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Crosscutting Areas

Market Entry and Competition Under Network Effects

Yinbo Feng,[a,*] Ming Hu[b]

[a] Research Institute for Interdisciplinary Sciences, School of Information Management and Engineering, Shanghai University of Finance and Economics, Shanghai 200433, China; [b] Rotman School of Management, University of Toronto, Toronto, Ontario M5S 3E6, Canada

*Corresponding author

Contact: feng.yinbo@mail.shufe.edu.cn, https://orcid.org/0000-0002-6771-0483 (YF); ming.hu@rotman.utoronto.ca, https://orcid.org/0000-0003-0900-7631 (MH)

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Abstract. We consider a three-stage game in which, first, a large number of potential firms make entry decisions, then those who choose to stay in the market decide on the investment (quality) level in each product, and last, customers with heterogeneous preferences arrive sequentially to make (random) purchase decisions based on product quality and historical sales under the network effect according to a discrete choice model. We characterize such a random purchase process and show that a growing network effect always contributes to more sales concentration ex post on a small number of products. Perhaps surprisingly, we further show several phase-changing phenomena regarding equilibrium outcomes with respect to the network effect's strength. In particular, the equilibrium product variety (respectively, quality investment) first decreases (respectively, increases) and then increases (respectively, decreases) as the network effect grows. Specifically, when the strength of the network effect is below a threshold, an increasing network effect would shift more sales toward those products with higher quality, preventing more products from entering the market ex ante and inducing firms to adopt the high-budget equilibrium strategy by making a small number of high-quality products, which is consistent with the blockbuster phenomenon. When the strength of the network effect is above the threshold, the network effect would easily cause the market to be concentrated on a few products ex post; even some low-quality products may have a chance to become a "hit." Interestingly, in this case, when the network effect is growing, the ex ante equilibrium product variety will be wider, and firms adopt the low-budget equilibrium strategy by making a (relatively) large number of low-quality products, a finding consistent with the long tail theory. We then establish the robustness of the previous main insights by accounting for endogenized pricing and multiproducts carried by each firm.

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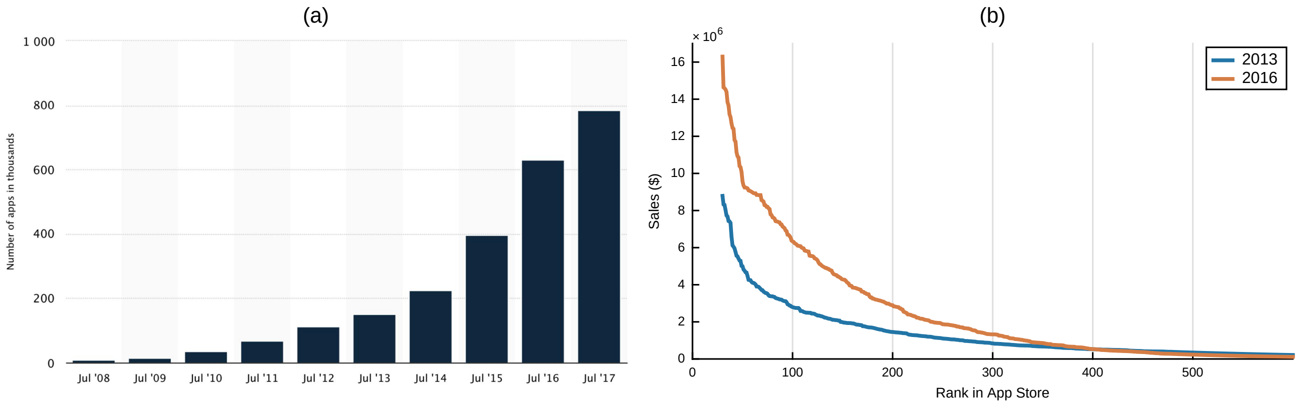
1. Introduction

The "long tail" theory was celebrated by *BusinessWeek* as the biggest idea of the year 2004, soon after the book *"The Long Tail"* by Chris Anderson was published (Anderson 2006). The book argues that our culture and economy are shifting away from a relatively small number of "hit" products toward a huge number of niche products. In other words, demand will become less concentrated, and the tail of the sales rank distribution will become *longer* and *fatter* over time.[1]

The long tail theory calls for applying a *low-budget strategy*—producing a (relatively) large number of products with (relatively) low investment levels. For example, in today's music business, producers and artists may spend less time on each song or track. Ramon Ibanga, Jr. (aka Illmind), a hip hop producer, in a recent interview with National Public Radio (NPR), said it might take him "about 20 minutes" to create one track of beats and sounds. He further said "some days I'll spend eight hours, and I'll create maybe 10 tracks, 10 lottery tickets. […] because every beat, every piece of music that I make has potential to get placed somewhere" (NPR 2017). For another example, toy makers such as Hasbro and Mattel are creating toys with a "next-to-nothing" budget that yet can be fueled by social media, for example, slime-making kits and fidget-spinners, as opposed to higher-priced franchise toys, for example, those tied to movie properties (Ziobro 2018).

Figure 1. (Color online) Variety and Sales of Gaming Apps in the iTunes App Store



Notes. (a) Variety (in thousands). (b) Annual sales (in dollars). Data collected from statista.com and ThinkGaming.com.

However, some other cultural industries may tell a different story. The concentration of the most popular titles in the video game industry is growing (Figure 1(b)), a phenomenon known as the *blockbuster phenomenon*. This phenomenon suggests that firms may adopt a different strategy. Wingfield (2013) reported that every publicly traded publisher of video games talked about a "bigger, better, fewer" strategy. We refer to this strategy as a *high-budget strategy*—producing a (relatively) small number of products with (relatively) high investment levels.

In this paper, we analytically study the impact of a network effect on entry decisions and investment strategies (i.e., the high-budget versus low-budget strategies) adopted by competing firms, based on which we further provide a theory that links the ex post sales volume concentration with the ex ante product variety in a market under network effects. The (positive) *network effect* refers to the following: the consumption of a good increases the value of that product to other consumers (Economides 1996). In some cases, the utility that a consumer derives from a good depends directly on its total number of adoptions. The detailed mechanisms behind the network effect can also be the influence of peers (e.g., through word of mouth or observational learning), social media, review websites, or self-enforcing recommendation systems.

Intuitively, the blockbuster phenomenon can have much to do with the (positive) network effect. For one thing, people who are likely to buy the popular product because of the network effect will make the product even more popular. For another thing, producers, like studios, are willing to use more resources, for example, for advertising and production, to produce a product that may be a hit due to the network effect. However, no one can accurately predict what will become a blockbuster. A high-budget product may turn out to be a flop. Conversely, a low-budget product may be a surprising blockbuster if it becomes popular early enough due to market uncertainty and a strong network effect. This possibility may attract more and more low-budget products to enter the market, although very few of them can gain a significant market share. In this sense, the (positive) network effect seemingly can also contribute to the phenomenon of the long tail.

Both the blockbuster and long tail phenomena coexist in some industries, for example, the online game industry, in which the network effect is essential. In the iTunes App Store, the downloads are more concentrated on a handful of top games, whereas a rapidly increasing number of games sell very rarely or never. Roughly estimated, the number of available gaming apps in the iTunes App Store increased from 151,460 in 2013 to 631,091 in 2016 (Figure 1(a)). Annual sales of the 100th game increased from \$2.8 million in 2013 to \$6.3 million in 2016, but sales of the 500th game decreased from \$0.38 million in 2013 to \$0.25 million in 2016 (Figure 1(b)). Around the 5,000th position, sales were very close to zero from 2013 to 2016.

We show that one key factor leading to the coexistence of the two phenomena is the (positive) network effect. In particular, we analyze a three-stage game where first, a large number of potential firms make an entry decision, then those who choose to stay in the market decide on the investment level in its product, and lastly, customers arrive sequentially to make (random) purchase decisions that are based on product quality and historical sales subject to the positive network effect. More specifically, in the last stage, sequentially arriving customers, upon arrival, make a choice from the set of products available in the market, according to a customer discrete choice model. We assume each firm carries a single product in the base model and consider multiple products carried by each firm in Section 5.2. The attraction value for each product depends on two attributes: the intrinsic quality[2] of the product determined by the firm before it released the product and the current market share of the product. In this paper, a product that is of high (respectively, low) quality because of high-budget (respectively, low-budget) production has a high (respectively, low) probability of being bought by a customer in the absence of the network effect. Moreover, the network benefit from consuming a product is determined by the *current* (instead of the anticipated future) market share of the product and a parameter measuring the network effect strength (denoted by β in our model). In the base model, the network effect parameter β is assumed to be the same for all products in the industry. We adopt a special type of stochastic process called *nonlinear Pólya urn process* to estimate the random, long-term market share for each product. Before the (random) sales process starts, in the first stage, firms decide on whether to enter the market with an entry cost, and in the second stage, all firms entering the market simultaneously decide on the quality of their product in anticipation of future competitive market dynamics.

Using this analytical model, we show that an increasing network effect (e.g., propelled by social media) always contributes to the sales concentration on a small number of products ex post, regardless of whether the prevailing network effect is weak or strong. This supports the blockbuster phenomenon. Perhaps surprisingly, we further show several phase-changing phenomena regarding equilibrium outcomes with respect to the network effect's strength. In particular, the equilibrium product variety (respectively, quality investment) first decreases (respectively, increases) and then increases (respectively, decreases) as the network effect grows. Specifically, when the network effect parameter is lower than a threshold, the increasing network effect would shift more sales toward the products with higher quality levels, ex ante requiring larger investment and preventing more products from entering the market. When the network effect parameter is higher than that threshold, the increasing network effect would easily cause the market to be concentrated on a few products ex post; even some low-quality products may have a chance to become "hits." Interestingly, in this case, the ex ante equilibrium product variety will be wider if the network effect becomes stronger as more firms bet on their products becoming "hits." That is, an increasing network effect not only intensifies the sales concentration but also leads to a longer tail, as evidenced in Figure 1(a). This result analytically confirms the long tail theory, noting that with more products available ex ante in the market, the extreme portion of the tail in the sales rank distribution would become longer ex post due to the additionally ranked products.

We then establish the robustness of the above main insights by accounting for endogenized pricing, multiproducts carried by each firm, an outside option for customers, and firms' heterogeneity in the cost structure.

2. Literature Review

In the economics literature, Doganoglu (2003), Mitchell and Skrzypacz (2006), and Cabral (2011) study committed or dynamic price competition among firms under network effects over an infinite horizon. Geng et al. (2022) study a two-stage duopoly price competition under two different network effects in which both firms select quality levels in the first stage simultaneously and engage in a two-period committed or dynamic price competition in the second stage. In the operations literature on competition among products that exhibit network effects, Bimpikis et al. (2016), Chen et al. (2018) (a competition version of Candogan et al. 2012), Chen and Chen (2021), and Jiang et al. (2022) study duopoly/oligopoly competition of advertising, market share or pricing, and inventory under network effects. The aforementioned papers go beyond our work along some dimensions, such as compatibility, (local) network structures, and dynamic competition. However, they do not endogenize the number of competing firms. In practice, as in the online game industry, the studio can decide whether to produce a game and how much to invest. Indeed, we focus on firms' entry decisions that determine the product variety in an industry and their quality decisions that influence the ex post sales distribution among products. Moreover, unlike papers such as Chen and Chen (2021) and Geng et al. (2022) in which customers either simultaneously decide their purchase decision or dynamically consume a product in a deterministic fashion, we allow the customer purchase process to be random in which the current customer's purchase follows a discrete choice model depending on the quality levels of the available products and their recent cumulative sales. The randomness in the purchase process adds significant complexity to the ex ante equilibrium analysis because firms not only need to identify the potential limiting market behavior but also need to characterize the likelihood of those limits when deciding on entry and quality decisions ex ante. We view the latter characterization, through adopting the exponential embedding method and utilizing hyperexponential distribution properties, as our main technical contribution.

More broadly, our paper contributes to the study of the long tail theory and blockbuster phenomenon. The long tail theory, popularized by Anderson (2006), predicts that niche products will gain increasing market shares, and as a result, the demand for popular or hit products will tend to decrease (the "fat" tail part of the theory). Moreover, the theory also predicts there will be more niche products in the market (the "long" tail part of the theory). The long tail theory has attracted widespread interest in academia. Brynjolfsson et al. (2010) give a review of the research on the long tail effect. They categorize the plausible drivers of the long tail effect into demand-side and supply-side drivers. Cachon et al. (2008) analytically show that a lower search cost can further encourage firms to enlarge their assortment to satisfy customers' heterogeneous preferences. Zentner et al. (2013) and Zhang (2018) empirically demonstrate that online stores exhibit a lower sales concentration on a small set of products because of the lower search cost for customers. We show that when the network effect is already strong enough, as the network effect further grows, firms have the incentive to reduce the production budget for a product but invest in a larger number of products, a practice that can lead to the tail of the ex post sales rank distribution being longer.

Conversely, studies of some industries highlight the fact that despite the promise of digitization and the long tail effect, the highest revenue continues to be concentrated in a small percentage of blockbusters and not in the long tail (Elberse and Oberholzer-Gee 2007, Elberse 2008). Hinz et al. (2011) and Tan et al. (2017) empirically evaluate the effect of increasing product variety on sales concentration. Unlike those studies, we build a theoretical model and consider the network effect as a main driver of sales concentration in an industry. Moreover, we show that when products' entry and quality decisions are endogenized, the growing network effect by itself can lead to the blockbuster and the long tail phenomena at the same time.

Bar-Isaac et al. (2012) study a similar research question as ours. They consider a competitive setting as we do, where firms adopt a richer set of strategies to design their products, and consumers search among a variety of products. Both papers mentioned previously provide a rationale for the blockbuster and long tail phenomena. They highlight the search cost as a key driver building on the literature on consumer search behavior. In contrast, we develop a theory by identifying the network effect as a key driver. Moreover, unlike their study, our paper considers endogenous product variety; that is, the entry decision for each firm is endogenized. This allows our model to study the phenomenon of product variety expansion (which leads to the long tail phenomenon) or shrinkage from the supply side. In our setting, the blockbuster or long tail phenomenon is driven by the interaction between the demand and supply sides.

With regard to the methodology, there is a stream of literature modeling consumers' dynamic purchase process through urn models (Ceyhan et al. 2011, Hentenryck et al. 2016, Maldonado et al. 2018, Kohli et al. 2020). The third stage of our sequential game is close to those models, and most of our third-stage results are built on their works. Ceyhan et al. (2011) and Maldonado et al. (2018) provide a complete characterization of the equilibrium of the market-share dynamics for the case where the network effect parameter β is less than or equal to one. Ceyhan et al. (2011) also provide some general convergence results under general assumptions for both $\beta \le 1$ and $\beta > 1$. For $\beta > 1$, Maldonado et al. (2018) and Ceyhan et al. (2011) indicate the existence of multiple equilibria of the market-share dynamics without specifying the properties of these equilibria or how they change with respect to the parameter β. The main technical contribution of our paper is to investigate these properties, as they are crucial for studying the first- and second-stage games, namely the impact of network effects on price/quality competition and the level of market entry, and our managerial contribution lies in discovering that the network effect has nonmonotonic impacts on quality competition and the level of market entry. Finally, our paper differs from Hu et al. (2016), which also adopts an urn model to capture the purchase process under the network effect in several aspects. First, their model is only applicable to the case of two substitutable products, whereas our model is applicable to any number of substitute products. Second, their paper considers a scenario where a monopolistic firm operates two substitutable goods, whereas we consider a situation where multiple firms compete with each other, with each firm operating one or more products. These two reasons make it impossible to answer the questions addressed in our paper using their model, namely how the quality investment strategy and the level of entry depend on the strength of network effects in a competitive market. Moreover, our model is more mathematically challenging to solve compared with theirs, which necessitates the use of different techniques to solve our model.

3. Model

We consider a market with a large number of potential competitive firms that make entry decisions simultaneously.[3] Any firm, say i, which sells a single product incurs a fixed cost K and a cost, cq_i, in development and production that is proportional to the chosen quality level $q_i \ge 0$. The assumption that each firm sells only one product helps to illustrate the logic behind our main qualitative results. In Section 5.2, we study the case in which a firm can carry multiple products, and for this case, we show that each firm carries one single product in equilibrium when the entry decisions are endogenized. We slightly abuse the term "quality" in our setting and interpret it as the *intrinsic appeal* of a product. A higher quality means that the product has a higher probability of being bought by a randomly chosen customer in the absence of network effects. For simplicity, the parameters $K \ge 0$ and $c > 0$ are assumed to be identical for all firms. The heterogeneous marginal costs are considered in Section 5.4. Suppose that a video game studio wishes to release a game. Then, the fixed cost K can be thought of as the cost of story selection and market interests research, and the cost of the quality improvement cq_i can be thought of as the cost of content design, production, and visual effects. The marginal cost of distributing digital products is relatively low, especially compared with their up-front production expenses. Hence, we assume $K + cq_i$ is the total cost for firm i. We consider a convex form of the cost function in Section 5.4. A higher-quality product would require more investment; then, we sometimes refer to a high-quality (respectively, low-quality) product as a high-budget (respectively, low-budget) product. If there are n firms entering the market, without loss of generality, we always index them as $i = 1, 2, \ldots, n$ and denote by n the number of competing firms, which is known by all firms. Let $\mathbf{q} = (q_1, \ldots, q_n)$ denote the vector of product quality levels. In the base model, we assume that the price p for customers is exogenously given and is the same for all products. This assumption allows us to focus on the competition among products from the single dimension of quality design. In Section 5.1, we study the extension with endogenous prices. In this paper, the total market size is normalized to be 1. With the exogenous price p, any firm can at most get a revenue p. Thus, we assume the exogenous price $p > K$ to avoid the trivial case that no firm wants to join the market.

We consider an infinite-period dynamic model where customers arrive sequentially, one per time period. The infinite-horizon assumption will be numerically relaxed in Section 5.5. When a customer enters the market and sees the n products in the market, she is interested in buying one from the n number of choices. For simplicity, we do not consider the "no purchase" option; however, our main insights carry over to the case with a constant no-purchase option. The customer's purchase decision is affected by both the quality of the n products and the network effect. More specifically, upon arrival at time $t+1$, a customer is able to observe the aggregate purchase decisions of her predecessors, denoted by $\mathbf{d}(t) = (d_1(t), \ldots, d_n(t))$, with $\mathbf{d}(0) = (1, 1, \ldots, 1)$. Let $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))$ be the vector of the market share of the n products at time t; that is, $x_i(t) = d_i(t)/(d_1(t) + \cdots + d_n(t))$. A customer's utility from purchasing one unit of product i at time $t+1$ is assumed to be

$$U_i(q_i, d_i(t)) = \alpha \ln q_i + \beta \ln d_i(t) - p + \xi_i.$$

The first and the last terms combined represent the customer's stand-alone valuation (intrinsic utility) of buying the product, where ξ_i is a random variable that represents customer-specific idiosyncrasies. The second term represents the customer's utility from the network effect (respectively, negative network effect), that is, the effect that an increased number of people or participants improve the value of a good when $\beta > 0$ (respectively, reduce the value when $\beta < 0$). Under the assumption that ξ_i's are independent and identically distributed, following a Gumbel distribution, the probability that the customer selects product i is given by the following discrete choice model where the attraction value of a product, $q_i^\alpha x_i^\beta(t)$, is in the Cobb-Douglas form[4]:

$$\phi_i(\mathbf{x}(t)) = \frac{q_i^\alpha d_i^\beta(t)}{\sum_{i=1}^n q_i^\alpha d_i^\beta(t)} = \frac{q_i^\alpha x_i^\beta(t)}{\sum_{i=1}^n q_i^\alpha x_i^\beta(t)}. \tag{1}$$

We see from (1) that parameters α and β have a critical impact on customers' choice behavior. The parameter α (respectively, β) measures to what extent customer choice behavior is affected by intrinsic quality levels (respectively, historical sales). We assume $\alpha > 0$. We use the parameter β to measure the strength of the network effect, with a higher value of β corresponding to a stronger network effect. Note that β is allowed to take values from $-\infty$ to $+\infty$. Later in the paper, we show that when β takes different values, the market dynamics may behave very differently (see Section 4.1).

If the customer selects product i at time $t+1$, the vector of sales volumes is updated as

$$d_j(t+1) = \begin{cases} d_j(t) + 1 & \text{if } j = i, \\ d_j(t) & \text{otherwise,} \end{cases}$$

and the vector of market shares is correspondingly updated as

$$\mathbf{x}(t+1) - \mathbf{x}(t) = \frac{1}{t+1+n}[\phi(\mathbf{x}(t)) - \mathbf{x}(t) + \mathbf{e}(t+1) - \mathbb{E}[\mathbf{e}(t+1)|\mathbf{x}(t)]], \tag{2}$$

where $\phi(\mathbf{x}(t)) = (\phi_1(\mathbf{x}(t)), \ldots, \phi_n(\mathbf{x}(t)))$, and $\mathbf{e}(t+1)$ is the random unit vector whose ith entry is one if product i is purchased at time $t+1$ and otherwise is zero. As the total market size is normalized to one, firm i's market share in the limit as $t \to \infty$ is equivalent to its sales.

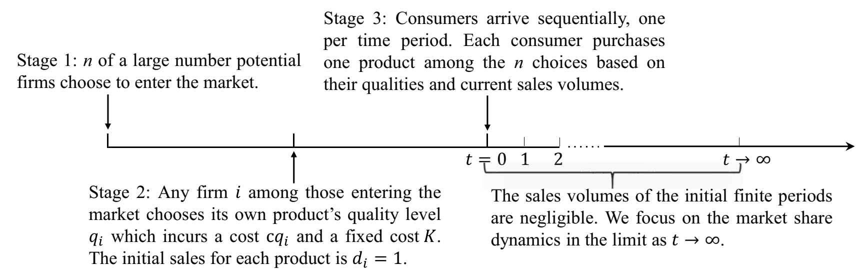
To conclude this section, we summarize the sequence of events as follows (Figure 2):

Stage 1 (Entry). Each firm decides whether to enter the market. A firm will exit the market if the expected profit is negative. The product variety is the number of firms or products entering the market.

Stage 2 (Quality Investment). Suppose n firms are entering the market. Given that n is commonly known, all firms choose their quality levels simultaneously so as to maximize their own expected profit in the competitive selling stage.

Stage 3 (Demand Process with Network Effect). The customers arrive sequentially to choose one product to maximize their utilities at the time of their arrival. Each

Figure 2. Sequence of Events



arriving customer is able to observe the quality levels of all products and their current market shares.

4. Equilibrium Analysis

In this section, we study the subgame-perfect equilibrium of the three-stage game using the backward induction approach. We begin our analysis with the third stage, in which the price p, all firms' quality $\mathbf{q}$, quality sensitivity α, and the network effect parameter β are given and known. Note that $\mathbf{d}(t)$ is what is referred to as a nonlinear Pólya urn process, and $\mathbf{x}(t)$ is a corresponding discrete stochastic process. By Nevel'son and Has'minskii (1973, theorem 7.3, chapter 2), $\mathbf{x}(t)$ almost surely converges to the set of fixed points for $\phi(\mathbf{x}) = \mathbf{x}$, as $t \to \infty$. In this paper, the vector of market shares in the limit $\lim_{t\to\infty} \mathbf{x}(t)$, which may be a point or a random variable, is defined as the demand vector for the n firms under all firms' joint quality level vector $\mathbf{q}$, the quality sensitivity parameter α, and the strength of the network effect β. Then, we study the equilibrium at the second stage, in which all incumbent firms simultaneously choose their quality levels according to their anticipated demand, which will be formed in the third consumption stage. Finally, we study the market entry stage, which determines how many products will be available in the market. We aim to obtain qualitative insights into how the network effect β affects the sales distribution in the industry and hence affects the firms' entry decisions that shape the product variety of the industry.

4.1. Sales Concentration Under Network Effect

At the consumption stage, customers whose individual purchase decisions depend on the aggregate sales of their predecessors but are independent of future sales arrive sequentially at the market, one per period. In this section, we study the limit of market shares dynamics under the network effect given that there are n firms in the market and their quality levels are $\mathbf{q} = (q_1, \dots, q_n)$.

The literature on urn processes (Zhu 2009) often focuses on the limiting behavior of the stochastic process (2) in three cases: $-\infty < \beta < 1$, $\beta = 1$, and $1 < \beta < +\infty$, because the process converges to different points or random variables in different cases as t approaches to infinity. Hu et al. (2016) was the first to study the effects of the three cases of network effect on a newsvendor problem. We also study three such cases, but we give heterogeneous attributes (i.e., quality levels) to products, which differs from Hu et al. (2016) and the large body of literature on urn processes.

Given the *asymmetric* product quality vector $\mathbf{q} = (q_1, \dots, q_n)$ and the symmetric initial sales volumes $\mathbf{d}(0) = (1, \dots, 1)$, the first customer bases her purchase decision only on the intrinsic utility she will obtain from each product. However, from the second customer on, each customer's choice decision depends not only on the intrinsic utility of each product for her but also depends on the (random) social utility from the (random) sales volumes. Thus, the asymptotic distribution of the vector of market shares must be a function of the vector of intrinsic quality $\mathbf{q}$ and the parameters α and β. We show that the limit of $\mathbf{x}(t)$, as $t \to \infty$, must exist, and we provide a closed-form expression for it in the following theorem, where we assume $n \geq 2$. The case $n = 1$ is trivial because, in our model, the monopoly product always captures the whole market.

Lemma 1 (Limit of Consumption Dynamics). *Suppose $n \geq 2$. Given any product quality vector $\mathbf{q} = (q_1, \dots, q_n) \neq \mathbf{0}$ and the initial sales volumes $\mathbf{d}(0) = (1, \dots, 1)$, the consumption dynamics $\mathbf{x}(t)$ converges to $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ almost surely, and $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ can be characterized by the following cases:*

i. Theorem 5.1 *in Maldonado et al.* (2018) (*if* $\beta < 1$, $\mathbf{x}^*(\mathbf{q}^\alpha, \beta) = (q_1^{\frac{\alpha}{1-\beta}}, \dots, q_n^{\frac{\alpha}{1-\beta}}) / \sum_{i=1}^n q_i^{\frac{\alpha}{1-\beta}}$).

ii. Theorem 3 *in Hentenryck et al.* (2016) *and* theorem 2.1 *in Chung et al.* (2003) (*if* $\beta = 1$, $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ *is uniformly distributed on the set* $\{\mathbf{x} \in [0, 1]^n : \sum_{i\in L} x_i = 1, \sum_{i\notin L} x_i = 0\}$, *where* $L = \{i : q_i$ *is the largest element of* $\mathbf{q}\}$).

iii. *If* $\beta > 1$, $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ *follows a multipoint distribution with support on* $\{\mathbf{e}_1, \dots, \mathbf{e}_n\}$ *where* $\mathbf{e}_i$ *is the unit vector with ith entry being one and zero otherwise, and*

$$\mathbb{P}\{\mathbf{x}^*(\mathbf{q}^\alpha, \beta) = \mathbf{e}_i\} = \int_0^{+\infty} f_{q_i^\alpha, \beta}(s) \prod_{\substack{j=1 \\ j \neq i}}^{n} \overline{F}_{q_j^\alpha, \beta}(s) \mathrm{d}s, \qquad (3)$$

where $f_{q^\alpha, \beta}(\cdot)$ is the probability density function of the sum of an infinite number of independent and identically distributed *(i.i.d.) exponential random variables with rates* $q^\alpha 1^\beta$, $q^\alpha 2^\beta, q^\alpha 3^\beta, \ldots,$ *and* $\overline{F}_{q^\alpha,\beta}(s) = 1 - \int_0^s f_{q^\alpha,\beta}(u)\mathrm{d}u$ *is its survival function.*

We categorize network effects into three scenarios by the parameter β: $\beta < 1$, $\beta = 1$, and $\beta > 1$. Lemma 1 shows that for a different network effect parameter β, both the market share dynamics' long-term and short-term behaviors exhibit dramatically different properties.

Lemma 1(i) says that if $\beta < 1$, the stochastic process $\mathbf{x}(t)$ has a *unique* limit regardless of what the aggregate sales distribution in any finite period is. This limit of market evolution depends solely on the quality levels of the n products. A product that is unpopular in the short run can always win back its market share over the long run as long as it has a higher quality. In this case, the market evolution has a predictable outcome in the end. The higher the quality of a product, the more market share it will eventually gain. A special case is when $\beta \to -\infty$, in which case $\mathbf{x}(t) \to (1/n, \ldots, 1/n)$. That is, the demand tends to be evenly distributed on each product when the negative network effect is strong enough. Another special case is when β approaches one from the left. In this case, the product(s) with the highest quality level will dominate all other products whose (total) market share approximates one in the end.

Lemma 1(ii) is a boundary case and says that when $\beta = 1$, the unique product with the highest quality level has a full market share of 1. If two or more products achieve the same highest quality level, they divide the market randomly, and the remaining firms obtain zero market share. (Since this result is about the limiting market behavior, it implies that over a finite horizon, there are one or multiple blockbusters, and the rest products capture very little sales. This remark also applies to Lemma 1(iii).) This scenario shows the importance of product quality in the sense that the market eventually evolves into a "winner-takes-all" outcome in which the products with the highest quality get everything, but the others have nothing.

Lemma 1(iii) says that if $\beta > 1$, the market is eventually dominated by one *unique* product, and other products all obtain zero fraction of the market. That is because if one of the products has a large enough market share at some point in time (which is referred to as the *attraction time*), then all subsequent customers will buy that product from that time onward. In the Online Appendix, Theorem A.4 shows that such an attraction time is finite. When $\beta > 1$, if a product fails to become popular within a certain length of time, it will never be able to do so. Interestingly, any product can be the winner regardless of its quality, and there is one and only one winner, in contrast with scenarios (i) and (ii). Even for a product with the lowest quality, it may be bought by enough customers at an early stage that it becomes the sole winner. The strong network effect reinforces this good start such that no other product will ever be able to win over. Similar phase-changing phenomena are also observed by Ceyhan et al. (2011), Hentenryck et al. (2016), Hu et al. (2016), and Maldonado et al. (2018). Therefore, we believe that the nonmonotonic impacts of the network effect result from the problem's nature rather than our specific assumptions.

The three scenarios have one thing in common—an increasing network effect always contributes to the sales concentration on a small number of products. The difference, however, is that the growing network effect will shift more demand toward the products with higher quality when the prevailing network effect is generally weak (or negative), that is, $\beta \le 1$; but it will more easily cause the market to be concentrated on one product, which can be a low-quality one when the prevailing network effect is strong, that is, $\beta > 1$. To further highlight this difference, we refer to the case $\beta \le 1$ as the case with a *dominated* network effect in which the sales concentration is *predictable* by simply observing the initial qualities of all products, and the case $\beta > 1$ as the case with a *dominant* network effect in which sales concentration is *unpredictable* due to the random nature of the sales process amplified by the strong network effect. We conclude that the dominated network effect intensifies the advantage of a high-quality product, while the dominant network effect has a different impact. The following lemma supplements this point.

Lemma 2. *The expected market shares in equilibrium satisfy the following properties:*

i. *For* $n \ge 2$, $\beta \ne 1$, *and* $\mathbf{q} > \mathbf{0}$, $\mathbb{E}x_i^*(\mathbf{q}^\alpha, \beta)$ *for* $i = 1, \ldots, n$ *is increasing in* q_i, *and decreasing in* q_j *(for all* $j \ne i$*), holding all other variables constant.*

ii. *Suppose* $q_1 \ge q_i \ge q_n$ *for* $i = 2, 3, \ldots, n-1$. *Then* $\mathbb{E}x_1^*(\mathbf{q}^\alpha, \beta)$ *(respectively,* $\mathbb{E}x_n^*(\mathbf{q}^\alpha, \beta)$*) is increasing (respectively, decreasing) in* β *for* $\beta < 1$, *and decreasing (respectively, increasing) in* β *for* $\beta > 1$.

Lemma 2(i) says that the expected market share for each firm is increasing in its own quality level but decreasing in any competitor's quality level, regardless of the strength of the network effect. This means that the market always gives the higher-quality firm a larger expected market share in the end. For $\beta \le 1$, this insight can be clearly observed from Lemma 1 parts (i) and (ii). For $\beta > 1$, Lemma 2(i) demonstrates that the firm with a higher quality level has a greater probability of becoming the sole winner in the market. Lemma 2(ii) illustrates that the disadvantage of the lowest-quality product strengthens as β increases but remains below one and diminishes as β continues to increase when β is greater than one.

In the competitive market, each firm may want to improve the quality of its product to attract as many customers as possible and then hopefully take advantage of the self-perpetuating network effect. A natural question arises: how does an increase in the network effect influence each firm's incentive to improve its quality, given that the number of firms is fixed? We will address the question rigorously in the next section.

4.2. Quality Competition

If the expected demand for a product, as a function of its quality, is known, a rational firm will naturally choose its quality level to make a tradeoff between the expected revenue and the production cost so as to maximize its expected profit. In Section 4.1, we characterize the demand functions for the n firms by $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ in three scenarios. Using these results, in this section, we study the n firms' competition in choosing quality levels and model it as a simultaneous-move game in which the strategy space for the n firms is $E \triangleq [0, +\infty)^n$.

For $i = 1, \ldots, n$, given other products' qualities $\mathbf{q}_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$ and parameters α and β, firm i's problem is to choose a quality level q_i to maximize

$$\pi_i(q_i, \mathbf{q}_{-i}, \alpha, \beta, K) = \begin{cases} p\mathbb{E}[x_i^*(q_i^\alpha, \mathbf{q}_{-i}^\alpha, \beta)] - cq_i - K & \text{if } q_i > 0, \\ 0 & \text{if } q_i = 0, \end{cases} \tag{4}$$

where $\pi_i(\cdot)$ is firm i's expected profit at the second stage. When $q_i = 0$, firm i incurs no fixed cost K because it does not produce or sell anything in the market. In a Nash equilibrium (NE), the quality level chosen by any firm is the best response to the quality levels of other firms. We denote by $\mathcal{G} = \{\{1, \ldots, n\}, \{\pi_1, \ldots, \pi_n\}, E\}$ the second-stage game with n firms. Because we assume thus far all firms have identical parameters, it makes sense to focus on symmetric pure-strategy equilibria of the game $\mathcal{G}$. Let $q^*(n, \alpha, \beta)$ and $\pi^*(n, \alpha, \beta, K)$ denote each firm's quality and expected profit, respectively, in a symmetric pure-strategy equilibrium of $\mathcal{G}$.

Recall that at the consumption stage, the market evolves differently as the network effect parameter β varies. In this section, we study the quality competition for any given β and characterize the equilibrium in two separate cases: $\beta \le 1$ and $\beta > 1$. The differences between equilibria in different cases reflect the effect of the strength of the network effect on the quality competition among n firms. The following theorems summarize the equilibrium results of the game $\mathcal{G}$ in those two cases.

Theorem 1 (Dominated Network Effect). *For $\beta \le 1$, define*

$$\tilde{n}(\alpha, \beta, K) \triangleq \begin{cases} \left\lfloor \dfrac{2p\alpha}{\sqrt{(1-\alpha-\beta)^2 p^2 + 4p\alpha K(1-\beta)} - p(1-\alpha-\beta)} \right\rfloor & \text{if } K > 0 \text{ or } \alpha + \beta > 1, \\ +\infty & \text{otherrwise}, \end{cases} \tag{5}$$

and the equilibrium of $\mathcal{G}$ is characterized as follows:

i. *If $n = 1$, $q^*(1, \alpha, \beta)$ can be any arbitrarily small positive number.*

ii. *If $2 \le n \le \tilde{n}(\alpha, \beta, K)$, there exists a unique symmetric pure-strategy equilibrium where $q^*(n, \alpha, \beta) = \frac{p\alpha(n-1)}{cn^2(1-\beta)}$.*

iii. *If $n > \tilde{n}(\alpha, \beta, K)$, there is no symmetric pure-strategy equilibrium, and in any asymmetric pure-strategy equilibrium, the number of firms offering nonzero quality levels cannot exceed $\tilde{n}(\alpha, \beta, K)$.*

Theorem 2 (Dominant Network Effect). *For $\beta > 1$, define*

$$\tilde{n}(\alpha, \beta, K) \triangleq \max\left\{ n \in \mathbb{N} : \frac{1}{n} - \alpha \int_0^{+\infty} (sf_\beta(s))' \overline{F}_\beta^{n-1}(s) \mathrm{d}s \ge \frac{K}{p} \right\}, \tag{6}$$

where $f_\beta(\cdot) = f_{1,\beta}(\cdot)$ and $\overline{F}_\beta(\cdot) = \overline{F}_{1,\beta}(\cdot)$, and the equilibrium of $\mathcal{G}$ is characterized as follows:

i. *If $n = 1$, $q^*(1, \alpha, \beta)$ can be any arbitrarily small positive number.*

ii. *If $2 \le n \le \tilde{n}(\alpha, \beta, K)$, there exists a unique symmetric pure-strategy equilibrium, where*

$$q^*(n, \alpha, \beta) = \frac{p\alpha}{c} \int_0^{+\infty} (sf_\beta(s))' \overline{F}_\beta^{n-1}(s) \mathrm{d}s. \tag{7}$$

iii. *If $n > \tilde{n}(\alpha, \beta, K)$, there is no symmetric pure-strategy equilibrium, and in any asymmetric pure-strategy equilibrium, the number of firms offering nonzero quality levels cannot exceed $\tilde{n}(\alpha, \beta, K)$.*

In Theorems 1 and 2, the term $\tilde{n}(\alpha, \beta, K)$ can be interpreted as the *market capacity* for product variety, namely, the largest number of competitive products with nonzero quality levels that generate nonnegative profits in the symmetric pure-strategy equilibrium. A market with a larger market capacity can accommodate a larger variety of products. Once the number of competitive products entering the market is beyond that capacity, the competition among them is so intense that no firm is expected to earn a positive profit. Here, we introduce the notion of $\tilde{n}(\alpha, \beta, K)$ to facilitate our elaboration of the equilibrium of the quality competition.

Proposition 1. *Suppose $n \ge 2$. The equilibrium profit and quality have the following properties:*

i. *For $\beta \in (-\infty, +\infty)$ and $n < \tilde{n}(\alpha, \beta, K)$, $\pi^*(n, \alpha, \beta, K) \ge \pi^*(n+1, \alpha, \beta, K)$.*

ii. *For $\beta \in (-\infty, +\infty)$ and $n < \tilde{n}(\alpha, \beta, K)$, $q^*(n, \alpha, \beta)$ is increasing in α and $\pi^*(n, \alpha, \beta, K)$ is decreasing in α.*

iii. *(Dominated network effect) For $\beta < 1$ and $n \le \tilde{n}(\alpha, \beta, K)$, $q^*(n, \alpha, \beta)$ is increasing in β.*

iv. *(Dominant network effect) For $\beta > 1$ and $n \le \tilde{n}(\alpha, \beta, K)$, $q^*(n, \alpha, \beta)$ is decreasing in β.*

Proposition 1(i) shows that the expected equilibrium profit for each firm is decreasing in the number of competitive products no matter whether the network effect is strong or weak. The more products are in the market, the more intense the competition is and, hence, the lower the expected profit each firm can obtain since we assume the total market size is fixed. This result is fairly intuitive in view of the traditional economic theory. We verify it holds even with the network effect.

Proposition 1(ii) illustrates that the equilibrium quality level is increasing in the parameter α, and the expected equilibrium profit for each firm is decreasing in the parameter α. When the parameter becomes larger, each seller has more incentive to improve their quality levels, which leads to more intensified competition and lower equilibrium profit for each firm.

Proposition 1, (iii) and (iv), shows a phase-changing phenomenon on the equilibrium quality investment with respect to the network effect parameter. Proposition 1(iii) indicates that the equilibrium quality increases, implying intensified quality competition, as the network effect parameter increases under a prevailing *dominated* network effect. It is interesting to contrast this finding with Proposition 1(iv), which addresses the scenario of a prevailing *dominant* network effect. In this case, equilibrium quality decreases, implying alleviated quality competition as the network effect parameter increases.

From Proposition 1, (iii) and (iv), we can explore the impact of the increasing network effect on the quality investment competition. In this paper, investment strategies can be generally categorized into two types: high-budget and low-budget strategies. A high-budget strategy releases high-investment products, while a low-budget strategy aims at commercializing low-investment products. On the one hand, we find from Proposition 1(iii) that when the prevailing network effect is dominated but growing, each firm has a strong incentive to improve its product's quality because the higher quality will result in more demand. This induces the equilibrium investment strategies to shift toward the high-budget strategy as the network effect increases. On the other hand, we find from Proposition 1(iv) that when the prevailing network effect is dominant, each firm is satisfied with a relatively low quality of its product. That is because a small advantage in sales could be significantly amplified as long as the advantage occurs soon after the product is released. Then, it is reasonable to believe that even if multiple products per firm are allowed, every firm will offer a wider range of low-investment products, that is, shifting toward the low-budget strategy, as the network effect parameter increases. We study the multiproduct case in Section 5.2.

As mentioned, Proposition 1 has implications for quality investment in an industry with an increasing network effect. For some industries, for example, the online multiplayer game industry, the network effect is expected to become bigger over time (Wingfield 2013). In some other industries, however, the network effect may not be simply increasing but could evolve nonmonotonically over time. Our result may also shed light on those depending on how strong their current network effect is. We see that the equilibrium strategy shifts toward the high-budget strategy when the network effect parameter moves close to one from either side of one, but the equilibrium strategy shifts toward the low-budget strategy when the network effect parameter moves away from one in either direction. It is worth noting that when β moves away from one in different directions, the incentives for each firm to use the low-budget strategy are different. When β moves to the left of one, a firm tends to use the low-budget strategy because of the diversified customer tastes. However, when β moves to the right of one, the driving force becomes the unpredictable customer tastes.

4.3. Product Variety

In the first stage of the market entry competition, firms decide simultaneously whether to enter the market. A firm will enter the market if it expects to gain a nonnegative profit from the subsequent stages by offering a product with *nonzero* quality.

If n firms decide to enter the market, each firm's expected profit in the first stage is given by $\pi^*(n,\alpha,\beta,K)$. Hence, n firms will join the competition if and only if $\pi^*(n,\alpha,\beta,K) \geq 0$. The product variety in the market is defined by the number of products entering the market as the result of firms' ex ante entry decisions. We consider only the pure strategy equilibrium.

In Section 4.2, we denoted by $\tilde{n}(\alpha,\beta,K)$ the market capacity, namely, the largest number of products entering the market such that all products entering the market can earn a nonnegative expected profit in the symmetric pure-strategy equilibrium at the second stage. We have shown that in Theorems 1 and 2, the n firms are willing to enter the market or indifferent to whether they enter or not if $n \leq \tilde{n}(\alpha,\beta,K)$. In this section, we view $\tilde{n}(\alpha,\beta,K)$ as a measure of the product variety in the market, given the parameter α, the strength of network effect β, and the fixed cost K. The following proposition presents some properties of $\tilde{n}(\alpha,\beta,K)$.

Proposition 2 (Product Variety). *The market capacity $\tilde{n}(\alpha,\beta,K)$ satisfies the following properties:*

i. $\tilde{n}(\alpha,1,K) = 1$; *$\tilde{n}(\alpha,\beta,K)$ is decreasing in K and α.*

ii. *(Dominated Network Effect) $\tilde{n}(\alpha,\beta,K)$ is decreasing in β on $(-\infty,1)$.*

iii. *(Dominant Network Effect) $\tilde{n}(\alpha,\beta,K)$ is increasing in β on $(1,+\infty)$.*

Proposition 2(i) of this theorem is intuitive. The price p is not only the price but also the revenue for the whole industry because we normalize the total market size to be one. Because we assume $p > K$, at least one product will enter the market. The lower the entry cost, the more the product variety is in equilibrium in the industry. A low entry cost is often cited as a reason to support the long tail phenomenon. Our framework confirms that the entry cost does indeed play a significant role in an industry with network effects. Part (i) also shows that the product variety decreases as the parameter α increases. That is because, as α increases, the incentive to improve the quality rises, which intensifies the competition and thereby reduces the product variety.

Proposition 2, (ii) and (iii), characterizes a phase-changing phenomenon on the equilibrium product variety with respect to the network effect parameter. With the dominated network effect, an increasing network effect would prevent more products from entering the market ex ante since the firms would expect more intense competition as the network effect becomes stronger (see Proposition 1(iii)). In contrast, with the dominant network effect, the ex ante equilibrium product variety will be wider if the network effect becomes stronger since the firms will expect a less quality-competitive market (see Proposition 1(iv)).

The video game industry may serve as an illustrating example. In particular, on the one hand, single-player video games can be thought of as an example of the case with a dominated network effect (i.e., $\beta < 1$). On the other hand, multiplayer mobile games can fit the case with a dominant network effect (i.e., $\beta > 1$). In a multiplayer game, more players mean a shorter waiting time to match up with an opponent or form a team, more users to generate content like guides and online conversations, and more people to buy and trade items using in-game marketplaces. The number of players can have a larger impact on a buyer's adoption decision than the game's intrinsic quality. Nowadays, a single-player video game (e.g., Uncharted 4) often has high intrinsic quality because it often needs high-resolution visuals and motion-detecting capabilities. In contrast, designing a multiplayer mobile game (e.g., Pokéman Go) is often focused on providing social features from which a player's utility is mainly driven by her interactions with other players. Hence, compared with the single-player video game segment, the online multiplayer video game segment has a larger product variety, but the demand is more concentrated on a small number of products.

We have shown in Section 4.1 that an increasing network effect always contributes to sales concentration on a small number of products, for example, ex post blockbusters. As a complement, Proposition 2, (ii) and (iii), implies how the depth of the tail in the sales rank distribution varies as the network effect parameter increases. More specifically, when the network effect is dominated, an increasing network effect contributes to a shrinking tail in the sales rank distribution; interestingly, when the network effect is dominant, an increasing network effect contributes to an expanding tail in which most of the products gain a market share of zero. Here, a market share of zero does not mean a sales volume of zero because we are assuming that there is an infinite number of customers. Therefore, any finite sales volume corresponds to a market share of zero. One can imagine that when β is large enough and the number of customers is finite, only one product has a significantly high sales volume, and all the other products entering the market have very low sales volumes (not zero), which is consistent with both the blockbuster phenomenon and the long tail phenomenon.

In Section 4.2, we have studied the quality competition with network effects among multiple firms and shown that an increasing network effect can intensify or alleviate the competition. The following proposition further studies the quality competition with network effects, but this time, it incorporates the endogenized number of competing firms.

Proposition 3. *For $\alpha > 0$, $K \geq 0$ and $\beta \in (-\infty, +\infty)$, suppose there are $\tilde{n}(\alpha, \beta, K)$ firms entering the market. Let $q^*(\alpha, \beta, K)$ be the quality that they offer in the symmetric pure-strategy equilibrium at the second stage. Then:*

i. *(Dominated network effect) $q^*(\alpha, \beta, K)$ is increasing in α and β if $\beta < 1$ and $\tilde{n}(\alpha, \beta, K) \geq 2$.*

ii. *(Dominant network effect) There exists $\tilde{\beta} \geq 1$, such that $q^*(\alpha, \beta, K)$ is increasing in α and decreasing in β if $\beta > \tilde{\beta}$ and $\tilde{n}(\alpha, \beta, K) \geq 2$.*

We find from Proposition 3 that the main insights in Section 4.2 still hold when the firms' entry decisions are endogenized. We can see that an increasing parameter α always shifts the equilibrium strategy toward the high-budget strategy. However, an increasing network effect parameter β may influence the equilibrium quality decision in opposite directions. Specifically, the equilibrium quality increases, that is, the quality decision shifts toward the high-budget strategy as the network effect parameter increases if the prevailing network effect is dominated, but it decreases, that is, the equilibrium investment decision shifts toward the low-budget strategy, as the network effect parameter increases if the prevailing network effect is dominant.

4.4. Intuition Behind Cases $\beta < 1$ and $\beta > 1$

We analytically demonstrated two key relationships involving the network effect parameter β: the quality in equilibrium follows an inverted U-shaped pattern, while the product variety (i.e., the level of market entry) in equilibrium follows a U-shaped pattern. Now, we provide an intuitive explanation for these findings. Since the latter type of nonmonotonicity is mainly driven by the former type of nonmonotonicity, we will primarily focus on explaining the former.

A customer's purchase decision is influenced by three factors: product quality, historical sales, and the network effect. Obviously, higher product quality increases the purchase probability. Given the strength of the network effect, as past sales volume accumulates, the influence of quality weakens. However, the weakening trend differs in its magnitude for the two cases of $\beta < 1$ and $\beta > 1$.

When $\beta < 1$ (namely, when the network effect is dominated), the influence of quality weakens as sales volume accumulates but does not tend toward zero. If a low-quality product gains a significant advantage in sales volume, it will be favored by customers for a while due to the network effect. However, because customers never lose interest in high-quality products, the high-quality products gradually catch up from a disadvantageous situation (such a catching-up process can be very lengthy, especially when β approaches one). Consequently, high-quality roducts must have high sales volumes as long as there is a sufficiently large number of customers, and an increasing network effect always contributes to the sales concentration on high-quality products in the long run. That is why we refer to the network effect in this case as a *dominated* network effect. In this case, increasing the strength of the network effect must lead to a higher incentive for each firm to improve its quality.

When $\beta > 1$ (namely, when the network effect is dominant), the market share dynamics converge quickly because, after a finite time of traction, all customers' purchase behavior tends to conform. Hence, quality has a diminishing influence on customers' choice behavior as time goes to infinity. As β increases, the traction time becomes (stochastically) smaller so that the influence of quality is more diminishing and the network effect plays a more critical role in the market evolution, which explains why sales concentration is unpredictable and why we refer to the network effect in this case as a *dominant* network effect. In this case, increasing the strength of the network effect leads to a lower incentive for each firm to improve its quality.

The previous intuitions are rigorously shown in Lemma 1(ii). To gain a more intuitive understanding of the nonmonotonic impact of the network effect on quality investment competition, we consider three special cases: $\beta = 0$, $\beta = 1$, and $\beta \to +\infty$. We can see from Lemma 1, (i) and (iii), that

$$\lim_{\beta\to+\infty} \mathbb{E}x_i^*(\mathbf{q}^\alpha,\beta) = \int_0^{+\infty} q_i^\alpha \exp(-q_i^\alpha s)\prod_{\substack{j=1\\ j\neq i}}^{n}\exp(-q_j^\alpha s)\mathrm{d}s$$
$$= \frac{q_i^\alpha}{q_1^\alpha + \cdots + q_n^\alpha} = x_i^*(\mathbf{q}^\alpha, 0).$$

This implies that in the case $\beta \to +\infty$, each firm has the same incentive to improve its quality as in the case $\beta = 0$. In contrast, in the case $\beta = 1$, each firm has the most incentive to improve its quality, as shown in Lemma 1(ii).

5. Extensions

Through Sections 4.1–4.3, we have shown how the network effect influences the equilibrium investment strategy chosen by each firm. In this section, we establish the robustness of our main insights by showing that the qualitative results in the base model continue to hold under a variety of modeling variations. Moreover, we also obtain some novel insights under each variation. Whenever there are no conflicts, we adopt the same notation in these extensions.

Part of the results in this section are shown numerically due to intractability. To conduct the numerical experiments, we approximate $f_{q^\alpha,\beta}(\cdot)$ by using the density function of the sum of finite exponentials. As β approaches one, we require more exponentials to ensure computational precision.

5.1. Endogenous Pricing

In our base model, we assume that the price in the market is exogenously given. This section studies the situation where each firm that stays in the market decides not only how much to invest in its quality but also how to price its product. All prices and quality levels are assumed to be set simultaneously at the beginning of the second stage, i.e., after firms' entry decisions but before customer arrivals. The first stage game and the third stage game are the same as in our base model. In the following, we study the subgame-perfect equilibrium of the extended three-stage game. We show that our main insights in the base model are still intact, even under endogenous pricing.

5.1.1. Third Stage: Demand Process. Suppose n firms stay in the market at the third stage. Given prices $\mathbf{p}$, quality levels $\mathbf{q}$, and parameters α and β, the customer choice probability of product i at time $t+1$ can be expressed as

$$\phi_i(\mathbf{x}(t)) = \frac{q_i^\alpha \exp(-p_i)x_i^\beta(t)}{\sum_{i=1}^n q_i^\alpha \exp(-p_i)x_i^\beta(t)}.$$

Using the same argument as in Section 4.1, we can show that the consumption dynamics $\mathbf{x}(t)$ converges to $\mathbf{x}^*(\mathbf{q}^\alpha\exp(-\mathbf{p}),\beta)$ almost surely, where $\mathbf{q}^\alpha\exp(-\mathbf{p}) = (q_1^\alpha\exp(-p_1),\ldots,q_n^\alpha\exp(-p_n))$ and the expression of $\mathbf{x}^*(\cdot,\cdot)$ is specified in Lemma 1.

5.1.2. Second Stage: Joint Quality and Price Competition. For $i = 1,\ldots,n$, given other products' quality levels $\mathbf{q}$ and other products' prices $\mathbf{p}_{-i}$, firm i's problem at the second stage is to decide its quality level q_i and price p_i to maximize its expected profit, that is,

$$\pi_i(q_i,p_i,\mathbf{q}_{-i},\mathbf{p}_{-i},\alpha,\beta,K)$$
$$= p_i\mathbb{E}x_i^*(q_i^\alpha\exp(-p_i),\mathbf{q}_{-i}^\alpha\exp(-\mathbf{p}_{-i}),\beta) - cq_i - K.$$

We consider only symmetric pure-strategy equilibria. Let $\tilde{n}(\alpha,\beta,K)$ be the market capacity, which has the same meaning as in our base model. According to the proof of Proposition 4, we have that, for $\beta \le 1$,

$$\tilde{n}(\alpha,\beta,K) \triangleq \begin{cases} \left\lfloor \dfrac{2\alpha}{\sqrt{(K+1-\alpha-\beta)^2+4\alpha K}-(K+1-\alpha-\beta)} \right\rfloor & \text{if } K>0 \text{ or } \alpha+\beta>1+K, \\ +\infty & \text{otherwise;} \end{cases}$$

and, for $\beta > 1$,

$$\tilde{n}(\alpha,\beta,K) \triangleq \max\left\{ n \in \mathbb{N} : \int_0^{+\infty} (sf_\beta(s))' \overline{F}_\beta^{n-1}(s)\mathrm{d}s \le \frac{1}{n(nK+\alpha)} \right\}.$$

By following the methodology used to prove Theorems 1 and 2, we can establish that if $n \le \tilde{n}(\alpha,\beta,K)$, then there exists a unique symmetric pure-strategy equilibrium. The following proposition summarizes some properties of the equilibrium.

Proposition 4. *Suppose* $n \ge 2$.

i. *For* $\beta \in (-\infty, +\infty)$ *and* $n \le \tilde{n}(\alpha,\beta,K)$, $q^*(n,\alpha,\beta) = \alpha/(nc)$.

ii. *(Dominated network effect) For* $\beta < 1$ *and* $n \le \tilde{n}(\alpha, \beta,K)$, $p^*(n,\alpha,\beta)$ *is decreasing in* β.

iii. *(Dominant network effect) For* $\beta > 1$ *and* $n \le \tilde{n}(\alpha, \beta,K)$, $p^*(n,\alpha,\beta)$ *is increasing in* β.

Proposition 4 shows that (i) the equilibrium quality level is increasing in the parameter α and is independent of the network effect parameter β, (ii) the equilibrium price decreases as the network effect parameter β increases when the prevailing network effect is dominated, and (iii) the equilibrium price increases as the network effect parameter β increases when the prevailing network effect is dominant. Proposition 4 is consistent with Proposition 1. Both conclude that an increasing network effect intensifies the competition among firms when the prevailing network effect is dominated but alleviates the competition when the prevailing network effect is dominant.

5.1.3. First Stage: Product Variety.

In the first stage, firms simultaneously decide whether to enter the market. A firm will enter the market when expecting to gain a nonnegative profit from the second and third stages by offering a product with *nonzero* quality. In contrast to Propositions 2 and 3, we summarize the equilibrium results under endogenous pricing as follows.

Proposition 5 (Product Variety Under Endogenous Pricing). *The market capacity* $\tilde{n}(\alpha,\beta,K)$ *satisfies the following properties:*

i. $\tilde{n}(\alpha,1,K) = 1$; $\tilde{n}(\alpha,\beta,K)$ *is decreasing in* K *and* α.

ii. *(Dominated network effect)* $\tilde{n}(\alpha,\beta,K)$ *is decreasing in* β *on* $(-\infty,1)$.

iii. *(Dominant network effect)* $\tilde{n}(\alpha,\beta,K)$ *is increasing in* β *on* $(1, +\infty)$.

Proposition 6. *For* $\alpha > 0$, $K \ge 0$ *and* $\beta \in (-\infty, +\infty)$, *suppose there are* $\tilde{n}(\alpha,\beta,K)$ *firms entering the market. Let* $q^*(\alpha,\beta,K)$ *be the quality level that they offer in the symmetric pure-strategy equilibrium at the second stage. Then:*

i. $q^*(\alpha,\beta,K)$ *is increasing in* α *if* $\tilde{n}(\alpha,\beta,K) \ge 2$.

ii. *(Dominated network effect)* $q^*(\alpha,\beta,K)$ *is increasing in* β *for* $\beta < 1$ *and* $\tilde{n}(\alpha,\beta,K) \ge 2$.

iii. *(Dominant network effect)* $q^*(\alpha,\beta,K)$ *is decreasing in* β *for* $\beta > 1$ *and* $\tilde{n}(\alpha,\beta,K) \ge 2$.

Propositions 5 and 6 show that our main insights in the base model are robust when pricing is endogenized. As the parameter α increases, the equilibrium quality strategy always shifts toward the high-budget strategy, which thereby leads to a shrinking product variety. When the prevailing network effect parameter moves from $-\infty$ to one (respectively, from one to $+\infty$), fewer and fewer (respectively, more and more) firms join the competition, and their equilibrium strategy shifts toward the high-budget strategy (respectively, low-budget strategy).

5.2. Multiple Products Carried by Each Firm

In this section, we consider a case in which each firm can produce and sell multiple products. In the second stage game, a firm, say i, can decide the number of products it wants to produce, labeled as m_i, and the quality level q_i, which is assumed to be the same for all its products. Each product for firm i incurs a fixed cost K and a variable cost cq_i. The rest is the same as in the base.

Let $d_{ij}(t)$ be the aggregated sales of firm i's product j at the end of time t with $d_{ij}(0) = 1$, $\forall i,j$, and let $x_{ij}(t)$ be the corresponding market share. We assume the price p is exogenous, like in the base model. A customer's utility from purchasing one unit of the jth product carried by firm i at time $t+1$ is assumed to be $U_{ij}(q_i, d_{ij}(t)) = \alpha \ln q_i + \beta \ln d_{ij}(t) - p + \xi_{ij}$. This utility function has the same meaning as in our base model.

5.2.1. Third Stage: Demand Process.

Suppose n firms enter the market, and each firm, say i, sells m_i products. The probability that the customer at time $t+1$ selects jth product by firm i is given by $\phi_{ij}(\mathbf{x}(t)) = q_i^\alpha x_{ij}^\beta / \sum_{i=1}^n \sum_{j=1}^{m_i} q_i^\alpha x_{ij}^\beta$. When $m_1 = m_2 = \cdots = m_n = 1$, the choice model reduces to that in our base model. Let $\mathbf{Q}^\alpha$ be a vector in which the first m_1 entries are q_1^α, the next m_2 entries are q_2^α, and so on. Lemma 1, which is still valid here, shows that the consumption dynamics $\mathbf{x}(t)$ converges to $\mathbf{x}^*(\mathbf{Q}^\alpha,\beta)$ almost surely and provides the closed-form expression of $\mathbf{x}^*(\cdot,\cdot)$. Note that $x^*_{ij}(\mathbf{Q}^\alpha,\beta)$ is the market share of the jth product by firm i. We denote by $x^*_i(\mathbf{m},\mathbf{q}^\alpha,\beta)$ the total market share of all m_i products by firm i, where $\mathbf{m}=(m_1,\ldots,m_n)$, $\mathbf{q}^\alpha=(q^\alpha_1,\ldots,q^\alpha_n)$. In a symmetric equilibrium, we must have $\mathbb{E}x^*_i(\mathbf{m},\mathbf{q}^\alpha,\beta)=m_i\mathbb{E}x^*_{i1}(\mathbf{Q}^\alpha,\beta)$.

5.2.2. Second Stage: Joint Quality and Variety Competition.

For $i=1,\ldots,n$, given other firms' decisions on the number of products $\mathbf{m}_{-i}$ and quality levels $\mathbf{q}_{-i}$, firm i's problem at the second stage is to decide the number of products m_i and the quality level q_i to maximize its expected profit, that is, $\pi_i(m_i,q^\alpha_i,\mathbf{m}_{-i},\mathbf{q}^\alpha_{-i},\beta,K)=p\mathbb{E}x^*_i(m_i,q^\alpha_i,\mathbf{m}_{-i},\mathbf{q}^\alpha_{-i},\beta)-m_i(cq_i+K)$. In this section, for the sake of tractability, we assume the decision variable $m_i\geq 1$ and is a continuous variable for $i=1,\ldots,n$.[5] We consider only symmetric pure-strategy equilibria. Let $m^*(n,\alpha,\beta,K)$, $q^*(n,\alpha,\beta,K)$, and $\pi^*(n,\alpha,\beta,K)$ be the equilibrium number of products, the equilibrium quality level and the equilibrium expected profit for each firm. Let $\tilde{n}(\alpha,\beta,K)$ be the largest number of firms who decide to enter the market and sell at least one product with a nonzero quality level, that is, $\tilde{n}(\alpha,\beta,K)\triangleq\max\{n\geq 2: m^*(n,\alpha,\beta,K)\geq 1, q^*(n,\alpha,\beta,K)>0\}$, if there is a symmetric pure-strategy equilibrium in which $m^*(n,\alpha,\beta,K)\geq 1$ and $q^*(n,\alpha,\beta,K)>0$ for $n=2$, and $\tilde{n}(\alpha,\beta,K)\triangleq 1$ otherwise. We also assume $\tilde{n}(\alpha,\beta,K)$ is a continuous variable. In the proof of Theorems 3 and 4, we show that $\tilde{n}(\alpha,\beta,K)$ has the same expression as in (5) and (6). The following theorems summarize the equilibrium results.

Theorem 3 (Dominated Network Effect). *Suppose $n\geq 2$, $K>0$. For $\beta\leq 1$, the equilibrium of the second stage game is characterized as follows*:

i. *If $n\leq\tilde{n}(\alpha,\beta,K)$, then there exists a unique symmetric pure-strategy equilibrium, and the equilibrium can be solved by*

$$m^*(n,\alpha,\beta,K)=\begin{cases}\max\left\{\dfrac{p(n-1)(1-\alpha-\beta)}{Kn^2(1-\beta)},1\right\} & \text{if } \beta<1,\\ 1 & \text{if } \beta=1,\end{cases}$$

$$m^*(n,\alpha,\beta,K)q^*(n,\alpha,\beta,K)=\frac{p\alpha(n-1)}{cn^2(1-\beta)}.$$

In particular, if $n=\tilde{n}(\alpha,\beta,K)$, then $m^*(n,\alpha,\beta,K)=1$.

ii. *If $n>\tilde{n}(\alpha,\beta,K)$, there is no symmetric pure-strategy equilibrium, and in any asymmetric pure-strategy equilibrium, the number of firms offering nonzero quality levels cannot exceed $\tilde{n}(\alpha,\beta,K)$.*

Theorem 4 (Dominant Network Effect). *Suppose $n\geq 2$, $K>0$. For $\beta>1$, the equilibrium of the second stage game is characterized as follows*:

i. *If $n\leq\tilde{n}(\alpha,\beta,K)$, then there exists at least one symmetric pure-strategy equilibrium. Let M_0 be the set of real roots of the following equation with respect to m:*

$$\frac{n-1}{n^2m}-\alpha\int_0^{+\infty}(sf_\beta(s)\overline{F}_\beta^{m-1}(s))'\overline{F}_\beta^{(n-1)m}(s)\mathrm{d}s=\frac{K}{p}.$$

For each equilibrium, there must be a $m_0\in M_0$ such that the equilibrium can be solved by

$$m^*(n,\alpha,\beta,K)=\max\{m_0,1\},$$

$$q^*(n,\alpha,\beta,K)=\frac{p\alpha}{c}\int_0^{+\infty}(sf_\beta(s)\overline{F}_\beta^{m^*(n,\alpha,\beta,K)-1}(s))'\overline{F}_\beta^{(n-1)m^*(n,\alpha,\beta,K)}(s)\mathrm{d}s. \tag{8}$$

In each symmetric pure-strategy equilibrium, $\pi^*(n,\alpha,\beta,K)$ must be the same. In particular, if $n=\tilde{n}(\alpha,\beta,K)$, then there must be a unique symmetric pure-strategy equilibrium where $m^*(n,\alpha,\beta,K)=1$.

ii. *If $n>\tilde{n}(\alpha,\beta,K)$, there is no symmetric pure-strategy equilibrium, and in any asymmetric pure-strategy equilibrium, the number of firms offering nonzero quality levels cannot exceed $\tilde{n}(\alpha,\beta,K)$.*

As shown in Theorem 4(i), when $\beta>1$ and $2\leq n\leq\tilde{n}(\alpha,\beta,K)$, there may be multiple equilibria. Fortunately, the equilibrium expected profits must be the same even in different equilibria. Consequently, if there are multiple equilibria, for tractability, we always select the equilibrium such that $m^*(n,\alpha,\beta,K)=\max\{\max\{M_0\},1\}$, and $q^*(n,\alpha,\beta,K)$ is solved by Equation (8). In contrast to Proposition 1, the following proposition highlights the influence of the network effect on the equilibrium quality level and the equilibrium number of products offered by each firm.

Proposition 7. *Suppose $n\geq 2$, $K>0$.*

i. *For $\beta\in(-\infty,+\infty)$ and $n\leq\tilde{n}(\alpha,\beta,K)$, $m^*(n,\alpha,\beta,K)$ is decreasing in α and K.*

ii. *For $\beta\in(-\infty,+\infty)$ and $n\leq\tilde{n}(\alpha,\beta,K)$, $q^*(n,\alpha,\beta,K)$ is increasing in α and K.*

iii. *(Dominated network effect) For $\beta<1$ and $n\leq\tilde{n}(\alpha,\beta,K)$, $m^*(n,\alpha,\beta,K)$ is decreasing in β, and $q^*(n,\alpha,\beta,K)$ is increasing in β.*

iv. *(Dominant network effect) For $\beta>1$ and $n\leq\tilde{n}(\alpha,\beta,K)$, $m^*(n,\alpha,\beta,K)$ is increasing in β, and $q^*(n,\alpha,\beta,K)$ is decreasing in β.*

Proposition 7, (i) and (ii), shows that, as the parameter α or the fixed cost K increases, the equilibrium number of products and equilibrium quality level increases, that is, each firm's production and quality investment strategy always shifts toward the high-budget strategy, which thereby leads to a shrinking product variety in the market.

Proposition 7, (iii) and (iv), shows the impact of the network effect on each firm's production and quality investment strategy. Specifically, when the network effect parameter β moves away from one in either direction (respectively, close to one from either side), the equilibrium strategy shifts toward the low-budget strategy (respectively, high-budget strategy), that is, offering more products with lower quality (respectively, offering fewer products with higher quality). The following corollary further confirms the insights drawn from Proposition 7. More precisely, it provides sufficient conditions under which each seller would offer a single product versus multiple products.

Corollary 1. *The equilibrium number $m^*(n,\alpha,\beta,K)$ of products offered by each firm satisfies the following properties:*

i. *If $1-\alpha \le \beta < 1$ and $n \le \tilde{n}(\alpha,\beta,K)$, then $m^*(n,\alpha,\beta,K) = 1$.*

ii. *If $\beta > 1, \alpha \ge 1$ and $n \le \tilde{n}(\alpha,\beta,K)$, then $m^*(n,\alpha,\beta,K) = 1$.*

iii. *There exist $\hat{n} \ge 2, \hat{K} \ge 0, 0 < \hat{\alpha} < 1, \hat{\beta}_1 < 1$ and $\hat{\beta}_2 > 1$ such that if $2 \le n \le \hat{n}, K \le \hat{K}, \alpha \le \hat{\alpha}, \beta \le \hat{\beta}_1$ or $\beta \ge \hat{\beta}_2$, then $m^*(n,\alpha,\beta,K) > 1$.*

5.2.3. First Stage: Product Variety.

At the first stage, firms simultaneously decide whether to enter the market. From Theorems 3(i) and 4(i), we can see that when there are exactly $\tilde{n}(\alpha,\beta,K)$ firms entering the market, each firm offers only one product, and the equilibrium quality is the same as in the base model. This allows us to treat $\tilde{n}(\alpha,\beta,K)$ as the market capacity, namely, the largest number of products entering the market, like in the base model. Obviously, Propositions 2 and 3 continue to hold in the multiproduct case.

5.3. No-Purchase Outside Option

This section considers a case in which there is a no-purchase outside option for customers. We assume the outside option (indexed as "0") has quality q_0, price p, initial sales volume $d_0(0) = 1$, and exhibits the network effect. Specifically, a customer's utility from choosing the outside option at time $t+1$ is assumed to be

$$U_0(q_0, d_0(t)) = \alpha \ln q_0 + \beta \ln d_0(t) - p + \xi_0,$$

where ξ_0 represents customer-specific idiosyncrasies toward the outside option. The assumption that the outside option exhibits the network effect is reasonable. Consider a store selling n products. If more customers do not buy anything in the store (i.e., choose the no-purchase option 0) or fewer customers do in the store, then subsequent customers are more likely to leave the store. If we do not make this assumption, the outside option's value for customers will ultimately dwindle to zero as time t approaches infinity, hence rendering the model identical to the base model. In this subsection, we assume q_0 is exogenously given, and other modeling settings are the same as the base model.

5.3.1. Third Stage: Demand Process.

Let $\mathbf{x}^*(\mathbf{q}^\alpha, \beta)$ denote the limit of consumption dynamics as t goes to infinity, where $\mathbf{q}^\alpha = (q_0^\alpha, q_1^\alpha, \ldots, q_n^\alpha)$. The expression of $\mathbf{x}^*(\cdot,\cdot)$ is given by Lemma 1.

5.3.2. Second Stage: Quality Competition.

In this stage, each firm aims to maximize its expected profit by choosing its quality. We consider only symmetric pure-strategy equilibria. Suppose q is a symmetric solution that meets the first-order conditions (FOCs) and second-order conditions (SOCs) of all firms' problems. The expressions of these conditions can be found in the proof of Proposition 8. Note that q may not be an equilibrium point, even if it is the unique solution satisfying the FOCs and SOCs. By following the proofs of Lemmas 1 and B.1(iv) (in the Online Appendix), we know that q is an equilibrium as long as q satisfies the FOCs and ensures that each firm earns a nonnegative profit. Define $\tilde{n}(\alpha,\beta,K,q_0) \triangleq \max\{n \ge 1:$ the system of FOCs of the n-firm game has a symmetric solution q such that $p\mathbb{E}[x_1^*((q_0/q)^\alpha, \mathbf{1}, \beta)] - cq - K \ge 0\}$ if firm 1 can gain a nonnegative expected profit by offering a positive quality for $n = 1$, and $\tilde{n}(\alpha,\beta,K,q_0) = 0$ otherwise. Then we can have that there is no symmetric pure-strategy equilibrium if $n > \tilde{n}(\alpha,\beta,K,q_0)$. The proof of Proposition 8 contains rigorous proof demonstrating both the existence and uniqueness of equilibrium. As a convention, we refer to $\tilde{n}(\alpha,\beta,K,q_0)$ as the market capacity.

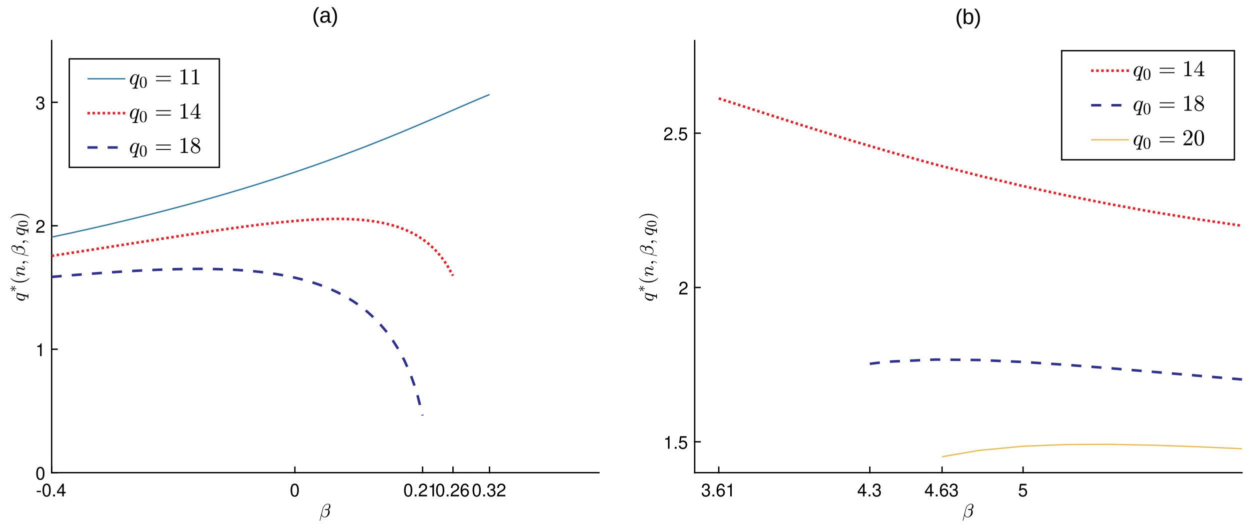
Proposition 8. *For $n \ge 2$, $\alpha > 0$, $K \ge 0$, $c > 0$ and $q_0 > 0$:*

i. *(Dominated network effect) There exists $\tilde{q}_0$ such that $q^*(n,\alpha,\beta,q_0)$ is increasing in β on $\{\beta: \beta < 1, n \le \tilde{n}(\alpha,\beta,K,q_0)\}$ if $q_0 \le \tilde{q}_0$, and $q^*(n,\alpha,\beta,q_0)$ is first-increasing-then-decreasing in β on $\{\beta: \beta < 1, n \le \tilde{n}(\alpha,\beta,K,q_0)\}$ if $q_0 > \tilde{q}_0$.*

ii. *(Dominant network effect) There exists $\hat{q}_0$ such that $q^*(n,\alpha,\beta,q_0)$ is decreasing in β on $\{\beta: \beta > 1, n \le \tilde{n}(\alpha,\beta,K,q_0)\}$ if $q_0 \le \hat{q}_0$; there exists $\hat{\beta}$ such that $q^*(n,\alpha,\beta,q_0)$ is decreasing in β if $\beta > \hat{\beta}$ and $n \le \tilde{n}(\alpha,\beta,K,q_0)$.*

According to Proposition 8, if q_0 is sufficiently small, $q^*(n,\alpha,\beta,q_0)$ is increasing in β for $\beta < 1$ and decreasing in β for $\beta > 1$. This makes sense because when q_0 is small, the model is similar to the base model without an outside option. However, if q_0 is not small enough, $q^*(n,\alpha,\beta,q_0)$ is no longer strictly increasing on $(-\infty, 1)$ or strictly decreasing on $(1, +\infty)$. Proposition 8 also indicates that $q^*(n,\alpha,\beta,q_0)$ exhibits an increasing behavior when β is below a certain threshold and a decreasing behavior when β exceeds a certain threshold. In Figure 3, we investigate the monotonicity of $q^*(n,\alpha,\beta,q_0)$ in β for different values of q_0 through numerical experimentation.

Figure 3. (Color online) Quality in Equilibrium with Outside Option



Notes. (a) $\beta < 1$. (b) $\beta > 1$. $n = 2$, $p = 1$, $\alpha = 0.8$, $K = 0.0002$, and $c = 0.05$. We have marked some important scales on the x axis. For examples, $0.21 = \max\{\beta : \tilde{n}(\alpha, \beta, K, q_0) \geq 2, \beta < 1, q_0 = 18\}$, and $4.3 = \min\{\beta : \tilde{n}(\alpha, \beta, K, q_0) \geq 2, \beta > 1, q_0 = 18\}$. Our numerical experiments compute equilibrium outcomes through the following approach: Consider a market with infinitely many potential firms indexed as $1, 2 \ldots$. We calculate $\tilde{n}(\alpha, \beta, K, q_0)$ by finding the largest integer i such that all firms from one up to i can gain nonnegative expected profits in equilibrium when they enter the market. For any given n, the equilibrium quality is determined by solving the system of FOCs and SOCs. In Figures 4 and 5 and Tables 1 and 2, the equilibrium outcomes are computed by a similar approach.

5.3.3. First Stage: Product Variety. Proposition 8 provides sufficient conditions for both increasing and decreasing equilibrium product quality with respect to parameter β. When quality decreases in equilibrium, which means that the quality competition is alleviated, firms earn more profit, and hence, it is natural for more of them to enter the market. Conversely, when quality increases in equilibrium, firms profit less, and fewer of them have an incentive to enter the market. Therefore, it is intuitive that if q_0 is sufficiently small, the product variety is increasing in β for $\beta < 1$ and decreasing in β for $\beta > 1$, just as what we have shown in the base model. In the following, we investigate the case where q_0 is not sufficiently small through numerical experiments. The numerical results are shown in Table 1. Table 1 confirms our qualitative insights that the product variety in equilibrium has a U-shaped relationship with the strength of the network effect, even in the model with an outside option.

Table 1. Product Variety in Equilibrium with Outside Option ($\tilde{n}(\alpha, \beta, K, q_0)$)

	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 2$	$\beta = 4$	$\beta = 6$
$q_0 = 11$	381	31	2	0	2	3	15
$q_0 = 14$	337	20	0	0	0	2	12
$q_0 = 18$	281	6	0	0	0	1	10
$q_0 = 20$	253	0	0	0	0	0	8

Note. $p = 1$, $\alpha = 0.8$, $K = 0.0002$, and $c = 0.05$.

5.4. Heterogeneous Firms

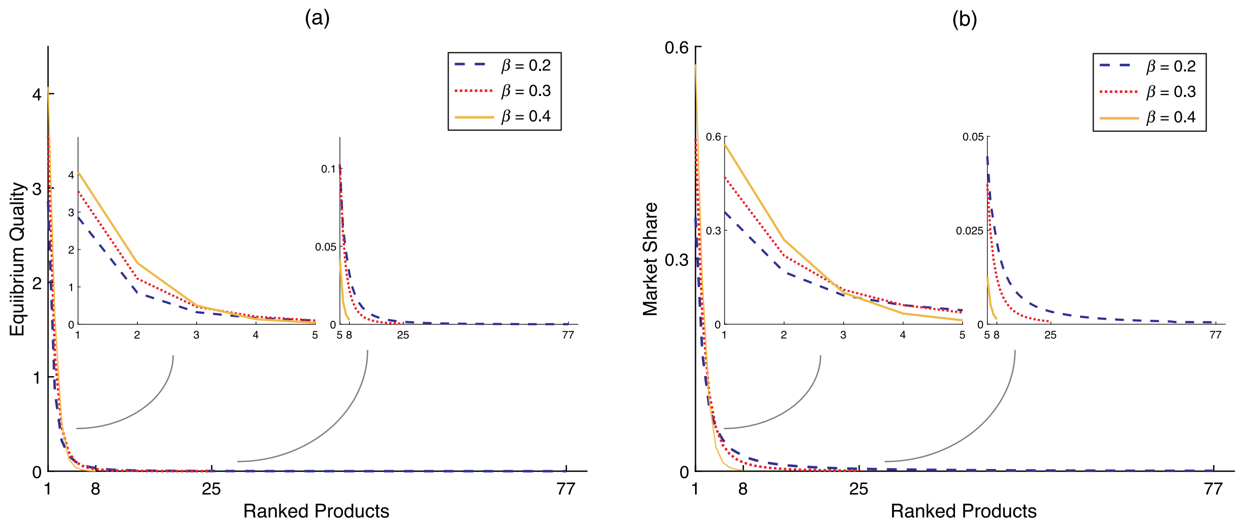
Unlike Section 4 and Sections 5.1–5.4, which focus on the case with homogeneous firms, this section studies the case in which the marginal cost of improving the quality among firms is heterogeneous.

Specifically, we incorporate the heterogeneity of $\mathbf{c}$ into our model. Without loss of generality, We assume $0 < c_1 \leq c_2 \leq \cdots \leq c_n$.

5.4.1. Third Stage: Demand Process. Because the third-stage game is independent of the parameter vector $\mathbf{c}$, all firms' expected market shares can be seen from Lemma 1.

5.4.2. Second Stage: Quality Competition. In the base model, Theorems 1 and 2 demonstrate that the number of products of nonzero quality levels cannot exceed a certain threshold in any pure-strategy equilibrium in the second stage. This threshold is referred to as the market capacity, which we can define in the same way in this subsection. However, in the case of asymmetric firms, there may be multiple equilibria, and the number of nonzero-quality products in each equilibrium may differ. For example, if we consider $n = 4$, there may be multiple equilibria. In one equilibrium, only products 1 and 2 have positive quality levels, and in another equilibrium, products 2, 3, and 4 have positive quality levels. Throughout this section, we focus only on the pure-strategy equilibria in which there exists an integer j $(1 \leq j \leq n)$ such that products $1, \ldots, j$ have positive

Figure 4. (Color online) Quality/Expected Market Share Distributions in Asymmetric Equilibria When $\beta < 1$



Notes. (a) Quality distributions. (b) Expected market share distributions. $p = 1$, $\alpha = 0.5$, $K = 0.0002$, and $c_i = 0.05i$, $i = 1, 2, \ldots$.

quality levels and products $j+1, \ldots, n$ have zero quality levels. Thus, we can define the market capacity as the largest integer $\tilde{n}$ such that firms $1, \ldots, \tilde{n}$ offer nonzero quality levels and earn nonnegative expected profits in a pure-strategy equilibrium if they enter the market.

Lemma 3. *Suppose $n \geq 2$. If $\mathbf{q}^* = (q_1^*, q_2^*, \ldots, q_n^*)$ satisfies the system of FOCs and SOCs, that is, $\frac{\partial \pi_i}{\partial q_i}\big|_{\mathbf{q}^*} = 0$ and $\frac{\partial^2 \pi_i}{\partial q_i^2}\big|_{\mathbf{q}^*} \leq 0$, $i = 1, 2, \ldots, n$, then $q_1^* \geq q_2^* \geq \cdots \geq q_n^* > 0$. Additionally, if firm n's expected profit at $\mathbf{q} = \mathbf{q}^*$ is nonnegative, then $\mathbf{q}^*$ is a pure-strategy equilibrium.*

If $\mathbf{q}^* = (q_1^*, q_2^*, \ldots, q_n^*)$ is a pure-strategy equilibrium, then all its positive entries satisfy the corresponding FOCs and SOCs. By Lemma 3, these entries are positive and sorted in descending order. From the proofs of Theorem 1 and Lemma B.1(iv) (in the Online Appendix), we see that for $n \geq 2$, SOCs hold at some point $\mathbf{q}$ if FOCs hold at $\mathbf{q}$ and each firm earns a nonnegative profit at $\mathbf{q}$. Therefore, for $n \geq 2$, $\mathbf{q}(> \mathbf{0})$ is a pure-strategy equilibrium as long as FOCs hold and firm n earns a nonnegative profit at $\mathbf{q}$.

Lemma 4. *For $\alpha > 0$, $\beta \in (-\infty, +\infty)$, $K \geq 0$, there exists an integer $\tilde{n}(\alpha, \beta, K)$ such that:*

i. *If $2 \leq n \leq \tilde{n}(\alpha, \beta, K)$, then there exists at least one pure-strategy equilibrium with all products having nonzero quality levels.*[6]

ii. *If $n > \tilde{n}(\alpha, \beta, K)$, in any pure-strategy equilibrium (if it exists), the number of firms offering nonzero quality levels cannot exceed $\tilde{n}(\alpha, \beta, K)$.*

We use $\tilde{n}(\alpha, \beta, K)$, specified in Lemma 4, to denote the market capacity. Formally, the market capacity can also be defined as $\tilde{n}(\alpha, \beta, K) \triangleq \max\{n \geq 2 :$ The system of FOCs of the n-firm game has a solution $\mathbf{q}$ such that $\pi_i \geq 0$ for $i = 1, \ldots, n\}$ if there is a pure-strategy equilibrium with all products having positive quality levels for $n = 2$, and $\tilde{n}(\alpha, \beta, K) = 1$ otherwise.

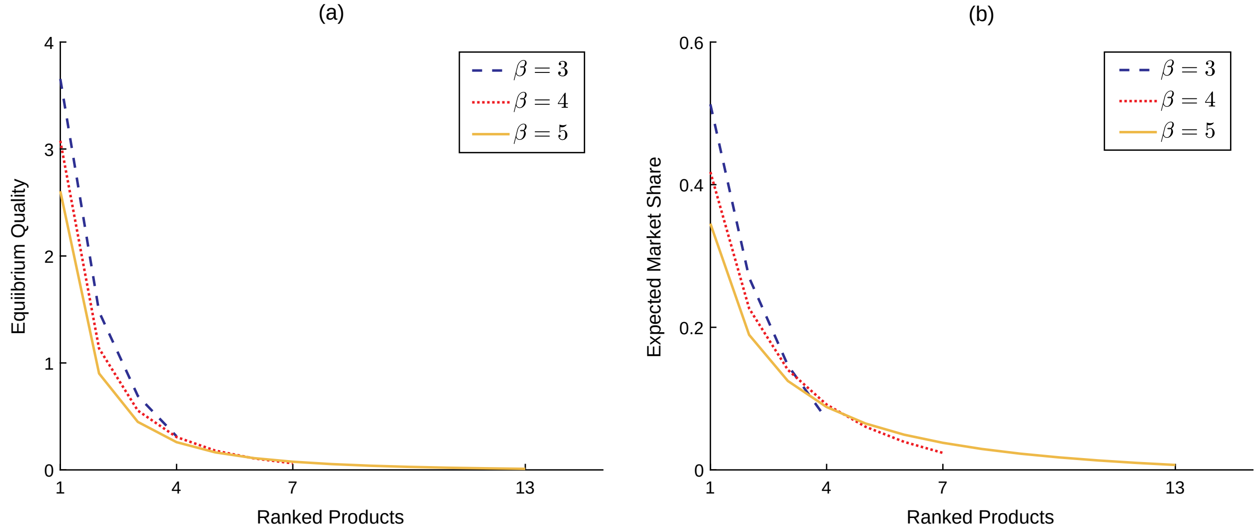
Proposition 9. *Suppose $n = 2$. There exist $\hat{\beta}_1 < 1$ and $\hat{\beta}_2 > 1$ such that the two-firm game has a unique pure-strategy equilibrium in which the two firms offer nonzero quality levels if $\beta \leq \hat{\beta}_1$ or $\beta \geq \hat{\beta}_2$. In the equilibrium, firm 1's equilibrium quality is increasing in β for $\beta < \hat{\beta}_1$ and decreasing in β for $\beta > \hat{\beta}_2$, and firm 2's expected profit is decreasing in β for $\beta < \hat{\beta}_1$ and increasing in β for $\beta > \hat{\beta}_2$.*

Proposition 9 shows that in the case of two firms, as β increases, the equilibrium budget strategy for the high-efficiency firm (i.e., firm 1) follows the same trends as those observed in the base model. For cases involving more than two firms, later in this section, we conduct numerical experiments to examine how firms' budget strategies vary as the network effect parameter changes (Figures 4 and 5).

5.4.3. First Stage: Product Variety. We can also see from Proposition 9 that the expected profit of the low-efficiency firm (i.e., firm 2) in equilibrium decreases as β moves to one from either side of one but increases as β moves away from one in either direction. Consequently, it is intuitive that the lower-efficiency firm (i.e., firm 2) has more incentive to join the competition, and hence, the product variety becomes wider as β moves away from one in either direction. Later in this section, we resort to numerical experiments to fully identify the monotonicity of the equilibrium product variety with respect to the network effect parameter (Table 2).

5.4.4. Numerical Experiments for First and Second Stages. In our numerical studies, we consider a market where there are infinitely many potential firms indexed

Figure 5. (Color online) Quality/Expected Market Share Distributions in Asymmetric Equilibria When $\beta > 1$



Notes. (a) Quality distributions. (b) Expected market share distributions. $p = 1$, $\alpha = 0.5$, $K = 0.0002$, and $c_i = 0.05i$, $i = 1, 2, \ldots$.

as $1, 2, \ldots$. For $i = 1, 2, \ldots, c_i = 0.05i$. Other parameters are $p = 1$, $\mu = 2$, $\alpha = 0.8$, and $K = 0.0002$. For each $n(\geq 2)$, we compute the equilibrium quality vector by solving the FOCs. If the solution to the FOCs can ensure firm n's expected profit is nonnegative, then the solution is a pure-strategy equilibrium. The market capacity $\tilde{n}(\alpha, \beta, K)$ can be obtained by finding the largest integer n such that the solution to the FOCs of the n-firm game in the second stage can ensure firm n's expected profit to be nonnegative. The numerical results are summarized in Table 2 and Figures 4 and 5.

In Table 2, we compute values of $\tilde{n}(\alpha, \beta, K)$ for a set of values of β. This table shows how the market capacity (i.e., product variety) depends on the strength of the network effect. In line with Proposition 2, (ii) and (iii), Table 2 further confirms the insight that the market capacity $\tilde{n}(\alpha, \beta, K)$ is decreasing in the network effect parameter β when $\beta < 1$ and increasing in β when $\beta > 1$. That is, the equilibrium product variety has a U-shaped relationship with the network effect parameter β.

Figure 4 depicts the evolution of the equilibrium quality rank distribution and the corresponding market share rank distribution as the network effect parameter β increases when $\beta < 1$. In each figure, the horizontal axis represents products ranked by their marginal costs for improving the quality from low to high. For example, 25 represents the product with parameter $c_{25} = 0.05 \times 25$.

The first observation from Figure 4 is that all products' quality ranks and expected market share ranks are consistent with the ranks of their marginal costs for improving the quality. Naturally, higher-efficiency firms, namely those with lower marginal costs for improving the quality, will offer higher quality and have a larger expected market share in the (asymmetric) equilibrium, regardless of the network parameter. The second observation is that when the network effect parameter increases from 0.2 to 0.4, in the quality and market share rank distributions, the head becomes bigger, and the tail becomes shorter, consistent with the blockbuster phenomenon. For each value of β, there are $\tilde{n}(\alpha, \beta, K)$ firms entering the market (see Table 2 for the value of $\tilde{n}(\alpha, \beta, K)$). These firms are relatively operationally efficient and have sufficiently low marginal costs for quality improvement. Third, we also numerically observe that, as β increases but is still below one, the quality levels, the market shares, and the profits of the first several most efficient firms become higher, and the low-efficiency firms are gradually forced out of the market.[7] However, the quality levels, the market shares, and the profits of the moderate-efficiency firms may become higher or lower as β increases, unlike the case of symmetric firms. Finally, we emphasize that when $\beta < 1$, the ranking of ex post expected market shares is indeed consistent with the ex ante quality ranking, that is, a firm with a higher quality level has a higher expected market share.

Next, we consider the equilibrium quality rank distribution and the expected market share rank distribution in the case when $\beta > 1$. The numerical results are

Table 2. Relationship Between Network Effect and Product Variety

	0.2	0.3	0.4	3	4	5
$\tilde{n}(\alpha, \beta, K)$	77	25	8	4	7	13

Note. $p = 1$, $\mu = 2$, $\alpha = 0.5$, $K = 0.0002$, and $c_i = 0.05i$, $i = 1, 2, \ldots$.

displayed in Figure 5. We have analytically shown in Lemma 1(iii) that the ex post market share ranking along a sample path is not necessarily consistent with the ex ante quality ranking, that is, the firm with a high-quality level can have zero market share, whereas the firm with a low-quality level can capture the entire market. Figure 5 illustrates that the *expected* market share ranking still matches with the quality ranking, that is, the expected market share for each firm is increasing in its own quality level, consistent with Lemma 2 for symmetric firms. Furthermore, one distinct difference between the cases of $\beta < 1$ and $\beta > 1$ is that when the network effect parameter increases from two to four, in the quality and expected market share rank distributions, the head becomes smaller, and the tail becomes longer.

5.5. How the Number of Customers Affects the Threshold of β

Thus far, our main results are obtained in an infinite-period model with the market outcome captured as in the limit. In the infinite-period model, we have classified network effects into dominated and dominant according to their strength measured by the parameter β. Thus, $\beta = 1$ is a cutoff between the two categories. On different sides of this cutoff, the equilibrium results in the quality competition and the entry game have different properties. In reality, a market has only a finite number of customer arrivals. This section studies how the number of customers affects the threshold of β and adopts the same assumption as in the base model that all firms are symmetric.

Let T be the total number of periods (or customers), and $\beta_{threshold}$ be the cutoff between two categories of network effects. The market share of product i is defined as the fraction of customers, out of the total T, who purchase the product. Let $x_i^*(\mathbf{q}^\alpha, \beta, T)$ denote the product i's market share, for $i = 1, \ldots, n$, and $q^*(n, \alpha, \beta, T)$ be the symmetric equilibrium quality. To show how $\beta_{threshold}$ depends on T, it suffices to plot curves of $\frac{\partial \mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$, instead of the equilibrium $q^*(n, \alpha, \beta, T)$, with different T. There are two reasons: (a) given n, $q^*(n, \alpha, \beta, T)$ may be undefined for some value of β because a pure-strategy equilibrium may not exist (see Theorems 1(iii) and 2(iii)), and (b) through the first-order conditions of all firms' profit maximization problems, we have $q^*(n, \alpha, \beta, T) = \frac{p}{c}\frac{\partial \mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$. Hence, $\frac{\partial \mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$ has the same monotonicity in β as $q^*(n, \alpha, \beta, T)$ (if it exists) that has an inverted U-shaped relationship with β. From this property, we can also see that one can directly compute the equilibrium results of the first and second stages and does not need to simulate the third-stage results.

Here, we will briefly introduce how to compute $\frac{\partial \mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$. Let ζ_t be the product selected at time t. By exhaustively enumerating all possible values of ζ_t, we can compute $\mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)$. Take $n = 2$, $T = 2$ as an example. Because $\mathbf{d}(0) = \mathbf{1}$, the total number of customers is four. We have

$$\mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T) = \mathbb{E}\frac{1 + \sum_{t=1}^T 1_{\{\zeta_t = 1\}}}{T + n} = \frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha}\frac{q_1^\alpha 2^\beta}{q_1^\alpha 2^\beta + q_2^\alpha}\frac{3}{4} + \frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha}\frac{q_2^\alpha}{q_1^\alpha 2^\beta + q_2^\alpha}\frac{1}{2} + \frac{q_2^\alpha}{q_1^\alpha + q_2^\alpha}\frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha 2^\beta}\frac{1}{2} + \frac{q_2^\alpha}{q_1^\alpha + q_2^\alpha}\frac{q_2^\alpha 2^\beta}{q_1^\alpha + q_2^\alpha 2^\beta}\frac{1}{4},$$

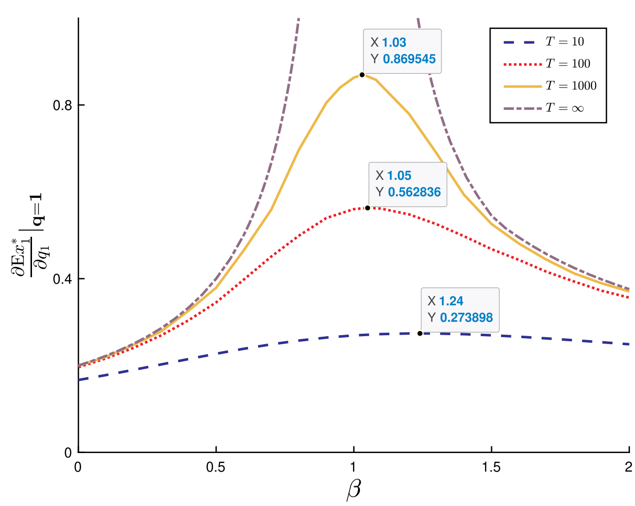
where $\frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha}\frac{q_1^\alpha 2^\beta}{q_1^\alpha 2^\beta + q_2^\alpha} = \mathbb{P}\{\zeta_1 = 1, \zeta_2 = 1 | \mathbf{q}\}$. Thus,

$$\frac{\partial \mathbb{E} x_1^*(\mathbf{q}^\alpha, \beta, T)}{\partial q_1}\bigg|_{\mathbf{q}=\mathbf{1}} = \mathbb{E}\left[\frac{1 + \sum_{t=1}^T 1_{\{\zeta_t = 1\}}}{T + n} \sum_{t=1}^T \left(1_{\{\zeta_t = 1\}} - \mathbb{P}\{\zeta_t = 1 | \mathbf{q} = \mathbf{1}, \mathbf{d}(t-1)\}\right)\right].$$

We randomly draw a sufficiently large number of samples (say, 10^8 samples) and then approximate the previous expectation using the sample mean. The number of all possible values for ζ_t can be extremely large when n and T are large, approximately on the order of $O(n^T)$. Therefore, when n and T are small, this approximation method yields good results.

From Figure 6, we see that $\beta_{threshold}$ is decreasing in T and converges to one as $T \to \infty$. Table 3 confirms this observation and, in addition, demonstrates that $\beta_{threshold}$ is increasing in n. These results imply that when we say a network effect is dominated (or dominant), we need

Figure 6. (Color online) How $\frac{\partial \mathbb{E} x_1^*}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$ Depends on β and T



Notes. $\alpha = 0.8$ and $n = 2$. The maximum point of each curve is highlighted. When $T = \infty$, $\frac{\partial x_1^*}{\partial q_1}\big|_{\mathbf{q}=\mathbf{1}}$ goes to ∞ as $\beta \to 1$.

Table 3. Cutoff $\beta_{threshold}$

	$T = 2$	$T = 10$	$T = 100$	$T = \infty$
$n = 2$	1.58	1.24	1.05	1
$n = 3$	2.00	1.48	1.15	1
$n = 4$	2.32	1.67	1.23	1

to consider the number of firms n and the market size T besides the parameter β.

6. Conclusion

We formulate a three-stage game to study how the growing network effect may affect the evolution of the market structure in cultural markets such as the online game industry. We show that an increasing network effect always contributes to the sales concentration on a small number of products regardless of whether the current network effect is weak or strong (which is the blockbuster phenomenon). We further show several phase-changing phenomena on equilibrium outcomes with respect to the network effect's strength. Specifically, the equilibrium product variety has a U-shaped relationship, and the equilibrium quality has an inverted U-shaped relationship with the strength of the network effect. This can be used to unify the long tail theory and the blockbuster phenomenon to some extent. Our findings have implications for the emerging creator economy, for example, how to incentivize creators to implement the high-budget or low-budget strategy through moderating the network effect, such as modifying the ranking or recommendation algorithms.

We choose the urn model over other models for the following reasons. When the network effect is particularly strong, meaning that network benefits grow rapidly as the sales increase, the market outcome often has multiple equilibria (Fudenberg and Tirole 2000, Jackson and Yariv 2007, Hu et al. 2020). Many studies, in order to ensure the uniqueness of equilibrium, typically assume a relatively weak network effect (Wang and Wang 2017). Our model, based on the urn model, is capable of capturing the probability distribution of multiple equilibrium points when the network effect is strong. This technical advantage aids in addressing our research questions. Some models, such as Hu et al. (2016) built on the Hotelling model and Fleder and Hosanagar (2009) built on the random-walk model, can also depict the probability distribution of multiple equilibrium points, but these models are primarily suitable for situations involving only two products.

We validate the robustness of the nonmonotonic impacts of the network effect by exploring various modeling variations in Section 5. Additionally, Ceyhan et al. (2011) also observe a phase-changing phenomenon similar to the one identified in Lemma 1, but under other general models such as MNL choice models. Because this phase-changing phenomenon serves as the cornerstone for our primary findings, we believe that the nonmonotonic impacts of the network effect stem from the nature of the problem rather than our specific model and assumptions.

Our results have limitations. First, we can demonstrate the existence of more products in the market, that is, the longer tail, due to the growing network effect. However, the impact on the market share rank distributions (see Endnote 1) remains unclear. Second, for tractability, we assume in the base model that firms are ex ante symmetric, and there is no outside option for customers. When these assumptions are relaxed, we demonstrate analytically that some of our insights obtained from the base model still hold, and we also provide numerical evidence that supports other insights. However, it would be ideal to show those insights analytically in a more general setting. Third, we do not consider customer heterogeneity in tastes in the sense that customers in different segments may have different choice behaviors.

Furthermore, we assume that firms make entry decisions simultaneously. In reality, firms may enter the market at different times, which means that some entrants may compete with incumbents who have already established a presence. Studying this scenario analytically within the framework of urn models poses challenges, as it requires solving a four-stage game instead of the three-stage game presented in the paper. However, our existing findings can shed some light on the sequential problem. In Section 5.3, for instance, if we view the outside option as a product offered by an incumbent, we observe that new entrants' entry decisions exhibit the same trends (not necessarily so for their quality decisions) as in the base model when the parameter β increases, for any given decision q_0 made by the incumbent. When the incumbent's best response is taken into consideration, it remains unclear whether our main insights would still hold. Rigorous examination of this scenario may require alternative modeling approaches or solution techniques, which we leave for future research.

We further point out some future research directions. First, it can be a fruitful direction to study how firms would dynamically run advertisements/promotions to influence customers' choices in the stochastic sales process as a Pólya urn process. Second, in our model, we assume the strength of the network effect is exogenously given. In the platform economy, a platform of products or user-generated content can influence the strength of the network effect, for example, through recommendation/ranking algorithms and review systems. In a competitive content-creator economy, we expect in equilibrium, one platform may reinforce the winners-take-all phenomenon to attract head influencers while another platform may focus on nurturing small content creators. In addition, on these platforms, there are multiple stakeholders, such as users on both supply and demand sides and the platform itself. We leave it to future research on how a monopoly platform or competing platforms should moderate the network effect under different objectives that may take into account user welfare.

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Endnotes

[1] With a higher total sales or there is more product variety in a market over time, it may not be fair to compare the sales rank distributions, in which the horizontal axis has the individual products ranked according to their sales and the vertical axis corresponds to their corresponding sales. An alternative is to consider the market share (normalized) rank distribution, where the horizontal axis is the normalized rank position in percentage, and the vertical axis corresponds to the market share. Although the tail of the *sales rank distribution* becomes longer and fatter, the tail of the *market share rank distribution* can become thinner.

[2] The term "quality" in economics and marketing is often referred to as a measure of vertically differentiating a group of products in the sense that a high-quality product generates higher surplus than a low-quality product for all customers. We abuse the term slightly and interpret "quality" as the *intrinsic appeal* of a product.

[3] We numerically show that our main insight on the endogenized quality level holds for a sequential movement game between two competing firms.

[4] Ceyhan et al. (2011) have studied the traditional logit choice model (with an exponential form of attractions), but their model does not have closed-form results, thus making it challenging to build on their model to study our research questions. However, both their and our models reveal a phase-changing phenomenon (see Lemma 1). In fact, this phase-changing phenomenon is the key driver of our main qualitative results. Therefore, the main qualitative results of our paper do not hinge on the specific choice model.

[5] It may be hard to imagine how to derive the demand vector $\mathbf{x}^*(\mathbf{m}, \mathbf{q}^\alpha, \beta)$ under a discrete choice model when the number of choices is continuous. Actually, we can obtain the closed-form expression for $\mathbb{E}\mathbf{x}^*(\mathbf{m}, \mathbf{q}^\alpha, \beta)$ by Lemma 1 when $\mathbf{m}$ is discrete. Then we treat the arguments $\mathbf{m}$ of $\mathbb{E}\mathbf{x}^*(\mathbf{m}, \mathbf{q}^\alpha, \beta)$ as being continuous.

[6] In our numerical experiments, we have never observed multiple equilibria.

[7] Interestingly, when the network effect is large enough (in the case when $\beta > 1$), the low-efficiency firms find it profitable to enter the market again.

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Yinbo Feng is an associate professor at the Research Institute for Interdisciplinary Sciences, School of Information Management and Engineering, Shanghai University of Finance and Economics. His research interests lie in supply chain management and operations management in a social context.

Ming Hu is the University of Toronto Distinguished Professor of Business Operations and Analytics and a professor of operations management at the Rotman School of Management, University of Toronto.