Online Appendix to "Product and Pricing Decisions in Crowdfunding"

A. Simultaneous versus Sequential Models

Sequential mechanism assumes that two buyers arrive at the proposed project at different periods and that the second buyer can observe the first buyer's decision. This is a common feature of all of the well-known crowdfunding and group-buying sites. An alternative format is a simultaneous setting where each of the two buyers makes decisions without knowing the other's action. The two buyers may arrive at the project and make sign-up decisions simultaneously, or they may arrive and make decisions sequentially, but the second buyer is not informed of the first one's decision. In essence, simultaneous and sequential settings are two alternative information management mechanisms. The simultaneous mechanism is not commonly observed in practice, but it has been studied in the literature (see, e.g., Hu et al. 2013).

For margin and volume strategies, the creator's optimal prices and profits remain the same. We show that the intertemporal strategy is dominated by the other strategies in the simultaneous model. To avoid confusion, we use superscript "$\sim$" to denote strategies in the simultaneous model.

LEMMA A.1. *The intertemporal strategy is dominated by other strategies in the simultaneous model.*

Proof of Lemma A.1. The profit of the intertemporal strategy in the simultaneous model is the same as that of the sequential mode, i.e., $\pi^{\tilde{D}} = \alpha(H+L)$. We prove by contradiction. Assume that for some α, $\pi^{\tilde{D}} > \pi^{\tilde{M}}$ and $\pi^{\tilde{D}} > \pi^{\tilde{H}}$. Notice that $\pi^{\tilde{M}} \geq 2\alpha(H - \alpha H + L)$ (as given by Proposition A.1), then from $\pi^{\tilde{D}} > \pi^{\tilde{M}}$, we have $H + L < 2\alpha H$. Moreover, since $\pi^{\tilde{H}} = 2\alpha^2 H$, from $\pi^{\tilde{D}} > \pi^{\tilde{H}}$, we have $H + L > 2\alpha H$, which is a contradiction. □

Due to Lemma A.1, our analysis focuses on the menu strategy.

PROPOSITION A.1. *In the simultaneous model,*

(i) if $\frac{H+L}{2\alpha H} \geq 1$, the optimal menu strategy is $p_h^{\tilde{M}} = (1-\alpha)H + \alpha L$, $p_l^{\tilde{M}} = L$. The optimal strategy induces a pure strategy equilibrium of buyers, in which high-type buyers always choose $p_h^{\tilde{M}}$. The corresponding expected profit is $\pi^{\tilde{M}} = 2\alpha(H - \alpha H + L)$;

(ii) Otherwise, the optimal menu strategy is $p_h^{\tilde{M}} = \frac{H^2+L^2}{2H}$, $p_l^{\tilde{M}} = L$. The optimal strategy induces a mixed strategy equilibrium of buyers, in which high-type buyers choose $p_h^{\tilde{M}}$ with probability $\gamma = \frac{H+L}{2\alpha H}$. The corresponding expected profit is $\pi^{\tilde{M}} = \frac{(H+L)^2}{2H}$.

Proof of Proposition A.1. We consider pure strategy equilibrium first. High-end buyers always pay at $p_h^{\tilde{M}}$, and low-type buyers always pays at $p_l^{\tilde{M}}$. To avoid deviation from equilibrium, high-type buyers have no incentive to purchase at a low price, i.e., $H - p_h^{\tilde{M}} \geq \alpha(H - p_l^{\tilde{M}})$. Note that if one high-type buyer deviates and buys at low, she is hoping that the other buyer's type is high, at which the project will succeed. The strategy is optimal when the above inequality and $p_l^{\tilde{M}} \leq L$ are binding, yielding $p_h^{\tilde{M}} = (1-\alpha)H + \alpha L$ and $p_l^{\tilde{M}} = L$. Note that the creator's profit is computed differently than in the sequential model: $\pi^{\tilde{M}} = \alpha^2(2p_h^{\tilde{M}}) + 2\alpha(1-\alpha)(p_h^{\tilde{M}} + p_l^{\tilde{M}})$. That is because the creator may obtain more that the target $T^{TM} = p_h^{\tilde{M}} + p_l^{\tilde{M}}$ when both buyers hold high valuations.

Next, consider any pricing strategy with $(H - p_h^{\tilde{M}}) \leq \alpha(H - p_l^{\tilde{M}})$, $H \geq p_h^{\tilde{M}} \geq L \geq p_l^{\tilde{M}}$. Then it is straightforward that there is no symmetric pure-strategy Nash equilibrium. Suppose that a high-type buyer purchases at $p_h^{\tilde{M}}$ with probability γ, and $p_l^{\tilde{M}}$ with probability $1-\gamma$. Then, we have the indifference condition $H - p_h^{\tilde{M}} = \gamma\alpha(H - p_l^{\tilde{M}})$, which yields that $p_h^{\tilde{M}} = H - \alpha\gamma H + \alpha\gamma p_l^{\tilde{M}}$. The profit, as a function of γ, is $\pi^{\tilde{M}}(\gamma) = (\alpha\gamma)^2(2p_h^{\tilde{M}}) + 2\alpha\gamma(1-\alpha\gamma)(p_h^{\tilde{M}} + p_l^{\tilde{M}})$, which is maximized when $p_l = L$, yielding the expected profit $\pi^{\tilde{M}}(\gamma) = 2\alpha\gamma(H - \alpha\gamma H + L)$.

Note that $\frac{\partial \pi^{\tilde{M}}}{\partial \gamma} = 2\alpha(H - 2\alpha\gamma H + L)$. When $\frac{H+L}{2\alpha H} \geq 1$, the optimal strategy is in pure-strategies: $p_h^{\tilde{M}} = (1-\alpha)H + \alpha L, p_l^{\tilde{M}} = L, \pi^{\tilde{M}} = 2\alpha(H - \alpha H + L)$, otherwise, the optimal strategy is in mixed-strategies:

$$p_h^{\tilde{M}} = \frac{H^2 + L^2}{2H}, p_l^{\tilde{M}} = L, \pi^{\tilde{M}} = \frac{(H+L)^2}{2H}. \quad \square$$

Comparing Proposition A.1 with Lemma 1, we obtain the following profitability comparison under different information structures.

COROLLARY A.1. *$\pi^{\tilde{M}} \geq \pi^M$, with the inequality strict when $\alpha > 0$.*

Proof of Corollary A.1. When $\frac{H+L}{2\alpha H} \geq 1$, we have $\pi^{\tilde{M}} - \pi^M = (1-\alpha)\alpha^2(H-L) \geq 0$, and the inequality is strict when $\alpha > 0$. Otherwise, notice that $\pi^{\tilde{M}} - p^M \geq 2\alpha(H - \alpha H + L) - \pi^M = (1-\alpha)\alpha^2(H-L) \geq 0$. $\square$

Corollary A.1 shows that when the creator offers the menu strategy, disclosing cumulative purchase information is worse off than no disclosure.[A.1] This insight is similar to that of Varian (1994). In both sequential and simultaneous settings, the creator prices exactly at the same levels (i.e., $p_h^M = p_h^{\tilde{M}}$, $p_l^M = p_l^{\tilde{M}}$). In the sequential menu setting, a high-type later arrival B_2 may free-ride the purchase made by the earlier arrival. This buyer will always pay the low price if the first buyer B_1 has already paid the high price in the first period. However, in a simultaneous setting, where the previous contribution cannot be observed, such free-riding does *not* occur and the creator's

[A.1] Sequential mechanism can out-perform simultaneous mechanism in alternative model formulations. For example, if buyers experience the warm-glow effect discussed in Section 4.1, the free-riding incentive is diminished, and a sequential mechanism can lead to better outcomes (see, e.g., Romano and Yildirim 2001).

expected profit will be higher. This result is in contrast to Hu et al. (2013), where sequential information disclosure is more profitable. This is because in Hu et al. (2013), each consumer makes one unit of purchase and a price is fixed *exogenously*; as a result, such a free-riding incentive by paying "less" after knowing the other's contribution does not exist.

Proposition A.1 also shows that creator's profit can be higher under a mixed-strategy equilibrium of buyers. If $\alpha > \frac{H+L}{2H}$, the buyers are more likely to be of the high type. When a buyer expects the other one to have a high product valuation, she will have a greater incentive to choose the low-price option. Interestingly, the creator can deter such behavior and gain more by allowing the buyers to play mixed strategies. Of course, in reality, it may not be easy to induce the buyers to do so.

Next, in the simultaneous setting, we compare the menu strategy with the other three strategies and obtain the following corollary.

COROLLARY A.2. *In the simultaneous setting, the optimal strategy is*

(i) volume strategy, if $\frac{H}{L} \le \frac{1}{\alpha}$*;*

(ii) menu strategy, if $\frac{1}{\alpha} \le \frac{H}{L}$ *and* $\alpha \le \frac{1}{2}$*, or* $\frac{1}{\alpha} \le \frac{H}{L} \le \frac{1}{2\alpha-1}$*;*

(iii) margin strategy, if $\alpha > \frac{1}{2}$ *and* $\frac{H}{L} \ge \frac{1}{2\alpha-1}$.

Proof of Corollary A.2. When $\frac{H+L}{2\alpha H} \ge 1$, i.e., $\frac{H}{L} \le \frac{1}{2\alpha-1}$ or $\alpha < \frac{1}{2}$, $\pi^{\tilde{M}} - \pi^L = 2(1-\alpha)(\alpha H - L)$, resulting in that the menu strategy outperforms the volume strategy when $\frac{H}{L} \le \frac{1}{\alpha}$. When $\frac{H+L}{2\alpha H} \le 1$, $\pi^{\tilde{M}} - \pi^L \ge 2(1-\alpha)(\alpha H - L) \ge (1-\alpha)(H-L) \ge 0$. Hence, the menu strategy is no worse than the volume strategy when $\frac{H}{L} \le \frac{1}{\alpha}$. Next we compare the menu strategy with the margin strategy. When $\frac{H+L}{2\alpha H} \le 1$, we have $\pi^{\tilde{M}} - \pi^H = \frac{(H+L)^2}{2H} - 2\alpha^2 H$, which is greater than 0 when $\frac{H}{L} \le \frac{1}{2\alpha-1}$, or $\alpha < \frac{1}{2}$. When $\frac{H+L}{2\alpha H} \ge 1$, $\pi^{\tilde{M}} - \pi^H = 2\alpha(H - 2\alpha H + L)$, which is greater than 0 when $\frac{H}{L} \le \frac{1}{2\alpha-1}$, or $\alpha < \frac{1}{2}$. Combining all above, we have the desired result. □

We plot the optimal strategy under different market conditions in Figure A.1. The dashed lines are the original dividing lines between the menu strategy and other strategies in the base model. Following our discussions earlier, the menu strategy becomes preferable over a *larger* parameter space in the simultaneous setting than in the sequential setting.

Different information management mechanisms (simultaneous versus sequential) may have implications on product line design, too. We let the creator decide the product qualities in the simultaneous setting and summarize the results in Proposition A.2.

PROPOSITION A.2. *In the simultaneous setting, when quality is endogenized,*

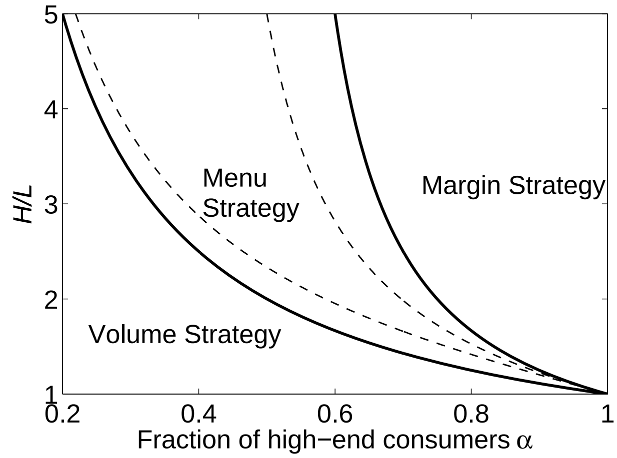

Figure A.1 Optimal Strategy with Different Values of H/L and α under Simultaneous Mechanism

(i) if $\alpha H \le L$, the creator's optimal decision under the menu strategy is

$$(Q_h^{\tilde{M}}, Q_l^{\tilde{M}}) = \Big(H, \frac{L-\alpha H}{1-\alpha}\Big), (P_h^{\tilde{M}}, P_l^{\tilde{M}}) = \Big(\frac{(1-\alpha+\alpha^2)H^2 - \alpha(1+\alpha)HL + \alpha L^2)}{1-\alpha}, \frac{(L-\alpha H)L}{1-\alpha}\Big).$$

The corresponding expected profit is

$$\Pi^{\tilde{M}} = \frac{\alpha((1-\alpha+\alpha^2)H^2 - 2\alpha HL + L^2)}{1-\alpha};$$

(ii) otherwise, the optimal menu strategy reduces to offering a single product.

Proof of Proposition A.2. Now we study the product line design problem in a simultaneous context. We first consider pure-strategy equilibrium. The creator sets qualities at $Q_l^{\tilde{M}} < Q_h^{\tilde{M}}$ and prices at $P_l^{\tilde{M}} < P_h^{\tilde{M}}$. Since only pure-strategy is considered, $HQ_h^{\tilde{M}} - P_h^{\tilde{M}} \ge \alpha(HQ_l^{\tilde{M}} - P_l^{\tilde{M}})$, and it is not difficult to calculate the creator's optimal pricing strategy: $P_l^{\tilde{M}} = LQ_l^{\tilde{M}}, P_h^{\tilde{M}} = \alpha LQ_l^{\tilde{M}} + HQ_h^{\tilde{M}} - \alpha HQ_l^{\tilde{M}}$. The profit function is as follows:

$$\Pi^{\tilde{M}} = \alpha^2(2P_h^{\tilde{M}} - (Q_h^{\tilde{M}})^2) + 2\alpha(1-\alpha)\big(P_h^{\tilde{M}} + P_l^{\tilde{M}} - \frac{(Q_h^{\tilde{M}})^2}{2} - \frac{(Q_l^{\tilde{M}})^2}{2}\big). \tag{A.1}$$

The first order conditions give the optimal solution

$$(Q_h^{\tilde{M}}, Q_l^{\tilde{M}}) = \Big(H, \frac{L-\alpha H}{1-\alpha}\Big), (P_h^{\tilde{M}}, P_l^{\tilde{M}}) = \Big(\frac{(1-\alpha+\alpha^2)H^2 - \alpha(1+\alpha)HL + \alpha L^2)}{1-\alpha}, \frac{(L-\alpha H)L}{1-\alpha}\Big),$$

$$\Pi^{\tilde{M}} = \frac{\alpha((1-\alpha+\alpha^2)H^2 - 2\alpha HL + L^2)}{1-\alpha}.$$

Next we consider mixed-strategies. Suppose now that the creator prices at $H \ge P_h^{\tilde{M}} \ge L \ge P_l^{\tilde{M}}$ such that $HQ_h^{\tilde{M}} - P_h^{\tilde{M}} \le \alpha(HQ_l^{\tilde{M}} - P_l^{\tilde{M}})$. The nonexistence of symmetric pure strategy Nash equilibrium is straightforward. Suppose that high-type buyers choose to pay at $P_h^{\tilde{M}}$ with probability

γ; then we have the indifference condition $H - P_h^{\tilde{M}} = \alpha\gamma(H - P_l^{\tilde{M}})$. Profit is still maximized when $P_l^{\tilde{M}} = LQ_l^{\tilde{M}}$. Solving for $P_h^{\tilde{M}}$ we have $P_h^{\tilde{M}} = HQ_h^{\tilde{M}} - \alpha\gamma HQ_l^{\tilde{M}} + \alpha\gamma LQ_l^{\tilde{M}}$. Replacing the related parameters to the profit function – which is still given by Equation (A.1) – we have the creator's profit. Then the first order conditions give the optimal qualities,

$$Q_l^{\tilde{M}} = \frac{L - \alpha\gamma H}{1 - \alpha\gamma}, Q_h^{\tilde{M}} = H,$$

where $\alpha\gamma H \leq L$ (otherwise we have $Q_l^{\tilde{M}} = 0$). Then, the expected profit function is

$$\Pi^{\tilde{M}} = \frac{\alpha\gamma((1 + \alpha\gamma(-1 + \alpha\gamma))H^2 - 2\alpha\gamma HL + L^2)}{1 - \alpha\gamma}.$$

Let $\eta = \alpha\gamma$ and we have $\Pi^{\tilde{M}} = \eta((1 - \eta + \eta^2)H^2 - 2\eta HL + L^2)/(1 - \eta)$. Note that $\partial\Pi^{\tilde{M}}/\partial\eta = 2HL + \frac{(H-L)^2}{(1-\eta)^2} - 2H^2\eta \geq 2H(L - H\eta)$, which implies that $\Pi^{\tilde{M}}$ is increasing in η when $L - H\eta \geq 0$. Moreover, note that when $L - H\eta \leq 0$, we have $\eta H = \alpha\gamma H \geq L$, which violates the assumption that $\alpha\gamma H \leq L$. Since α is fixed, a larger γ implies a higher η, and hence a higher payoff. When $\alpha H \leq L$, profit is maximized when $\gamma = 1$. Therefore we are back to a pure strategy. Therefore, the optimal strategy must be in pure strategy, which completes the proof. □

Given our emphasis on product-line and pricing decisions, $\alpha H \leq L$ is the more relevant case. Surprisingly, the creator's optimal product decisions are exactly the same as in the traditional model. In other words, the optimal quality levels in the simultaneous setting, as shown in the above proposition, are identical to those of the traditional setting shown in Lemma 2. Hence, the optimal quality gap for the product line is *greater* in the simultaneous setting than that in the sequential setting (i.e., $Q_h^{\tilde{M}} - Q_l^{\tilde{M}} \geq Q_h^M - Q_l^M$). As discussed earlier, a free-riding incentive exists in the sequential setting, but not in the simultaneous setting. In the absence of free riding, the creator extracts more surplus from high-type buyers in the simultaneous setting. Here the creator keeps the low-option product quality $Q_l^{\tilde{M}}$ lower to sustain a sufficiently high price $P_h^{\tilde{M}}$ for the high-option product.

Although the optimal quality levels in simultaneous setting are *identical* to those in the traditional setting, the optimal prices are different. Specifically, $P_l^{\tilde{M}} = P_l^T$, $P_h^{\tilde{M}} - P_h^T = (H - L)(L - \alpha H) \geq 0$. In other words, the optimal price of a high-quality product is *higher* in simultaneous crowdfunding than in the traditional setting, even though the same levels of product quality are offered. This difference is caused by the nature of the crowdfunding mechanism. In addition to the original forces that lead to product differentiation in the traditional model of product line design, here the buyers can also be persuaded to pay more to meet the common goal of project success.

B. Overfunding

When the second cohort has a random size, consider now the case $n_2^L \leq n_1 \leq n_2^H$. Under this case, the size of the second cohort can be either less than or greater than that of the first cohort. If the creator wishes to induce the first cohort to pay the high price to ensure success, she sets $T^{M_o} = n_2^L p_h^{M_o} + n_1 p_l^{M_o}$. And to induce self-selection, it is sufficient (not always necessary) that

$$n_1 H - n_2^L(p_h^{M_o} - p_l^{M_o}) - n_1 p_l^{M_o} \geq (\alpha + \beta - \alpha\beta) n_1 (H - p_l^{M_o}),$$

which yields that

$$p_h^{M_o} = \min(H, (1-\alpha)(1-\beta)(H-L)\frac{n_1}{n_2^L} + L), p_l^{M_o} = L.$$

Overpayment always occurs when the first cohort has high valuation and $N_2 = n_2^H$.

Suppose now that the size of the first cohort is random, following a two-point distribution: $N_1 = n_1^H$ with probability β, and $N_1 = n_1^L$ with probability $1-\beta$. The size of the second cohort is a constant n_2 such that $n_1^L \leq n_2 \leq n_1^H$. Suppose that the creator prices the menu at $p_h^{M_o}, p_l^{M_o}$ and sets a target $T^{M_o} = n_1^L p_h^{M_o} + n_2 p_l^{M_o}$. Then, to motivate the first cohort to choose the high price $p_h^{M_o}$ when $N_1 = n_1^L$, we have

$$n_1(H - p_h^{M_o}) \geq \alpha n_1 (H - p_l^{M_o}),$$

which yields that $p_l^{M_o} = L, p_h^{M_o} = (1-\alpha)H + \alpha L$. But note that when $N_1 = n_1^H$, the first cohort of buyers do not always choose $p_h^{M_o}$. Overpayment always occurs when $N_1 = n_1^H$ and $n_1^H p_l^{M_o} > n_1^L p_h^{M_o}$.

C. Creator's Incentive to Commit

In this section we show that when a menu strategy is chosen, it is indeed an equilibrium outcome for the creator to commit to the provision point mechanism, i.e., not to carry out the project if the target is not met.

Let $\eta \in [0,1]$ be the probability that the creator will carry on the project under the condition that the target is not met. This probability, which characterizes the belief about the creator's behavior, is common knowledge between the creator and the buyers in order to ensure belief consistency in equilibrium.

Given that the first buyer pays p_l^M, to ensure that the second buyer chooses p_h^M when her type is high, we need to impose

$$H - p_h^M \geq \eta(H - p_l^M).$$

In the first period, a high-type consumer's IC condition becomes

$$H - p_h^M \geq (\eta(1-\alpha) + \alpha)(H - p_l^M).$$

Solving these two IC conditions yields the creator's optimal pricing strategy

$$p_h^M = (1-\alpha)(1-\eta)H + (\alpha+\eta-\alpha\eta)L, p_l^M = L.$$

Note that when $\eta = 1$, the strategy becomes the volume strategy; and when $\eta = 0$, the strategy is the menu strategy.

With probability $2\alpha - \alpha^2$, the creator obtains $p_h + p_l$; and with probability $\eta(1-\alpha)^2$, the creator obtains $2p_l^M$. Therefore, the creator's expected profit from the above strategy without commitment is

$$\pi(\eta) = (2\alpha-\alpha^2)(p_h^M + p_l^M) + \eta(1-\alpha)^2(2p_l^M).$$

Simple algebra yields that

$$\frac{\partial \pi}{\partial \eta} = (1-\alpha)((\alpha^2 - 2\alpha)H + (2-\alpha^2)L),$$

which is a constant, either positive, negative or zero. Therefore, the creator optimizes the expected profit either when $\eta = 0$ or when $\eta = 1$. In other words, either the volume strategy is optimal or the menu strategy with commitment is optimal. Hence, when the menu strategy is optimal, the creator has no incentive to carry on the project when it fails.

References

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