm{U}} \equiv \begin{cases} \bar{N}_1^{\mathrm{U}} & H/L \leqslant 1/\gamma, \\ \bar{N}_2^{\mathrm{U}} & H/L > 1/\gamma, \end{cases}$$
$$\bar{N}_1^{\mathrm{U}} \equiv \frac{2(\theta-1)L\lambda}{c} + 1,$$
$$\bar{N}_2^{\mathrm{U}} \equiv \frac{2\left[L+(\theta-2)H\gamma\right]\lambda}{c} + 1.$$

$\bar{N}_1^{\mathrm{U}}$ increases in L. For a given L, $\bar{N}_2^{\mathrm{U}}$ increases in H/L when $\theta > 2$ while decreasing in H/L when $1 < \theta \leqslant 2$. Besides, $\bar{N}_1^{\mathrm{U}} > \bar{N}_2^{\mathrm{U}}$ if and only if $H/L < 1/\gamma$.

Figure D.1 illustrates the regions in the parameter space where it is optimal to offer group buying, and the resulting market segmentation.

Figure D.1 Optimal Unobservable Group-Buying Strategy and Customer Segmentation



(a) $\theta > 2$ (b) $1 < \theta \leqslant 2$

□

Proof of Proposition C.2. By comparing the equilibrium profits of the observable case (see Proposition 2) and the unobservable case (see Proposition C.1), we can prove that when $N \leqslant \max\{\bar{N}_1, \bar{N}_2\}$, $\pi^{\mathrm{U}} > \pi^{\mathrm{G}}$ always holds. As for $N > \max\{\bar{N}_1, \bar{N}_2\}$, we find a sufficient condition of $\pi^{\mathrm{U}} > \pi^{\mathrm{G}}$. Specifically, when $\theta \to +\infty$, $\lim_{\theta\to+\infty}\pi^{\mathrm{U}} = +\infty > cN\left[\gamma H + (1-\gamma)L\right] / \left[(1-\gamma)(H-L)\right] + L\lambda = \lim_{\theta\to+\infty}\pi^{\mathrm{G}}$, implying that there exists a positive threshold $\bar{\theta}_0^{\mathrm{U}}$ such that $\pi^{\mathrm{U}} > \pi^{\mathrm{G}}$ always holds for $\theta > \bar{\theta}_0^{\mathrm{U}}$ due to the continuity. In addition, by comparing the values of thresholds in Theorems 2 and C.1, the second part of the corollary can be proved. In detail, $\bar{N}_1^{\mathrm{U}} > \bar{N}_4$ if and only if $\theta > \bar{\theta}_1^{\mathrm{U}}$; $\bar{N}_2^{\mathrm{U}} > \bar{N}_5$ if and only if $\theta > \bar{\theta}_2^{\mathrm{U}}$; $\bar{N}_2^{\mathrm{U}} > \bar{N}_1$ if and only if $\theta > \bar{\theta}_3^{\mathrm{U}}$; and $\bar{N}_2^{\mathrm{U}} > \bar{N}_6$ if and only if $\theta > \bar{\theta}_4^{\mathrm{U}}$, where $\bar{\theta}_4^{\mathrm{U}}$ is the unique positive solution to the equation $\bar{N}_2^{\mathrm{U}} = \bar{N}_6$, and other thresholds for θ are defined as

$$\bar{\theta}_1^{\mathrm{U}} \equiv \frac{c(1-\gamma)}{[L-(H-L)\gamma]\,\lambda} + 1,$$
$$\bar{\theta}_2^{\mathrm{U}} \equiv 2 - \frac{L}{H\gamma},$$
$$\bar{\theta}_3^{\mathrm{U}} \equiv \frac{2(H\gamma - L)\lambda - c\gamma}{[2H\gamma - L(1-\gamma)]\,\lambda} + 1,$$
$$\bar{\theta}^{\mathrm{U}} \equiv \begin{cases} \bar{\theta}_1^{\mathrm{U}} & H/L \leqslant 1/\gamma, \\ \max\{\bar{\theta}_2^{\mathrm{U}}, \bar{\theta}_3^{\mathrm{U}}, \bar{\theta}_4^{\mathrm{U}}\} & H/L > 1/\gamma. \quad \square \end{cases}$$

E. Heterogeneous Waiting Costs

We study group buying with customers' heterogeneous waiting costs, with the superscript "D" denoting the equilibrium outcome.

Lemma E.1 (Customers' Sign-up Behavior with Heterogeneous Waiting Costs). *If customers have heterogeneous waiting costs, for any $\theta \neq 1$ and N, if high-end (low-end) customers sign up at state $n\,(1 \leqslant n \leqslant N)$, then all high-end (low-end) customers sign up at any subsequent state $n'\,(1 \leqslant n' \leqslant n)$.*

Proof of Lemma E.1. We prove the lemma by induction. For any state $n\,(1 \leqslant n \leqslant N)$, there are three possible cases depending on which segment signs up: high-end segment only (Case 1); low-end segment only (Case 2); and both segments (Case 3). Suppose a high-end customer signs up at state n (Case 1), implying that the following IR and IC constraints must be satisfied:

$$IR_H : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant 0,$$
$$IC_H : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant H - r^{\mathrm{D}}.$$

Regardless of what the prices p^{D} and r^{D} are, since $w^{\mathrm{D}}(n)$ is monotonously increasing in n, and the surpluses of purchasing the regular product remain the same for the high-end customers (i.e., $H - r^{\mathrm{D}}$), $\theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n-1) > H - r^{\mathrm{D}}$ always holds. Thus, a high-end customer who arrives at state $n-1$ would also sign up, because the following constraints can always be satisfied:

$$IR_H : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n-1) > 0,$$
$$IC_H : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n-1) > H - r^{\mathrm{D}}.$$

The analysis above ensures that once a high-end customer signs up at the specific state n, all the following high-end customers will also join the group buying. On the other hand, if no high-end customer signs up at the specific state n, it implies that $\theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) < \max\{H - r^{\mathrm{D}}, 0\}$. This cannot guarantee that the next arriving high-end customer will also be unwilling to sign up. Cases 2 and 3 can be proved analogously. $\square$

Proof of Proposition D.1. By Lemma E.1, if the high-end customers sign up first, there are three possible scenarios: $\{H\}$, $\{H; H+L\}$, and $\{H+L\}$, which are defined as Case I for ease of analysis. Similarly, if the low-end customers sign up first, there are also three possible scenarios: $\{L\}$, $\{L; H+L\}$, and $\{L+H\}$, which are defined as Case II. We use the tipping state $\bar{x}^{\mathrm{D}}$ to stand for different scenarios. To be specific, in Case I, $\bar{x}^{\mathrm{D}}=0$, $1 \leqslant \bar{x}^{\mathrm{D}} < N$, and $\bar{x}^{\mathrm{D}}=N$ represent scenarios $\{H\}$, $\{H; H+L\}$, and $\{H+L\}$, respectively. In Case II, $\bar{x}^{\mathrm{D}}=0$, $1 \leqslant \bar{x}^{\mathrm{D}} < N$, and $\bar{x}^{\mathrm{D}}=N$ represent scenarios $\{L\}$, $\{L; H+L\}$, and $\{L+H\}$, respectively.

For Case II, the IR and IC constraints are

$$\begin{cases} IR_L : \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_L : \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant L - r^{\mathrm{D}} & 1 \leqslant n \leqslant N, \\ IR_{H1} : H - r^{\mathrm{D}} \geqslant 0 & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IC_{H1} : H - r^{\mathrm{D}} \geqslant \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IR_{H2} : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ IC_{H2} : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant H - r^{\mathrm{D}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{D}} < N$ and the expected waiting time $w^{\mathrm{D}}(n)$ is

$$w^{\mathrm{D}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ \frac{\bar{x}^{\mathrm{D}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{D}}-1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{D}} < n \leqslant N. \end{cases}$$

In equilibrium, IR_L is binding at state $n=N$. Thus, $p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \theta L - c_L \cdot w^{\mathrm{D}}(N)$. Define two prices $\bar{r}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) - (\theta-1)H + c_H \cdot w^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, and $\bar{r}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) - (\theta-1)L + c_L \cdot w^{\mathrm{D}}(N)$. We can see that $\bar{r}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \leqslant \bar{r}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ holds if and only if $c_H \leqslant \bar{c}_H$, where

$$\bar{c}_H \equiv \frac{(\theta-1)(H-L)(1-\gamma)\lambda + c_L(N-1-\gamma\bar{x}^{\mathrm{D}})}{(1-\gamma)(\bar{x}^{\mathrm{D}}-1)}.$$

To satisfy IC_{H1} and IC_L, the price r^{D} should meet the constraints $r^{\mathrm{D}} < \bar{r}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ and $r^{\mathrm{D}} \geqslant \bar{r}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$. Therefore, when $c_H \leqslant \bar{c}_H$, since $\bar{r}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \leqslant \bar{r}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, it is impossible to satisfy these two constraints at the same time, and hence Case II cannot become the equilibrium. In contrast, when $c_H > \bar{c}_H$, since $\bar{r}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) > \bar{r}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, it is possible to satisfy these two constraints at the same time, and hence Case II can become the equilibrium.

Using similar logic, we can show that when $c_H \leqslant \bar{c}_H$, Case I can become the equilibrium; while when $c_H > \bar{c}_H$, Case I cannot become the equilibrium, which proves the proposition. □

Proof of Proposition D.2. Based on Proposition D.1, we consider the regions of $c_H \leqslant \bar{c}_H$ and $c_H > \bar{c}_H$ separately.

First, when $c_H \leqslant \bar{c}_H$, Case I can be the equilibrium, and the IR and IC constraints are

$$\begin{cases} IR_H: \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_H: \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant H - r^{\mathrm{D}} & 1 \leqslant n \leqslant N, \\ IR_{L1}: L - r^{\mathrm{D}} \geqslant 0 & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IC_{L1}: L - r^{\mathrm{D}} \geqslant \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IR_{L2}: \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ IC_{L2}: \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant L - r^{\mathrm{D}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{D}} \leqslant N$ and the expected waiting time $w^{\mathrm{D}}(n)$ is

$$w^{\mathrm{D}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ \frac{\bar{x}^{\mathrm{D}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{D}} - 1}{\gamma\lambda} & \bar{x}^{\mathrm{D}} < n \leqslant N. \end{cases}$$

In equilibrium, IR_{L1} is binding. Thus, $r^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = L$. Whether IC_H or IC_{L2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{D}}$. Since $w^{\mathrm{D}}(n)$ is monotonously increasing in n, no matter whether IC_H or IC_{L2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv \theta L - c_L \cdot w^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, $\bar{p}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv (\theta - 1)H + L - c_H \cdot w^{\mathrm{D}}(N)$, and we know that $\bar{p}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) < \bar{p}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ if and only if $\bar{x}^{\mathrm{D}} > \bar{x}_1^{\mathrm{D}}$, where

$$\bar{x}_1^{\mathrm{D}} \equiv \frac{c_H(N-1) + c_L\gamma - (\theta - 1)(H - L)\gamma\lambda}{\gamma c_L + (1-\gamma)c_H}.$$

Then, we can write the price p^{D} as the function of tipping state $\bar{x}^{\mathrm{D}}$:

$$p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \begin{cases} \bar{p}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) & 1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_1^{\mathrm{D}}, \\ \bar{p}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) & \bar{x}_1^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{D}, also as the function of tipping state $\bar{x}^{\mathrm{D}}$:

$$\begin{aligned} \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) &= \left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \gamma\lambda + r^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot (1-\gamma)\lambda\right] \cdot P(n > \bar{x}^{\mathrm{D}}) + \left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \lambda\right] \cdot P(n \leqslant \bar{x}^{\mathrm{D}}) \\ &= \frac{[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \gamma\lambda + L(1-\gamma)\lambda] \cdot \frac{N - \bar{x}^{\mathrm{D}}}{\gamma\lambda} + [p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \lambda] \cdot \frac{\bar{x}^{\mathrm{D}}}{\lambda}}{\frac{\bar{x}^{\mathrm{D}}}{\lambda} + \frac{N - \bar{x}^{\mathrm{D}}}{\gamma\lambda}} \\ &= \frac{p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \gamma\lambda N + L(1-\gamma)\lambda(N - \bar{x}^{\mathrm{D}})}{N - (1-\gamma)\bar{x}^{\mathrm{D}}}. \end{aligned}$$

For $\bar{x}_1^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N$, plugging $p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \bar{p}_1^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ into $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, we have

$$\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \frac{\theta L\gamma\lambda N + L(1-\gamma)\lambda(N - \bar{x}^{\mathrm{D}}) - c_L\gamma N(\bar{x}^{\mathrm{D}} - 1)}{N - (1-\gamma)\bar{x}^{\mathrm{D}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ w.r.t. $\bar{x}^{\mathrm{D}}$, we have

$$\frac{\partial \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial \bar{x}^{\mathrm{D}}} = \frac{\gamma N\left[L(1-\gamma)(\theta-1)\lambda - c_L(N - 1 + \gamma)\right]}{\left[N - (1-\gamma)\bar{x}^{\mathrm{D}}\right]^2}.$$

We can see that $\frac{\partial \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial \bar{x}^{\mathrm{D}}} \geqslant 0$ if and only if $N \leqslant \bar{N}_1^{\mathrm{D}}$. Therefore, for $\bar{x}_1^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N$, when $N \leqslant \bar{N}_1^{\mathrm{D}}$, the firm sets $\bar{x}^{\mathrm{D}} = N$; when $N > \bar{N}_1^{\mathrm{D}}$, the firm sets $\bar{x}^{\mathrm{D}} = \bar{x}_1^{\mathrm{D}}$. Note that $\bar{x}_1^{\mathrm{D}} > 0$ if and only if $N > \bar{N}_2^{\mathrm{D}}$.

For $1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_1^{\mathrm{D}}$, plugging $p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \bar{p}_2^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ into $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, we have

$$\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \frac{[(\theta-1)H + L]\gamma\lambda N + L(1-\gamma)\lambda(N-\bar{x}^{\mathrm{D}}) - c_H N(N-1) + c_H(1-\gamma)N\bar{x}^{\mathrm{D}}}{N-(1-\gamma)\bar{x}^{\mathrm{D}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ w.r.t. $\bar{x}^{\mathrm{D}}$, we have

$$\frac{\partial \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial \bar{x}^{\mathrm{D}}} = \frac{(1-\gamma)N\left[c_H + (\theta-1)H\gamma\lambda\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{D}}\right]^2} > 0.$$

Since $\frac{\partial \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial \bar{x}^{\mathrm{D}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_1^{\mathrm{D}}$, the firm always sets $\bar{x}^{\mathrm{D}} = \bar{x}_1^{\mathrm{D}}$.

Second, when $c_H > \bar{c}_H$, Case II can be the equilibrium, and the IR and IC constraints are

$$\begin{cases} IR_L : \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_L : \theta L - p^{\mathrm{D}} - c_L \cdot w^{\mathrm{D}}(n) \geqslant L - r^{\mathrm{D}} & 1 \leqslant n \leqslant N, \\ IR_{H1} : H - r^{\mathrm{D}} \geqslant 0 & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IC_{H1} : H - r^{\mathrm{D}} \geqslant \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) & \bar{x}^{\mathrm{D}} < n \leqslant N, \\ IR_{H2} : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ IC_{H2} : \theta H - p^{\mathrm{D}} - c_H \cdot w^{\mathrm{D}}(n) \geqslant H - r^{\mathrm{D}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{D}} \leqslant N$ and the expected waiting time $w^{\mathrm{D}}(n)$ is

$$w^{\mathrm{D}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{D}}, \\ \frac{\bar{x}^{\mathrm{D}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{D}} - 1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{D}} < n \leqslant N. \end{cases}$$

In equilibrium, IR_{H1} is binding. Thus, $r^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = H$. Whether IR_L or IC_{H2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{D}}$. Since $w^{\mathrm{D}}(n)$ is monotonously increasing in n, no matter whether IC_H or IC_{L2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_3^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv \theta L - c_L \cdot w^{\mathrm{D}}(N)$, $\bar{p}_4^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \equiv \theta H - c_H \cdot w^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, and we know that $\bar{p}_3^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) < \bar{p}_4^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ if and only if $\bar{x}^{\mathrm{D}} < \bar{x}_2^{\mathrm{D}}$, where

$$\bar{x}_2^{\mathrm{D}} \equiv \frac{c_L(N-1) + c_H(1-\gamma) + \theta(H-L)(1-\gamma)\lambda}{(c_H - c_L)(1-\gamma) + c_L}.$$

Then, we can write the price p^{D} as the function of tipping state $\bar{x}^{\mathrm{D}}$:

$$p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \begin{cases} \bar{p}_3^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) & 1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_2^{\mathrm{D}}, \\ \bar{p}_4^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) & \bar{x}_2^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{D}, also as the function of tipping state $\bar{x}^{\mathrm{D}}$:

$$\begin{aligned} \pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) &= \left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot (1-\gamma)\lambda + r^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \gamma\lambda\right] \cdot P(n > \bar{x}^{\mathrm{D}}) + \left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \lambda\right] \cdot P(n \leqslant \bar{x}^{\mathrm{D}}) \\ &= \frac{\left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot (1-\gamma)\lambda + H\gamma\lambda\right] \cdot \frac{N-\bar{x}^{\mathrm{D}}}{(1-\gamma)\lambda} + \left[p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot \lambda\right] \cdot \frac{\bar{x}^{\mathrm{D}}}{\lambda}}{\frac{\bar{x}^{\mathrm{D}}}{\lambda} + \frac{N-\bar{x}^{\mathrm{D}}}{(1-\gamma)\lambda}} \\ &= \frac{p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) \cdot (1-\gamma)\lambda N + H\gamma\lambda(N-\bar{x}^{\mathrm{D}})}{N-\gamma\bar{x}^{\mathrm{D}}}. \end{aligned}$$

For $\bar{x}_2^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N$, plugging $p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \bar{p}_4^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ into $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, we have

$$\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \frac{\theta H(1-\gamma)\lambda N + H\gamma\lambda(N-\bar{x}^{\mathrm{D}}) - c_H(1-\gamma)N(\bar{x}^{\mathrm{D}}-1)}{N-\gamma\bar{x}^{\mathrm{D}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ w.r.t. $\bar{x}^{\mathrm{D}}$, we have

$$\frac{\partial\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial\bar{x}^{\mathrm{D}}} = \frac{(1-\gamma)N\left[(\theta-1)H\gamma\lambda - c_H(N-\gamma)\right]}{\left[N-\gamma\bar{x}^{\mathrm{D}}\right]^2}.$$

We can see that $\frac{\partial\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial\bar{x}^{\mathrm{D}}} \geqslant 0$ if and only if $N \leqslant \bar{N}_3^{\mathrm{D}}$. Therefore, for $\bar{x}_2^{\mathrm{D}} < \bar{x}^{\mathrm{D}} \leqslant N$, when $N \leqslant \bar{N}_3^{\mathrm{D}}$, the firm sets $\bar{x}^{\mathrm{D}} = N$; when $N > \bar{N}_3^{\mathrm{D}}$, the firm sets $\bar{x}^{\mathrm{D}} = \bar{x}_2^{\mathrm{D}}$.

For $1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_2^{\mathrm{D}}$, plugging $p^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \bar{p}_3^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ into $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$, we have

$$\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}}) = \frac{\theta L(1-\gamma)\lambda N + H\gamma\lambda(N-\bar{x}^{\mathrm{D}}) - c_L N(N-1) + c_L\gamma N\bar{x}^{\mathrm{D}}}{N-\gamma\bar{x}^{\mathrm{D}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})$ w.r.t. $\bar{x}^{\mathrm{D}}$, we have

$$\frac{\partial\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial\bar{x}^{\mathrm{D}}} = \frac{(\theta L + H)\gamma(1-\gamma)\lambda N + c_L\gamma N}{\left[N-\gamma\bar{x}^{\mathrm{D}}\right]^2} > 0.$$

Since $\frac{\partial\pi^{\mathrm{D}}(\bar{x}^{\mathrm{D}})}{\partial\bar{x}^{\mathrm{D}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{D}} \leqslant \bar{x}_2^{\mathrm{D}}$, the firm always sets $\bar{x}^{\mathrm{D}} = \bar{x}_2^{\mathrm{D}}$.

Comparing the results above, we define three thresholds for the batch size N:

$$\bar{N}_1^{\mathrm{D}} \equiv \frac{(\theta-1)(1-\gamma)L\lambda}{c_L} + 1 - \gamma,$$
$$\bar{N}_2^{\mathrm{D}} \equiv \frac{(\theta-1)(H-L)\gamma\lambda - \gamma c_L}{c_H} + 1,$$
$$\bar{N}_3^{\mathrm{D}} \equiv \frac{(\theta-1)H\gamma\lambda}{c_H} + \gamma,$$

which determines the optimal tipping state $\bar{x}^{\mathrm{D}}$. Besides, $\bar{N}_1^{\mathrm{D}}$ increases in L, and $\bar{N}_3^{\mathrm{D}}$ increases in H. For a given L, $\bar{N}_2^{\mathrm{D}}$ increases in H/L. $\bar{N}_2^{\mathrm{D}} > \bar{N}_1^{\mathrm{D}}$ if and only if $H > [\gamma c_L + (1-\gamma)c_H]L/(\gamma c_L) - (c_H - c_L)/[(\theta-1)\lambda]$. Thus, the REE when offering group buying when customers have heterogeneous waiting costs is

(i) when $c_H \leqslant \bar{c}_H$,

  (a) when $H \leqslant [\gamma c_L + (1-\gamma)c_H]L/(\gamma c_L) - (c_H - c_L)/[(\theta-1)\lambda]$,

    (1) if $N \leqslant \bar{N}_1^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = N$, $r^{\mathrm{D}} = L$, $p^{\mathrm{D}} = \theta L - c_L(N-1)/\lambda$, and $\pi^{\mathrm{D}} = \theta L\lambda - c_L(N-1)$;

    (2) if $N > \bar{N}_1^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = \bar{x}_1^{\mathrm{D}}$, $r^{\mathrm{D}} = L$, $p^{\mathrm{D}} = \theta L - c_L(\bar{x}_1^{\mathrm{D}} - 1)/\lambda$, and $\pi^{\mathrm{D}} = \pi_1^{\mathrm{D}}$;

  (b) when $H > [\gamma c_L + (1-\gamma)c_H]L/(\gamma c_L) - (c_H - c_L)/[(\theta-1)\lambda]$,

    (1) if $N \leqslant \bar{N}_2^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = 0$, $r^{\mathrm{D}} = L$, $p^{\mathrm{D}} = (\theta-1)H + L - c_H(N-1)/(\gamma\lambda)$, and $\pi^{\mathrm{D}} = (\theta - 1)H\gamma\lambda + L\lambda - c_H(N-1)$;

    (2) if $N > \bar{N}_2^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = \bar{x}_1^{\mathrm{D}}$, $r^{\mathrm{D}} = L$, $p^{\mathrm{D}} = \theta L - c_L(\bar{x}_1^{\mathrm{D}} - 1)/\lambda$, and $\pi^{\mathrm{D}} = \pi_1^{\mathrm{D}}$;

(ii) when $c_H > \bar{c}_H$,

(1) if $N \leqslant \bar{N}_3^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = N$, $r^{\mathrm{D}} = H$, $p^{\mathrm{D}} = \theta H - c_H(N-1)/\lambda$, and $\pi^{\mathrm{D}} = \theta H\lambda - c_H(N-1)$;

(2) if $N > \bar{N}_3^{\mathrm{D}}$, $\bar{x}^{\mathrm{D}} = \bar{x}_2^{\mathrm{D}}$, $r^{\mathrm{D}} = H$, $p^{\mathrm{D}} = \theta H - c_H(\bar{x}_2^{\mathrm{D}}-1)/\lambda$, and $\pi^{\mathrm{D}} = \pi_2^{\mathrm{D}}$;

where

$$\pi_1^{\mathrm{D}} \equiv \frac{\theta L\gamma\lambda N + L(1-\gamma)\lambda(N-\bar{x}_1^{\mathrm{D}}) - c_L\gamma N(\bar{x}_1^{\mathrm{D}}-1)}{N-(1-\gamma)\bar{x}_1^{\mathrm{D}}},$$
$$\pi_2^{\mathrm{D}} \equiv \frac{\theta H(1-\gamma)\lambda N + H\gamma\lambda(N-\bar{x}_2^{\mathrm{D}}) - c_H(1-\gamma)N(\bar{x}_2^{\mathrm{D}}-1)}{N-\gamma\bar{x}_2^{\mathrm{D}}}. \quad \square$$

Proof of Corollary D.1. Since $\bar{N}_2^{\mathrm{D}} > \bar{N}_2$ always holds when $c_H \leqslant c_L$, and $\bar{N}_2^{\mathrm{D}} < \bar{N}_2$ always holds when $c_L < c_H \leqslant \bar{c}_H$, the corollary can be proved. $\square$

F. Inferior Group-Buying Product

We study group buying of a product that is inferior to the regular product, with the superscript "I" denoting the equilibrium outcome.

Proof of Proposition E.1. In similar fashion to the proof of Proposition 1, we define $\bar{x}^{\mathrm{I}}$ as the tipping state, where $0 \leqslant \bar{x}^{\mathrm{I}} \leqslant N$. Note that Lemma A.1 is also applicable to the case with an inferior group-buying product. Thus, there are five possible scenarios in total for customer segmentation. Define the following two cases for ease of analysis. In Case I, $\bar{x}^{\mathrm{I}} = 0$, $1 \leqslant \bar{x}^{\mathrm{I}} < N$, and $\bar{x}^{\mathrm{I}} = N$ represent scenarios $\{L\}$, $\{L; L+H\}$, and $\{L+H\}$, respectively. In Case II, $\bar{x}^{\mathrm{I}} = 0$ and $1 \leqslant \bar{x}^{\mathrm{I}} < N$ represent scenarios $\{H\}$, and $\{H; H+L\}$, respectively. In equilibrium, the prices p^{I} and r^{I} should satisfy the IR and IC constraints.

For Case II, the IR and IC constraints are

$$\begin{cases} IR_H : \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_H : \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(n) \geqslant H - r^{\mathrm{I}} & 1 \leqslant n \leqslant N, \\ IR_{L1} : L - r^{\mathrm{I}} \geqslant 0 & \bar{x}^{\mathrm{I}} < n \leqslant N, \\ IC_{L1} : L - r^{\mathrm{I}} \geqslant \theta L - p^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(n) & \bar{x}^{\mathrm{I}} < n \leqslant N, \\ IR_{L2} : \theta L - p^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \\ IC_{L2} : \theta L - p^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(n) \geqslant L - r^{\mathrm{I}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{I}} < N$ and the expected waiting time $w^{\mathrm{C}}(n)$ is

$$w^{\mathrm{C}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \\ \frac{\bar{x}^{\mathrm{I}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{I}}-1}{\gamma\lambda} & \bar{x}^{\mathrm{I}} < n \leqslant N. \end{cases}$$

The constraint IC_{L1} requires $p^{\mathrm{I}} \geqslant -(1-\theta)L + r^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{I}})$. The constraint IC_H requires $p^{\mathrm{I}} \leqslant -(1-\theta)H + r^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(N)$. Since $w^{\mathrm{C}}(\bar{x}^{\mathrm{I}}) \leqslant w^{\mathrm{C}}(N)$, $-(1-\theta)L + r^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(\bar{x}^{\mathrm{I}}) > -(1-\theta)H + r^{\mathrm{I}} - c\cdot w^{\mathrm{C}}(N)$ always holds, so the constraints IC_{L1} and IC_H can never be satisfied at the same time when $\theta < 1$. Therefore, Case II cannot become the equilibrium.

Using similar logic, we can show that Case I can become the equilibrium, which proves the proposition. Refer to the proof of Proposition E.2 for details of Case I. $\square$

Proof of Proposition E.2. We continue the detailed analysis for Case I in this part (see the definition in the proof of Proposition E.1). The IR and IC constraints are

$$\begin{cases} IR_L : \theta L - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_L : \theta L - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(n) \geqslant L - r^{\mathrm{I}} & 1 \leqslant n \leqslant N, \\ IR_{H1} : H - r^{\mathrm{I}} \geqslant 0 & \bar{x}^{\mathrm{I}} < n \leqslant N, \\ IC_{H1} : H - r^{\mathrm{I}} \geqslant \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(n) & \bar{x}^{\mathrm{I}} < n \leqslant N, \\ IR_{H2} : \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \\ IC_{H2} : \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(n) \geqslant H - r^{\mathrm{I}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{I}} \leqslant N$ and the expected waiting time $w^{\mathrm{I}}(n)$ is

$$w^{\mathrm{I}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{I}}, \\ \frac{\bar{x}^{\mathrm{I}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{I}} - 1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{I}} < n \leqslant N. \end{cases}$$

Suppose $r^{\mathrm{I}} \geqslant L$, then IR_L is binding at state $n = N$ in equilibrium. Thus, $p^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) = \theta L - c\cdot w^{\mathrm{I}}(N)$. Besides, the constraint IR_{H1} is weaker than IC_{H1}; the same relationship exists between IR_{H2} and IC_{H2}. In order to satisfy IC_{H1} and IC_{H2} at state $n = \bar{x}^{\mathrm{I}}$, the price r^{I} should meet the constraints $H - r^{\mathrm{I}} \geqslant \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ and $\theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) \geqslant H - r^{\mathrm{I}}$, implying that $H - r^{\mathrm{I}} = \theta H - p^{\mathrm{I}} - c\cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ in equilibrium. Therefore, $r^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) = (1-\theta)H + \theta L - c\cdot w^{\mathrm{I}}(N) + c\cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$. So far, we have written both $p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ and $r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ as the functions of $\bar{x}^{\mathrm{I}}$. To satisfy $r^{\mathrm{I}} \geqslant L$, the tipping state $\bar{x}^{\mathrm{I}} \geqslant \bar{x}^{\mathrm{I}}_1$, where $\bar{x}^{\mathrm{I}}_1 \equiv N - \gamma - (1-\gamma)(1-\theta)(H-L)\lambda/c$. We then derive the firm's long-run average profit π^{I}:

$$\begin{aligned} \pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) &= \left[p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot(1-\gamma)\lambda + r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot\gamma\lambda\right]\cdot P(n > \bar{x}^{\mathrm{I}}) + \left[p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot\lambda\right]\cdot P(n \leqslant \bar{x}^{\mathrm{I}}) \\ &= \frac{\left[p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot(1-\gamma)\lambda + r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot\gamma\lambda\right]\cdot\frac{N-\bar{x}^{\mathrm{I}}}{(1-\gamma)\lambda} + \left[p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot\lambda\right]\cdot\frac{\bar{x}^{\mathrm{I}}}{\lambda}}{\frac{\bar{x}^{\mathrm{I}}}{\lambda} + \frac{N-\bar{x}^{\mathrm{I}}}{(1-\gamma)\lambda}} \\ &= \frac{p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot(1-\gamma)\lambda N + r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})\cdot\gamma\lambda(N-\bar{x}^{\mathrm{I}})}{N-\gamma\bar{x}^{\mathrm{I}}}. \end{aligned}$$

Plugging $p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ and $r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ into the firm's long-run average profit π^{I}, we have

$$\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) = \frac{\theta L\lambda N + (1-\theta)H\gamma\lambda N - cN(N-1) - [\theta L + (1-\theta)H]\,\gamma\lambda\bar{x}^{\mathrm{I}} + c\gamma N\bar{x}^{\mathrm{I}} - c\gamma(N-\bar{x}^{\mathrm{I}}-\gamma)(N-\bar{x}^{\mathrm{I}})/(1-\gamma)}{N-\gamma\bar{x}^{\mathrm{I}}}.$$

Take the second-order derivative of $\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ w.r.t. $\bar{x}^{\mathrm{I}}$. Denote $\bar{N} \equiv N/\gamma > N$. We find that when $\bar{x}^{\mathrm{I}} < \bar{N}$, $\frac{\partial^2\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})}{\partial\bar{x}^{I2}} < 0$ always holds, implying that $\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ is concave in $\bar{x}^{\mathrm{I}}$; when $\bar{x}^{\mathrm{I}} > \bar{N}$, $\frac{\partial^2\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})}{\partial\bar{x}^{I2}} > 0$ always holds, implying that $\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ is convex in $\bar{x}^{\mathrm{I}}$. Besides, $\lim_{\bar{x}^{\mathrm{I}}\to\bar{N}^-} = -\infty$, $\lim_{\bar{x}^{\mathrm{I}}\to\bar{N}^+} = +\infty$. In this case, the optimal $\bar{x}^{\mathrm{I}}$ within the constraint $0 \leqslant \bar{x}^{\mathrm{I}} \leqslant N$, in which $\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ is concave in $\bar{x}^{\mathrm{I}}$, is determined by $\bar{x}^{\mathrm{I}} = \bar{x}^{\mathrm{I}}_2$, where $\bar{x}^{\mathrm{I}}_2$ is the unique reasonable solution to the first-order condition $\frac{\partial\pi^{\mathrm{I}}(\bar{x}^{\mathrm{I}})}{\partial\bar{x}^{\mathrm{I}}} = 0$.[4] It is easy to verify that $\bar{x}^{\mathrm{I}}_1 < N$ and $\bar{x}^{\mathrm{I}}_2 < N$ always hold; $\bar{x}^{\mathrm{I}}_2 \geqslant \bar{x}^{\mathrm{I}}_1$ if and only if $H/L \geqslant \bar{m}^{\mathrm{I}}$

[4] The other solution is ruled out because it is always greater than N and is not within the constraint $0 \leqslant \bar{x}^{\mathrm{I}} \leqslant N$.

or $N \leqslant \bar{N}_1^{\mathrm{I}}$; $\bar{x}_2^{\mathrm{I}} > 0$ if and only if $N > \bar{N}_2^{\mathrm{I}}$; and $\bar{x}_1^{\mathrm{I}} > 0$ if and only if $N > \bar{N}_3^{\mathrm{I}}$. Therefore, combining the analyses above, when $H/L \geqslant \bar{m}^{\mathrm{I}}$ or $N \leqslant \bar{N}_1^{\mathrm{I}}$, the firm sets $\bar{x}^{\mathrm{I}} = \max\{\bar{x}_2^{\mathrm{I}}, 0\}$; otherwise, the firm sets $\bar{x}^{\mathrm{I}} = \max\{\bar{x}_1^{\mathrm{I}}, 0\}$.

Suppose $r^{\mathrm{I}} < L$, then IC_L is binding at state $n = N$ in equilibrium. Thus, $p^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) = r^{\mathrm{I}} - (1-\theta)L - c \cdot w^{\mathrm{I}}(N)$. Likewise, in order to satisfy IC_{H1} and IC_{H2} at state $n = \bar{x}^{\mathrm{I}}$, the price r^{I} should meet the constraints $H - r^{\mathrm{I}} \geqslant \theta H - p^{\mathrm{I}} - c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ and $\theta H - p^{\mathrm{I}} - c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) \geqslant H - r^{\mathrm{I}}$, implying that $r^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) = (1-\theta)H + p^{\mathrm{I}}(\bar{x}^{\mathrm{I}}) + c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$. Plugging $p^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$ into the expression of $r^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$, we have $(1-\theta)(H-L) = c \cdot w^{\mathrm{I}}(N) - c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$, which implies that $\bar{x}^{\mathrm{I}} = \bar{x}_1^{\mathrm{I}}$. Hence, $r^{\mathrm{I}} = L$. Therefore, $r^{\mathrm{I}} < L$ cannot form the equilibrium in Case I.

Combining the results above, we define the following thresholds for the batch size N:

$$
\begin{aligned}
\bar{N}^{\mathrm{I}} &\equiv \begin{cases} \min\{\bar{N}_2^{\mathrm{I}}, \bar{N}_3^{\mathrm{I}}\} & H/L \leqslant \bar{m}^{\mathrm{I}}, \\ \bar{N}_2^{\mathrm{I}} & H/L > \bar{m}^{\mathrm{I}}, \end{cases} \\
\bar{m}^{\mathrm{I}} &\equiv 2 - \frac{c(1+\gamma)}{(1-\gamma)(1-\theta)L\lambda}, \\
\bar{N}_1^{\mathrm{I}} &\equiv \frac{\gamma\left[c\gamma + (1-\gamma)(1-\theta)(H-L)\lambda\right]^2}{c(1-\gamma)\left[(1-\gamma)(1-\theta)(2L-H)\lambda - c(1+\gamma)\right]}, \\
\bar{N}_2^{\mathrm{I}} &\equiv \frac{(1-\gamma)^2\left[(1-\theta)H\lambda - c\right]}{c(2-\gamma)}, \\
\bar{N}_3^{\mathrm{I}} &\equiv \frac{(1-\gamma)(1-\theta)(H-L)\lambda}{c} + \gamma,
\end{aligned}
$$

where $\bar{m}^{\mathrm{I}}$ is the threshold for the valuation heterogeneity H/L, which increases in L. For a given L, $\bar{N}_1^{\mathrm{I}}$ and $\bar{N}_3^{\mathrm{I}}$ increase in H/L, and $\bar{N}_2^{\mathrm{I}}$ increases in H. Thus, the REE when offering group buying of an inferior product is

(i) when $H/L \leqslant \bar{m}^{\mathrm{I}}$,

  (1) if $N \leqslant \min\{\bar{N}_2^{\mathrm{I}}, \bar{N}_3^{\mathrm{I}}\}$, $\bar{x}^{\mathrm{I}} = 0$, $r^{\mathrm{I}} = (1-\theta)H + \theta L - c(N-\gamma)/\left[(1-\gamma)\lambda\right]$, $p^{\mathrm{I}} = \theta L - c(N-1)/\left[(1-\gamma)\lambda\right]$, and $\pi^{\mathrm{I}} = (1-\theta)H\gamma\lambda + \theta L\lambda - c(N-1)/(1-\gamma) - c\gamma$;

  (2) if $\min\{\bar{N}_2^{\mathrm{I}}, \bar{N}_3^{\mathrm{I}}\} < N \leqslant \max\{\bar{N}_1^{\mathrm{I}}, \bar{N}_3^{\mathrm{I}}\}$, $\bar{x}^{\mathrm{I}} = \bar{x}_2^{\mathrm{I}}$, $r^{\mathrm{I}} = (1-\theta)H + \theta L - c \cdot w^{\mathrm{I}}(N) + c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$, $p^{\mathrm{I}} = \theta L - c \cdot w^{\mathrm{I}}(N)$, and $\pi^{\mathrm{I}} = \left[p^{\mathrm{I}} \cdot (1-\gamma)\lambda N + r^{\mathrm{I}} \cdot \gamma\lambda(N - \bar{x}^{\mathrm{I}})\right]/(N - \gamma\bar{x}^{\mathrm{I}})$;

  (3) if $N > \max\{\bar{N}_1^{\mathrm{I}}, \bar{N}_3^{\mathrm{I}}\}$, $\bar{x}^{\mathrm{I}} = \bar{x}_1^{\mathrm{I}}$, $r^{\mathrm{I}} = L$, $p^{\mathrm{I}} = \theta L - \gamma(1-\theta)(H-L) - c(N-1-\gamma)/\lambda$, and $\pi^{\mathrm{I}} = \left[p^{\mathrm{I}} \cdot (1-\gamma)\lambda N + r^{\mathrm{I}} \cdot \gamma\lambda(N - \bar{x}^{\mathrm{I}})\right]/(N - \gamma\bar{x}^{\mathrm{I}})$;

(ii) when $H/L > \bar{m}^{\mathrm{I}}$,

  (1) if $N \leqslant \bar{N}_2^{\mathrm{I}}$, $\bar{x}^{\mathrm{I}} = 0$, $r^{\mathrm{I}} = (1-\theta)H + \theta L - c(N-\gamma)/\left[(1-\gamma)\lambda\right]$, $p^{\mathrm{I}} = \theta L - c(N-1)/\left[(1-\gamma)\lambda\right]$, and $\pi^{\mathrm{I}} = (1-\theta)H\gamma\lambda + \theta L\lambda - c(N-1)/(1-\gamma) - c\gamma$;

  (2) if $N > \bar{N}_2^{\mathrm{I}}$, $\bar{x}^{\mathrm{I}} = \bar{x}_2^{\mathrm{I}}$, $r^{\mathrm{I}} = (1-\theta)H + \theta L - c \cdot w^{\mathrm{I}}(N) + c \cdot w^{\mathrm{I}}(\bar{x}^{\mathrm{I}})$, $p^{\mathrm{I}} = \theta L - c \cdot w^{\mathrm{I}}(N)$, and $\pi^{\mathrm{I}} = \left[p^{\mathrm{I}} \cdot (1-\gamma)\lambda N + r^{\mathrm{I}} \cdot \gamma\lambda(N - \bar{x}^{\mathrm{I}})\right]/(N - \gamma\bar{x}^{\mathrm{I}})$. $\square$

Proof of Theorem E.1. For $N > \bar{N}^{\mathrm{I}}$, $\bar{x}^{\mathrm{I}} = \max\{\bar{x}_1^{\mathrm{I}}, \bar{x}_2^{\mathrm{I}}\}$. While for $N \leqslant \bar{N}^{\mathrm{I}}$, $\bar{x}^{\mathrm{I}} = 0$. We can rigorously attest that the equilibrium profit with group-buying where $\bar{x}^{\mathrm{I}} = \bar{x}_1^{\mathrm{I}}$ or $\bar{x}^{\mathrm{I}} = 0$ is dominated by the equilibrium profit without group buying. For $\bar{x}^{\mathrm{I}} = \bar{x}_1^{\mathrm{I}}$, we have $r^{\mathrm{I}} = L$ and $p^{\mathrm{I}} < L$, hence $\pi^{\mathrm{I}} < L\lambda = \pi^{\mathrm{V}} \leqslant \max\{\pi^{\mathrm{V}}, \pi^{\mathrm{M}}\}$ always holds. For $\bar{x}^{\mathrm{I}} = 0$, we can prove that when $H/L > 1/\gamma$, $\pi^{\mathrm{I}} < \pi^{\mathrm{V}}$ always holds, and when $H/L \leqslant 1/\gamma$, $\pi^{\mathrm{I}} < \pi^{\mathrm{M}}$ always holds, therefore $\pi^{\mathrm{I}} < \max\{\pi^{\mathrm{V}}, \pi^{\mathrm{M}}\}$ in this case.

Only for the equilibrium where $\bar{x}^{\mathrm{I}} = \bar{x}_2^{\mathrm{I}}$, it is possible that $\pi^{\mathrm{I}} \geqslant \max\{\pi^{\mathrm{M}}, \pi^{\mathrm{V}}\}$. For $\bar{x}^{\mathrm{I}} = \bar{x}_2^{\mathrm{I}}$, we can prove that $\pi^{\mathrm{I}} \geqslant \pi^{\mathrm{V}}$ when $N \leqslant \bar{N}_4^{\mathrm{I}}$, and $\pi^{\mathrm{I}} \geqslant \pi^{\mathrm{M}}$ when $N \leqslant \bar{N}_5^{\mathrm{I}}$, where $\bar{N}_4^{\mathrm{I}}$ and $\bar{N}_5^{\mathrm{I}}$ are the unique positive solution to the equations $\pi^{\mathrm{I}} = \pi^{\mathrm{V}}$ and $\pi^{\mathrm{I}} = \pi^{\mathrm{M}}$, respectively. Besides, $\bar{N}_4^{\mathrm{I}} < \bar{N}_5^{\mathrm{I}}$ if and only if $H/L < 1/\gamma$. Hence, $\pi^{\mathrm{I}} \geqslant \max\{\pi^{\mathrm{M}}, \pi^{\mathrm{V}}\}$ if and only if $\bar{N}^{\mathrm{I}} < N \leqslant \min\{\bar{N}_4^{\mathrm{I}}, \bar{N}_5^{\mathrm{I}}\}$. When $\gamma \to 1$, $\lim_{\gamma \to 1} \pi^{\mathrm{I}} = +\infty > H\lambda = \pi^{\mathrm{M}} \geqslant \max\{\pi^{\mathrm{M}}, \pi^{\mathrm{V}}\}$, thus $\pi^{\mathrm{I}} > \max\{\pi^{\mathrm{M}}, \pi^{\mathrm{V}}\}$ always holds, implying that for a small interval of $\gamma \in [1-\epsilon, 1]$, the inequation $\pi^{\mathrm{I}} > \max\{\pi^{\mathrm{M}}, \pi^{\mathrm{V}}\}$ holds due to the continuity.

□

G. Horizontally Differentiated Products

We study group buying in the context of horizontally differentiated products, with the superscript "H" denoting the equilibrium outcome.

Proof of Proposition F.1. In similar fashion to the proof of Propositions 1 and 2, we define $\bar{x}^{\mathrm{H}}$ as the tipping state. There are three possible scenarios in total. We use $\bar{x}^{\mathrm{H}} = 0$, $1 \leqslant \bar{x}^{\mathrm{H}} < N$, and $\bar{x}^{\mathrm{H}} = N$ to represent scenarios $\{G\}$, $\{G; G+R\}$, and $\{G+R\}$, respectively. In equilibrium, the prices p^{H} and r^{H} should satisfy the IR and IC constraints:

$$
\begin{cases}
IR_G : u - L - p^{\mathrm{H}} - c \cdot w^{\mathrm{H}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\
IC_G : u - L - p^{\mathrm{H}} - c \cdot w^{\mathrm{H}}(n) \geqslant u - H - r^{\mathrm{H}} & 1 \leqslant n \leqslant N, \\
IR_{R1} : u - L - r^{\mathrm{H}} \geqslant 0 & \bar{x}^{\mathrm{H}} < n \leqslant N, \\
IC_{R1} : u - L - r^{\mathrm{H}} \geqslant u - H - p^{\mathrm{H}} - c \cdot w^{\mathrm{H}}(n) & \bar{x}^{\mathrm{H}} < n \leqslant N, \\
IR_{R2} : u - H - p^{\mathrm{H}} - c \cdot w^{\mathrm{H}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{H}}, \\
IC_{R2} : u - H - p^{\mathrm{H}} - c \cdot w^{\mathrm{H}}(n) \geqslant u - L - r^{\mathrm{H}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{H}},
\end{cases}
$$

where $0 \leqslant \bar{x}^{\mathrm{H}} \leqslant N$ and the expected waiting time $w^{\mathrm{H}}(n)$ is

$$
w^{\mathrm{H}}(n) = \begin{cases}
\frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{H}}, \\
\frac{\bar{x}^{\mathrm{H}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{H}} - 1}{\gamma\lambda} & \bar{x}^{\mathrm{H}} < n \leqslant N.
\end{cases}
$$

In equilibrium, IR_{R1} is binding. Thus, $r^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = u - L$. Whether IR_G or IC_{R2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{H}}$. Since $w^{\mathrm{H}}(n)$ is monotonously increasing in n, no matter IR_G or IC_{R2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_1^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \equiv u - L - c \cdot w^{\mathrm{H}}(N)$, $\bar{p}_2^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \equiv u - H - c \cdot w^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$, and we know that $\bar{p}_1^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) < \bar{p}_2^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$ if and only if $\bar{x}^{\mathrm{H}} < \bar{x}_1^{\mathrm{H}}$, where $\bar{x}_1^{\mathrm{H}} \equiv N - 1 + \gamma - (H-L)\gamma\lambda/c$. Then, we can write the price p^{H} as the function of tipping state $\bar{x}^{\mathrm{H}}$:

$$p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = \begin{cases} \bar{p}_1^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) & 1 \leqslant \bar{x}^{\mathrm{H}} \leqslant \bar{x}_1^{\mathrm{H}}, \\ \bar{p}_2^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) & \bar{x}_1^{\mathrm{H}} < \bar{x}^{\mathrm{H}} \leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{H}, also as the function of tipping state $\bar{x}^{\mathrm{H}}$:

$$\begin{aligned} \pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) &= \left[p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \cdot \gamma\lambda + r^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \cdot (1-\gamma)\lambda\right] \cdot P(n > \bar{x}^{\mathrm{H}}) + \left[p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \cdot \lambda\right] \cdot P(n \leqslant \bar{x}^{\mathrm{H}}) \\ &= \frac{p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) \cdot \gamma\lambda N + (u-L)(1-\gamma)\lambda(N - \bar{x}^{\mathrm{H}})}{N - (1-\gamma)\bar{x}^{\mathrm{H}}}. \end{aligned}$$

For $\bar{x}_1^{\mathrm{H}} < \bar{x}^{\mathrm{H}} \leqslant N$, plugging $p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = \bar{p}_2^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$ into $\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$, we have

$$\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = \frac{(u-H)\gamma\lambda N - c\gamma N(\bar{x}^{\mathrm{H}} - 1) + (u-L)(1-\gamma)\lambda(N - \bar{x}^{\mathrm{H}})}{N - (1-\gamma)\bar{x}^{\mathrm{H}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$ w.r.t. $\bar{x}^{\mathrm{H}}$, we have

$$\frac{\partial \pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})}{\partial \bar{x}^{\mathrm{H}}} = -\frac{\gamma N\left[(H-L)(1-\gamma)(\theta-1)\lambda + c(N-1+\gamma)\right]}{\left[N - (1-\gamma)\bar{x}^{\mathrm{H}}\right]^2} < 0.$$

Since $\frac{\partial \pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})}{\partial \bar{x}^{\mathrm{H}}} < 0$ always holds, for $\bar{x}_1^{\mathrm{H}} < \bar{x}^{\mathrm{H}} \leqslant N$, the firm always sets $\bar{x}^{\mathrm{H}} = \bar{x}_1^{\mathrm{H}}$. Note that $\bar{x}_1^{\mathrm{H}} > 0$ if and only if $N > \bar{N}^{\mathrm{H}}$.

For $1 \leqslant \bar{x}^{\mathrm{H}} \leqslant \bar{x}_1^{\mathrm{H}}$, plugging $p^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = \bar{p}_1^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$ into $\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$, we have

$$\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}}) = \frac{(u-L)\gamma\lambda N - c\gamma N\bar{x}^{\mathrm{H}} - cN(N - \bar{x}^{\mathrm{H}} - 1) + (u-L)(1-\gamma)\lambda(N - \bar{x}^{\mathrm{H}})}{N - (1-\gamma)\bar{x}^{\mathrm{H}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})$ w.r.t. $\bar{x}^{\mathrm{H}}$, we have

$$\frac{\partial \pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})}{\partial \bar{x}^{\mathrm{H}}} = \frac{c(1-\gamma)N}{\left[N - (1-\gamma)\bar{x}^{\mathrm{H}}\right]^2} > 0.$$

Since $\frac{\partial \pi^{\mathrm{H}}(\bar{x}^{\mathrm{H}})}{\partial \bar{x}^{\mathrm{H}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{H}} \leqslant \bar{x}_1^{\mathrm{H}}$, the firm always sets $\bar{x}^{\mathrm{H}} = \bar{x}_1^{\mathrm{H}}$.

Combining the results above, we define the following threshold for the batch size N:

$$\bar{N}^{\mathrm{H}} \equiv \frac{(H-L)\gamma\lambda}{c} + 1 - \gamma,$$

which determines the optimal tipping state $\bar{x}^{\mathrm{H}}$. Thus, the REE when offering group buying in the context of horizontally differentiated products is

(i) if $N \leqslant \bar{N}^{\mathrm{H}}$, $\bar{x}^{\mathrm{H}} = 0$, $r^{\mathrm{H}} = u - L$, $p^{\mathrm{H}} = u - L - c(N-1)/(\gamma\lambda)$, and $\pi^{\mathrm{H}} = (u-L)\lambda - c(N-1)$;

(ii) if $N > \bar{N}^{\mathrm{H}}$, $\bar{x}^{\mathrm{H}} = \bar{x}_1^{\mathrm{H}}$, $r^{\mathrm{H}} = u - L$, $p^{\mathrm{H}} = u - H - c(\bar{x}_1^{\mathrm{H}} - 1)/\lambda$, and $\pi^{\mathrm{H}} = [(u-H)\gamma\lambda N + (u-L)(1-\gamma)\lambda(N - \bar{x}_1^{\mathrm{H}}) - c\gamma N(\bar{x}_1^{\mathrm{H}} - 1)] / [N - (1-\gamma)\bar{x}_1^{\mathrm{H}}]$. $\square$

Proof of Theorem F.1. In the product-line strategy, the firm sets prices $p^{\mathrm{P}} = r^{\mathrm{P}} = u - L$, and the firm's long-run average profit is

$$\pi^{\mathrm{P}} = \frac{Np^{\mathrm{P}} + N\frac{1-\gamma}{\gamma}r^{\mathrm{P}} - \frac{hN(N+1)}{2\gamma\lambda}}{\frac{N}{\gamma\lambda}} = (u-L)\lambda - \frac{(N+1)h}{2}.$$

The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the inventory holding cost h:

$$\bar{h}_1^{\mathrm{H}} \equiv \frac{2c(N-1)}{N+1},$$
$$\bar{h}_2^{\mathrm{H}} \equiv \frac{2(u-L)\lambda - 2\pi_1^{\mathrm{H}}}{N+1},$$
$$\bar{h}^{\mathrm{H}} \equiv \max\left\{\bar{h}_1^{\mathrm{H}}, \bar{h}_2^{\mathrm{H}}\right\},$$

where $\pi_1^{\mathrm{H}} = [(u-H)\gamma\lambda N + (u-L)(1-\gamma)\lambda(N - \bar{x}_1^{\mathrm{H}}) - c\gamma N(\bar{x}_1^{\mathrm{H}} - 1)] \,/\, [N - (1-\gamma)\bar{x}_1^{\mathrm{H}}]$. $\square$

H. No Regular Product

We study group buying without the regular product, with the superscript "R" denoting the equilibrium outcome. Note that each customer decides whether to join the group buying or exit with no purchase upon arrival.

PROPOSITION H.1 (REE WITHOUT REGULAR PRODUCT). *Without the regular product, for any given $\theta > 1$ and N, there exist two thresholds for the batch size, $\bar{N}_1^{\mathrm{R}} \equiv \theta(1-\gamma)L\lambda/c + 1 - \gamma$ and $\bar{N}_2^{\mathrm{R}} \equiv \theta(H-L)\gamma\lambda/c + 1 - \gamma$, such that*

(i) when $H/L \leqslant 1/\gamma$, the firm sets prices so that

(i-1) if $N \leqslant \bar{N}_1^{\mathrm{R}}$, $\{H+L\}$ is an REE;

(i-2) if $N > \bar{N}_1^{\mathrm{R}}$, $\{H; H+L\}$ is an REE;

(ii) when $H/L > 1/\gamma$, the firm sets prices so that

(ii-1) if $N \leqslant \bar{N}_2^{\mathrm{R}}$, $\{H\}$ is an REE;

(ii-2) if $N > \bar{N}_2^{\mathrm{R}}$, $\{H; H+L\}$ is an REE.

COROLLARY H.1 (EFFECT OF NO REGULAR PRODUCT ON SIGN-UP TIME). *Without the regular product, in intertemporal segmentation the low-end customers join the group buying later than in the base model.*

Proof of Proposition H.1. In similar fashion to the proof of Propositions 1 and 2, we define $\bar{x}^{\mathrm{R}}$ as the tipping state. There are five possible scenarios in total. Define the following two cases for ease of analysis. In Case I, $\bar{x}^{\mathrm{R}} = 0$, $1 \leqslant \bar{x}^{\mathrm{R}} < N$, and $\bar{x}^{\mathrm{R}} = N$ represent scenarios $\{H\}$, $\{H; H+L\}$, and $\{H+L\}$, respectively. In Case II, $\bar{x}^{\mathrm{R}} = 0$ and $1 \leqslant \bar{x}^{\mathrm{R}} < N$ represent scenarios $\{L\}$ and $\{L; H+L\}$, respectively. In equilibrium, the prices p^{R} and r^{R} should satisfy the IR and IC constraints.

For Case I, the IR and IC constraints are

$$\begin{cases} IR_H, IC_H: \theta H - p^{\mathrm{R}} - c\cdot w^{\mathrm{R}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IR_{L1}: 0 \geqslant 0 & \bar{x}^{\mathrm{R}} < n \leqslant N, \\ IC_{L1}: 0 \geqslant \theta L - p^{\mathrm{R}} - c\cdot w^{\mathrm{R}}(n) & \bar{x}^{\mathrm{R}} < n \leqslant N, \\ IR_{L2}, IC_{L2}: \theta L - p^{\mathrm{R}} - c\cdot w^{\mathrm{R}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{R}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{R}} \leqslant N$ and the expected waiting time $w^{\mathrm{R}}(n)$ is

$$w^{\mathrm{R}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{R}}, \\ \frac{\bar{x}^{\mathrm{R}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{R}}-1}{\gamma\lambda} & \bar{x}^{\mathrm{R}} < n \leqslant N. \end{cases}$$

Whether IC_H or IC_{L2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{R}}$. Since $w^{\mathrm{R}}(n)$ is monotonously increasing in n, no matter whether IC_H or IC_{L2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_1^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) \equiv \theta L - c\cdot w^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$, $\bar{p}_2^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) \equiv \theta H - c\cdot w^{\mathrm{R}}(N)$, and we know that $\bar{p}_1^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) < \bar{p}_2^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$ if and only if $\bar{x}^{\mathrm{R}} > \bar{x}_1^{\mathrm{R}}$, where $\bar{x}_1^{\mathrm{R}} \equiv N - 1 + \gamma - \theta(H-L)\gamma\lambda/c < \bar{x}_1^{\mathrm{G}}$ always holds. Then, we can write the price p^{R} as the function of tipping state $\bar{x}^{\mathrm{R}}$:

$$p^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) = \begin{cases} \bar{p}_2^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) & 1 \leqslant \bar{x}^{\mathrm{R}} \leqslant \bar{x}_1^{\mathrm{R}}, \\ \bar{p}_1^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) & \bar{x}_1^{\mathrm{R}} < \bar{x}^{\mathrm{R}} \leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{R}, also as the function of tipping state $\bar{x}^{\mathrm{R}}$:

$$\begin{aligned} \pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) &= \left[p^{\mathrm{R}}(\bar{x}^{\mathrm{R}})\cdot\gamma\lambda\right]\cdot P(n > \bar{x}^{\mathrm{R}}) + \left[p^{\mathrm{R}}(\bar{x}^{\mathrm{R}})\cdot\lambda\right]\cdot P(n \leqslant \bar{x}^{\mathrm{R}}) \\ &= \frac{p^{\mathrm{R}}(\bar{x}^{\mathrm{R}})\cdot\gamma\lambda N}{N-(1-\gamma)\bar{x}^{\mathrm{R}}}. \end{aligned}$$

For $\bar{x}_1^{\mathrm{R}} < \bar{x}^{\mathrm{R}} \leqslant N$, plugging $p^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) = \bar{p}_1^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$ into $\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$, we have

$$\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) = \frac{\gamma N\left[\theta L\gamma - c(\bar{x}^{\mathrm{R}}-1)\right]}{N-(1-\gamma)\bar{x}^{\mathrm{R}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$ w.r.t. $\bar{x}^{\mathrm{R}}$, we have

$$\frac{\partial\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})}{\partial\bar{x}^{\mathrm{R}}} = \frac{\gamma N\left[(\theta L(1-\gamma)\lambda - c(N-1+\gamma)\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{R}}\right]^2}.$$

We can see that $\frac{\partial\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})}{\partial\bar{x}^{\mathrm{R}}} \geqslant 0$ if and only if $N \leqslant \bar{N}_1^{\mathrm{R}}$. Therefore, for $\bar{x}_1^{\mathrm{R}} < \bar{x}^{\mathrm{R}} \leqslant N$, when $N \leqslant \bar{N}_1^{\mathrm{R}}$, the firm sets $\bar{x}^{\mathrm{R}} = N$; when $N > \bar{N}_1^{\mathrm{R}}$, the firm sets $\bar{x}^{\mathrm{R}} = \bar{x}_1^{\mathrm{R}}$. Note that $\bar{x}_1^{\mathrm{R}} > 0$ if and only if $N > \bar{N}_2^{\mathrm{R}}$.

For $1 \leqslant \bar{x}^{\mathrm{R}} \leqslant \bar{x}_1^{\mathrm{R}}$, plugging $p^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) = \bar{p}_2^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$ into $\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$, we have

$$\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}}) = \frac{\theta H\gamma\lambda N - cN(N-1) + c(1-\gamma)N\bar{x}^{\mathrm{R}}}{N-(1-\gamma)\bar{x}^{\mathrm{R}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})$ w.r.t. $\bar{x}^{\mathrm{R}}$, we have

$$\frac{\partial\pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})}{\partial\bar{x}^{\mathrm{R}}} = \frac{(1-\gamma)N(\theta H\gamma\lambda + c)}{\left[N-(1-\gamma)\bar{x}^{\mathrm{R}}\right]^2} > 0.$$

Since $\frac{\partial \pi^{\mathrm{R}}(\bar{x}^{\mathrm{R}})}{\partial \bar{x}^{\mathrm{R}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{R}} \leqslant \bar{x}_1^{\mathrm{R}}$, the firm always sets $\bar{x}^{\mathrm{R}} = \bar{x}_1^{\mathrm{R}}$.

For Case II, the IR and IC constraints are

$$\begin{cases} IR_L, IC_L: \theta L - p^{\mathrm{R}} - c \cdot w^{\mathrm{R}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IR_{H1}: 0 \geqslant 0 & \bar{x}^{\mathrm{R}} < n \leqslant N, \\ IC_{H1}: 0 \geqslant \theta H - p^{\mathrm{R}} - c \cdot w^{\mathrm{R}}(n) & \bar{x}^{\mathrm{R}} < n \leqslant N, \\ IR_{H2}, IC_{H2}: \theta H - p^{\mathrm{R}} - c \cdot w^{\mathrm{R}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{R}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{R}} < N$ and the expected waiting time $w^{\mathrm{R}}(n)$ is

$$w^{\mathrm{R}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{R}}, \\ \frac{\bar{x}^{\mathrm{R}}}{\lambda} + \frac{n - \bar{x}^{\mathrm{R}} - 1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{R}} < n \leqslant N. \end{cases}$$

For $\bar{x}^{\mathrm{R}} < n \leqslant N$, it is impossible to satisfy IC_L and IC_{H1} constraints at the same time. Hence, Case II cannot become the equilibrium.

Comparing the results above, we define two thresholds for the batch size N:

$$\bar{N}_1^{\mathrm{R}} \equiv \frac{\theta(1-\gamma)L\lambda}{c} + 1 - \gamma,$$
$$\bar{N}_2^{\mathrm{R}} \equiv \frac{\theta(H-L)\gamma\lambda}{c} + 1 - \gamma,$$

which determines the optimal tipping state $\bar{x}^{\mathrm{R}}$. $\bar{N}_1^{\mathrm{R}}$ increases in L. For a given L, $\bar{N}_2^{\mathrm{R}}$ increases in H/L. Besides, $\bar{N}_2^{\mathrm{R}} > \bar{N}_1^{\mathrm{R}}$ if and only if $H/L > 1/\gamma$. Thus, the REE when offering group buying without the regular product is

(i) when $H/L \leqslant 1/\gamma$,

(1) if $N \leqslant \bar{N}_1^{\mathrm{R}}$, $\bar{x}^{\mathrm{R}} = N$, $p^{\mathrm{R}} = \theta L - c(N-1)/\lambda$, and $\pi^{\mathrm{R}} = \theta L\lambda - c(N-1)$;

(2) if $N > \bar{N}_1^{\mathrm{R}}$, $\bar{x}^{\mathrm{R}} = \bar{x}_1^{\mathrm{R}}$, $p^{\mathrm{R}} = \theta L + \theta\gamma(H-L) - c(N-2+\gamma)/\gamma$, and $\pi^{\mathrm{R}} = \pi_1^{\mathrm{R}}$;

(ii) when $H/L > 1/\gamma$,

(1) if $N \leqslant \bar{N}_2^{\mathrm{R}}$, $\bar{x}^{\mathrm{R}} = 0$, $p^{\mathrm{R}} = \theta H - c(N-1)/(\gamma\lambda)$, and $\pi^{\mathrm{R}} = \theta H\gamma\lambda - c(N-1)$;

(2) if $N > \bar{N}_2^{\mathrm{R}}$, $\bar{x}^{\mathrm{R}} = \bar{x}_1^{\mathrm{R}}$, $p^{\mathrm{R}} = \theta L + \theta\gamma(H-L) - c(N-2+\gamma)/\gamma$, and $\pi^{\mathrm{R}} = \pi_1^{\mathrm{R}}$;

where

$$\pi_1^{\mathrm{R}} \equiv \frac{c\gamma N\left[\theta L\gamma - c(N-2+\gamma) + \theta(H-L)\gamma\lambda\right]}{c\left[1 + \gamma(N-2+\gamma)\right] + \theta(H-L)(1-\gamma)\gamma\lambda}.$$

Since $\bar{N}_1^{\mathrm{R}} > \bar{N}_1$ and $\bar{N}_2^{\mathrm{R}} > \bar{N}_2$ always hold, the region of scenario $\{H; H+L\}$ shrinks against that of scenarios $\{H\}$ and $\{H+L\}$. Besides, $p^{\mathrm{R}} \geqslant p^{\mathrm{G}}$ always holds. □

Proof of Corollary H.1. Since $\bar{x}_1^{\mathrm{R}} < \bar{x}_1^{\mathrm{G}}$ always holds, the corollary can be proved. □

I. Fit Uncertainty

We study group buying with fit uncertainty about the group-buying product, with the superscript "F" denoting the equilibrium outcome.

PROPOSITION I.1 (REE UNDER FIT UNCERTAINTY). *When there is fit uncertainty about the group-buying product, for any given $\theta > 1$, $0 < \chi < 1$, and N, there exist two thresholds for the batch size, $\bar{N}_1^{\mathrm{F}} \equiv (\theta-1)(1-\gamma)L\lambda/c + 1-\gamma$ and $\bar{N}_2^{\mathrm{F}} \equiv (\theta-1)(H-L)\gamma\lambda/c + 1-\gamma$, such that*

(i) when $H/L \leqslant 1/\gamma$, the firm sets prices so that

(i-1) if $N \leqslant \bar{N}_1^{\mathrm{F}}$, $\{H+L\}$ is an REE;

(i-2) if $N > \bar{N}_1^{\mathrm{F}}$, $\{H; H+L\}$ is an REE;

(ii) when $H/L > 1/\gamma$, the firm sets prices so that

(ii-1) if $N \leqslant \bar{N}_2^{\mathrm{F}}$, $\{H\}$ is an REE;

(ii-2) if $N > \bar{N}_2^{\mathrm{F}}$, $\{H; H+L\}$ is an REE.

THEOREM I.1 (PROFIT COMPARISON UNDER FIT UNCERTAINTY). *Suppose $\theta > 1$ and $0 < \chi < 1$. When there is fit uncertainty about the group-buying product, there exists a threshold for the fit probability, $\bar{\chi}^{\mathrm{F}}$, below which it is optimal for the firm to offer the product line via flexible-duration group buying rather than doing so noncontingently, and above which vice versa.*

Proof of Proposition I.1. In similar fashion to the proof of Propositions 1 and 2, we define $\bar{x}^{\mathrm{F}}$ as the tipping state. There are five possible scenarios in total for customer segmentation. Define the following two cases for ease of analysis. In Case I, $\bar{x}^{\mathrm{F}} = 0$, $1 \leqslant \bar{x}^{\mathrm{F}} < N$, and $\bar{x}^{\mathrm{F}} = N$ represent scenarios $\{H\}$, $\{H; H+L\}$, and $\{H+L\}$, respectively. In Case II, $\bar{x}^{\mathrm{F}} = 0$ and $1 \leqslant \bar{x}^{\mathrm{F}} < N$ represent scenarios $\{L\}$ and $\{L; H+L\}$, respectively. In equilibrium, the prices p^{F} and r^{F} should satisfy the IR and IC constraints.

For Case I, the IR and IC constraints are

$$\begin{cases} IR_H : \theta H - p^{\mathrm{F}} - c\cdot w^{\mathrm{F}}(n) \geqslant 0 & 1 \leqslant n \leqslant N, \\ IC_H : \theta H - p^{\mathrm{F}} - c\cdot w^{\mathrm{F}}(n) \geqslant H - r^{\mathrm{F}} & 1 \leqslant n \leqslant N, \\ IR_{L1} : L - r^{\mathrm{F}} \geqslant 0 & \bar{x}^{\mathrm{F}} < n \leqslant N, \\ IC_{L1} : L - r^{\mathrm{F}} \geqslant \theta L - p^{\mathrm{F}} - c\cdot w^{\mathrm{F}}(n) & \bar{x}^{\mathrm{F}} < n \leqslant N, \\ IR_{L2} : \theta L - p^{\mathrm{F}} - c\cdot w^{\mathrm{F}}(n) \geqslant 0 & 1 \leqslant n \leqslant \bar{x}^{\mathrm{F}}, \\ IC_{L2} : \theta L - p^{\mathrm{F}} - c\cdot w^{\mathrm{F}}(n) \geqslant L - r^{\mathrm{F}} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{F}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{F}} \leqslant N$ and the expected waiting time $w^{\mathrm{F}}(n)$ is

$$w^{\mathrm{F}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1 \leqslant n \leqslant \bar{x}^{\mathrm{F}}, \\ \frac{\bar{x}^{\mathrm{F}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{F}}-1}{\gamma\lambda} & \bar{x}^{\mathrm{F}} < n \leqslant N. \end{cases}$$

In equilibrium, IR_{L1} is binding. Thus, $r^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) = L$. Whether IC_H or IC_{L2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{F}}$. Since $w^{\mathrm{F}}(n)$ is monotonously increasing in n, no matter whether IC_H or IC_{L2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_1^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) \equiv \theta L - c\cdot w^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$, $\bar{p}_2^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) \equiv (\theta-1)H + L - c\cdot w^{\mathrm{F}}(N)$, and we know that $\bar{p}_1^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) < \bar{p}_2^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$ if and only if $\bar{x}^{\mathrm{F}} > \bar{x}_1^{\mathrm{F}}$, where $\bar{x}_1^{\mathrm{F}} \equiv N-1+\gamma-(\theta-1)(H-L)\gamma\lambda/c$. Then, we can write the price p^{F} as the function of tipping state $\bar{x}^{\mathrm{F}}$:

$$p^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) = \begin{cases} \bar{p}_2^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) & 1 \leqslant \bar{x}^{\mathrm{F}} \leqslant \bar{x}_1^{\mathrm{F}}, \\ \bar{p}_1^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) & \bar{x}_1^{\mathrm{F}} < \bar{x}^{\mathrm{F}} \leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{F}, also as the function of tipping state $\bar{x}^{\mathrm{F}}$:

$$\begin{aligned}\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) &= \left[p^{\mathrm{F}}(\bar{x}^{\mathrm{F}})\cdot\gamma\lambda + r^{\mathrm{F}}(\bar{x}^{\mathrm{F}})\cdot(1-\gamma)\lambda\right]\cdot P(n>\bar{x}^{\mathrm{F}})\cdot\chi + \left[p^{\mathrm{F}}(\bar{x}^{\mathrm{F}})\cdot\lambda\right]\cdot P(n\leqslant\bar{x}^{\mathrm{F}})\cdot\chi + r^{\mathrm{F}}\cdot\lambda\cdot(1-\chi) \\ &= \frac{p^{\mathrm{F}}(\bar{x}^{\mathrm{F}})\cdot\gamma\lambda\chi N + L(1-\gamma)\lambda\chi(N-\bar{x}^{\mathrm{F}})}{N-(1-\gamma)\bar{x}^{\mathrm{F}}} + L\lambda(1-\chi).\end{aligned}$$

For $\bar{x}_1^{\mathrm{F}} < \bar{x}^{\mathrm{F}} \leqslant N$, plugging $p^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) = \bar{p}_1^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$ into $\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$, and then, taking the first-order derivative of $\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$ w.r.t. $\bar{x}^{\mathrm{F}}$, we have

$$\frac{\partial\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})}{\partial\bar{x}^{\mathrm{F}}} = \frac{\gamma\chi N\left[L(1-\gamma)(\theta-1)\lambda - c(N-1+\gamma)\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{F}}\right]^2}.$$

We can see that $\frac{\partial\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})}{\partial\bar{x}^{\mathrm{F}}} \geqslant 0$ if and only if $N \leqslant \bar{N}_1^{\mathrm{F}}$. Therefore, for $\bar{x}_1^{\mathrm{F}} < \bar{x}^{\mathrm{F}} \leqslant N$, when $N \leqslant \bar{N}_1^{\mathrm{F}}$, the firm sets $\bar{x}^{\mathrm{F}} = N$; when $N > \bar{N}_1^{\mathrm{F}}$, the firm sets $\bar{x}^{\mathrm{F}} = \bar{x}_1^{\mathrm{F}}$. Note that $\bar{x}_1^{\mathrm{F}} > 0$ if and only if $N > \bar{N}_2^{\mathrm{F}}$.

For $1 \leqslant \bar{x}^{\mathrm{F}} \leqslant \bar{x}_1^{\mathrm{F}}$, plugging $p^{\mathrm{F}}(\bar{x}^{\mathrm{F}}) = \bar{p}_2^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$ into $\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$, and then, taking the first-order derivative of $\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})$ w.r.t. $\bar{x}^{\mathrm{F}}$, we have

$$\frac{\partial\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})}{\partial\bar{x}^{\mathrm{F}}} = \frac{(1-\gamma)\chi N\left[c+(\theta-1)H\gamma\lambda\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{F}}\right]^2} > 0.$$

Since $\frac{\partial\pi^{\mathrm{F}}(\bar{x}^{\mathrm{F}})}{\partial\bar{x}^{\mathrm{F}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{F}} \leqslant \bar{x}_1^{\mathrm{F}}$, the firm always sets $\bar{x}^{\mathrm{F}} = \bar{x}_1^{\mathrm{F}}$.

As in the proof of Proposition 1, we know that Case II cannot become the equilibrium. Hence, we define two thresholds for the batch size N:

$$\begin{aligned}\bar{N}_1^{\mathrm{F}} &\equiv \frac{(\theta-1)(1-\gamma)L\lambda}{c} + 1 - \gamma, \\ \bar{N}_2^{\mathrm{F}} &\equiv \frac{(\theta-1)(H-L)\gamma\lambda}{c} + 1 - \gamma,\end{aligned}$$

which determines the optimal tipping state $\bar{x}^{\mathrm{F}}$. Besides, $\bar{N}_1^{\mathrm{F}}$ increases in L. For a given L, $\bar{N}_2^{\mathrm{F}}$ increases in H/L. $\bar{N}_2^{\mathrm{F}} > \bar{N}_1^{\mathrm{F}}$ if and only if $H/L > 1/\gamma$. Thus, when fit uncertainty about the group-buying product exists, the REE when offering group buying is

(i) when $H/L \leqslant 1/\gamma$,

  (1) if $N \leqslant \bar{N}_1^{\mathrm{F}}$, $\bar{x}^{\mathrm{F}} = N$, $r^{\mathrm{F}} = L$, $p^{\mathrm{F}} = \theta L - c(N-1)/\lambda$, and $\pi^{\mathrm{F}} = L\lambda[1+(\theta-1)\chi] - c\chi(N-1)$;

  (2) if $N > \bar{N}_1^{\mathrm{F}}$, $\bar{x}^{\mathrm{F}} = \bar{x}_1^{\mathrm{F}}$, $r^{\mathrm{F}} = L$, $p^{\mathrm{F}} = \theta L + \gamma(\theta-1)(H-L) - c(N-2+\gamma)/\lambda$, and $\pi^{\mathrm{F}} = \pi_1\chi + L\lambda(1-\chi)$;

(ii) when $H/L > 1/\gamma$,

(1) if $N \leqslant \bar{N}_2^{\mathrm{F}}$, $\bar{x}^{\mathrm{F}} = 0$, $r^{\mathrm{F}} = L$, $p^{\mathrm{F}} = (\theta - 1)H + L - c(N-1)/(\gamma\lambda)$, and $\pi^{\mathrm{F}} = (\theta-1)H\gamma\lambda\chi + L\lambda - c\chi(N-1)$;

(2) if $N > \bar{N}_2^{\mathrm{F}}$, $\bar{x}^{\mathrm{F}} = \bar{x}_1^{\mathrm{F}}$, $r^{\mathrm{F}} = L$, $p^{\mathrm{F}} = \theta L + \gamma(\theta-1)(H-L) - c(N-2+\gamma)/\lambda$, and $\pi^{\mathrm{F}} = \pi_1\chi + L\lambda(1-\chi)$;

where π_1 is given in the proof of Proposition 2. $\square$

Proof of Theorem I.1. In the product-line strategy, the firm's long-run average profit is

$$\begin{aligned}
\pi^{\mathrm{P}} &= \frac{\left[Np^{\mathrm{P}} + N\frac{1-\gamma}{\gamma}r^{\mathrm{P}} - \frac{hN(N+1)}{2\gamma\lambda}\right]\chi}{\frac{N}{\gamma\lambda}} + (L\lambda - hN)(1-\chi) \\
&= (\theta-1)H\gamma\lambda\chi + L\lambda - Nh + \frac{(N-1)h\chi}{2}.
\end{aligned}$$

The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the fit probability χ:

$$\begin{aligned}
\bar{\chi}_1^{\mathrm{F}} &\equiv \frac{Nh}{(c+h/2)(N-1) + (\theta-1)(H\gamma - L)\lambda}, \\
\bar{\chi}_2^{\mathrm{F}} &\equiv \frac{Nh}{(c+h/2)(N-1)}, \\
\bar{\chi}_3^{\mathrm{F}} &\equiv \frac{Nh}{(\theta-1)H\gamma\lambda + L\lambda - \pi_1 + (N-1)h/2}, \\
\bar{\chi}^{\mathrm{F}} &\equiv \max\left\{\bar{\chi}_1^{\mathrm{F}}, \bar{\chi}_2^{\mathrm{F}}, \bar{\chi}_3^{\mathrm{F}}\right\}. \quad \square
\end{aligned}$$

J. Endogenized Batch Size

It is technically challenging to simultaneously optimize the batch size and make the pricing decision in a flexible-duration group-buying campaign. Besides, as we do in our work, most of the literature on group buying assumes an exogenous batch size for analytical tractability (see, e.g., Jing and Xie 2011, Hu et al. 2013, 2015, Liu and Tunca 2019), or can only resort to a numerical study when endogenizing the batch size (see, e.g., Surasvadi et al. 2017, Marinesi et al. 2018).

Without any doubt, a model in which both the price and batch size in flexible-duration group buying are endogenously determined would be very desirable. Here we use numerical studies to explore what might happen when the batch size is endogenized. To endogenize the batch size of the group-buying product, we assume that the firm incurs a fixed setup cost $K > 0$ for each batch and a variable cost $\delta > 0$ for each unit of product. Hence, in a group-buying campaign with batch size N, the long-run average production cost, $(K + \delta N)/\left(\frac{\bar{x}^{\mathrm{G}}}{\lambda} + \frac{N - \bar{x}^{\mathrm{G}}}{\gamma\lambda}\right)$, should be subtracted from the firm's long-run average profit, π^{G}, which is characterized in the proof of Proposition 2. The following table summarizes the optimal batch size and the corresponding optimal customer segmentation in group buying under different values of parameters.

Table J.1 Numerical Analysis for Optimal Batch Size in Group Buying

	Parameter value	Optimal batch size	Optimal customer segmentation
	$H=6$ $(H/L=1.2)$	$N^*=46$	$\{H;H+L\}$
	$H=7$ $(H/L=1.4)$	$N^*=48$	$\{H;H+L\}$
Impact of market	$H=8$ $(H/L=1.6)$	$N^*=51$	$\{H;H+L\}$
heterogeneity H/L	$H=9$ $(H/L=1.8)$	$N^*=54$	$\{H;H+L\}$
$(K=200,\ \delta=1)$	$H=10$ $(H/L=2.0)$	$N^*=32$	$\{H\}$
	$H=11$ $(H/L=2.2)$	$N^*=32$	$\{H\}$
	$H=12$ $(H/L=2.4)$	$N^*=32$	$\{H\}$
	$K=100$	$N^*=23$	$\{H\}$
	$K=200$	$N^*=32$	$\{H\}$
Impact of fixed cost K	$K=300$	$N^*=39$	$\{H\}$
$(H=15,\ \delta=1)$	$K=400$	$N^*=45$	$\{H\}$
	$K=500$	$N^*=50$	$\{H\}$
	$K=600$	$N^*=55$	$\{H\}$
	$\delta=1$	$N^*=32$	$\{H\}$
	$\delta=2$	$N^*=32$	$\{H\}$
Impact of variable cost δ	$\delta=3$	$N^*=32$	$\{H\}$
$(K=200,\ H=15)$	$\delta=4$	$N^*=32$	$\{H\}$
	$\delta=5$	$N^*=32$	$\{H\}$
	$\delta=6$	$N^*=32$	$\{H\}$

Note: Other parameters are set as follows: $\lambda=5$, $\gamma=0.5$, $L=5$, $c=0.5$, and $\theta=3$.

Table J.1 shows that as the valuation heterogeneity H/L increases, the optimal batch size of the group-buying product does not always increase. The underlying reason is as follows. When the valuation heterogeneity H/L is low, the firm has an incentive to invite both high- and low-end customers to sign up for the group buying, through intertemporal segmentation $\{H;H+L\}$. Since there are many customers who are interested in group buying, it would not take a long time for a group-buying campaign to succeed, and thus, the firm can afford to set a relatively large batch size. On the contrary, when the valuation heterogeneity H/L is high, it is more profitable for the firm to charge a higher price for the group-buying product and hence invite only high-end customers to sign up for group buying (corresponding to customer segmentation $\{H\}$). In this case, the expected time for a group-buying campaign to succeed would be longer, as only high-valuation customers would be interested in signing up. Consequently, the firm cannot choose a large batch size. Taken together, the optimal batch size of the group-buying product can drop significantly as the valuation heterogeneity H/L increases. The driving force behind this relationship is the intertemporal customer segmentation we capture as the main insight in this paper. In addition, the impact of production cost on the optimal batch size is consistent with common intuition.

Specifically, as indicated by Table J.1, the optimal batch size always increases in the fixed setup cost K, while it remains invariant as the variable cost δ changes.

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