informs
http://pubsonline.informs.org/journal/mnsc
MANAGEMENT SCIENCE
Vol. 67, No. 5, May 2021, pp. 2711–2728
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Joint vs. Separate Crowdsourcing Contests

Ming Hu,[a] **Lu Wang**[b,*]

[a] Rotman School of Management, University of Toronto, Toronto, Ontario, Canada M5S 3E6; [b] College of Business, Shanghai University of Finance and Economics, Shanghai 200433, China

*Corresponding author

Contact: ming.hu@rotman.utoronto.ca, https://orcid.org/0000-0003-0900-7631 (MH); wang.lu@mail.shufe.edu.cn, https://orcid.org/0000-0002-6825-6921 (LW)

Received: January 3, 2017
Revised: May 29, 2019; February 17, 2020
Accepted: March 28, 2020
Published Online in Articles in Advance: October 5, 2020

https://doi.org/10.1287/mnsc.2020.3683

Abstract. In a crowdsourcing contest, innovation is outsourced by a firm to an open crowd that competes in generating innovative solutions. Given that the projects typically consist of multiple attributes, how should the firm optimally design a crowdsourcing contest for such a project? We consider two alternative mechanisms. One is a *joint* contest, where the best solution is chosen from the joint solutions across attributes submitted by all contestants. The other is multiple *separate* parallel subcontests, with each dedicated to one attribute of the project. It is intuitive that the separate contest has the advantage of potentially creating a "cooperative" final solution contributed by different contestants. However, somewhat surprisingly, we show that the separate contest may reduce the incentive for the crowd to exert effort, resulting in the joint contest becoming the optimal scheme. The comparison of the expected *best* performances in the two contests depends on the project's characteristics. For example, if contestants' performances have a sufficiently high (respectively, low) level of randomness, the separate (respectively, joint) contest is optimal. If the number of contestants is large (respectively, small) enough, the separate (respectively, joint) contest is optimal. Moreover, we find that when the prize is endogenized, the optimal amount of the prize in the joint contest is no less than that in the separate contest. Finally, we extend the model to account for contestants with heterogeneous types.

History: Accepted by Gad Allon, operations management.
Funding: This work was supported by the National Natural Science Foundation of China [Grant 71901136] and the Natural Sciences and Engineering Research Council of Canada [Grant RGPIN-2015-06757].
Supplemental Material: The online appendices are is available at https://doi.org/10.1287/mnsc.2020.3683.

Keywords: game theory • crowdsourcing contest • tournament design

1. Introduction

The crowdsourcing contest has been widely adopted by firms, nonprofit organizations, and governments to solicit innovative solutions to complex problems. In a typical crowdsourcing contest, an organizer outsources its project to those who compete to provide solutions and offers a prize for the winning solution.

Because a project often has multiple attributes or dimensions, one method of crowdsourcing is to run a *separate* contest in which multiple subcontests are launched, each dealing with one attribute or dimension of the project. For example, in 2013, the Pentagon launched a contest, through a web portal called Vehicleforge.mil, for the design of an amphibious vehicle for the U.S. Marines. The first subcontest, with a $1 million prize, involved mobility and drive-train subsystems for the vehicle. About six months later came a subcontest for the design of the chassis and other subsystems, a contest with another $1 million prize (see Lohr 2012).

An alternative crowdsourcing mechanism is to run a *joint* contest, in which every contestant is required to submit his or her solution for the whole project all at once, even though the project may require contestants to deal with a number of different attributes. For example, a class of so-called reduction-to-practice challenges on InnoCentive.com, a leading innovation crowdsourcing platform, requires contestants to submit a prototype that shows an idea in actual practice—in other words, an aggregate solution that combines the theoretical work of generating ideas and the practical work of presenting physical evidence. As another example, right after the separate contest for the military vehicle mentioned earlier, the Pentagon launched a contest with a $2 million prize in 2014. In contrast to the separate contest held in 2013, this joint contest required contestants to submit a single solution for an entire vehicle (see Lohr 2012). One would assume that the separate contest had the benefit of allowing the best solution for each component to be combined, thus leading to an overall best solution. Therefore, it is puzzling why the Pentagon switched to the joint mechanism.

Motivated by the examples mentioned earlier, we study the optimal crowdsourcing contest design for projects with multiple attributes. We consider two alternative mechanisms that can be implemented by the contest organizer, hereafter referred to as *the firm*. One is to run a joint contest where the best solution is chosen from the aggregate solutions submitted by contestants. The other is to run multiple separate subcontests, each dealing with one attribute of the project; the final design is made up of the best design for each attribute.

1.1. Practical Examples

In this section, we provide more examples of contests in a separate or joint form. Figure 1 illustrates the terminology and the main focus of this study.

Modular Separate Contest. In a separate contest, if designs for individual attributes can be put together as modules, we refer to such a contest as a *modular contest*.[1] The Pentagon's vehicle design contest mentioned earlier is a modular contest. Some crowdsourcing contests on data analysis also belong to this category, such as contests on Kaggle, the platform for predictive modeling and analytics competitions. Programming solutions are largely modular because, for software design, application programming interfaces (APIs) are the methods of communication between different components of a complete solution. The API allows the solution programs to work together even if solutions are written by different contestants with different programming languages. For example, Google launched the Open Images Challenge in 2018 with two tracks. Both were parallel subcontests with the same data set. One was for object detection, and the other was for detecting the visual relationship between the detected objects. The solutions for the two tracks were modular.[2]

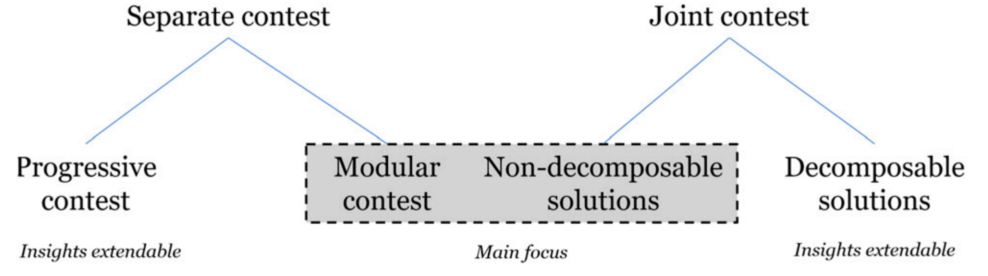
Joint Contest with Nondecomposable Solutions. In a joint contest, aggregate solutions can be *nondecomposable*. Here *decomposable* means that the firm is able to separate an aggregate solution from the joint contest into component parts so that it can combine the best parts from different solutions to form a solution that may not be achieved by any single contestant. However, not all solutions can be treated this way. For example, General Electric (GE) Hospital Quest on Kaggle requires contestants to improve the efficiencies in hospitals and to make hospital visits hassle free. Many factors come together to avoid things such as long wait times, poor communication, and repetitive paperwork. Contestants are required to submit comprehensive solutions; these are not necessarily decomposable because the approaches to specific issues are interdependent.

Modular Separate Contest and Its Joint Counterpart with Nondecomposable Solutions. It is possible for the joint counterpart of a separate modular contest to have an aggregate solution that is nondecomposable. For example, in object-oriented programming, the well-known open/closed principle requires that software entities (classes, modules, functions, etc.) should be open for extension but closed for modification. This means that it could be challenging, at the very least, to decompose programs because programmers may hide functions or features in their solutions. Hence, the firm may either be unable to decompose the solutions or prefer not to because of the high cost.[3]

1.2. Overview of Main Results

Our analysis shows that, other things being equal, there are two opposing forces affecting the comparison between the two contest mechanisms. On the one hand, because each subcontest of the separate contest focuses on one attribute of the project, the best aggregate performance is made up of the best performances in each of those subcontests. However, in the joint contest, contestants submit a single solution for all the attributes; thus, the best performance is the one submitted by a single contestant. Therefore, the combination of the best performances on different attributes in the separate contest is more likely to have a high value than the best performance in the joint contest. We call this the *combination effect*. On the other hand, the contestants' effort depends on the marginal expected gain from the contest. In the joint contest,

Figure 1. Contest Classification



the random factors that affect the performance or evaluation criteria across multiple attributes have been pooled together in the solutions provided by contestants, which leads to reduced variability under mild assumptions. As a result, the marginal expected gain from each bit of extra effort on any attribute in the joint contest is more "worthwhile" than that in the corresponding subcontest of the separate contest. This is an intuitive explanation from an individual contestant's perspective; the exact reasoning is more involved because contestants compete with each other and need to take into account competitors' behavior. We call this the *pooling effect*, driven by the pooling of multiple performances subject to random factors across different attributes in the joint contest. We show that *pooling*, as a common theme in the operations literature, has a notable application to crowdsourcing contest design.

The comparison of the expected best performances in the two contest mechanisms boils down to a comparison of those two opposing effects.[4] The pooling effect occurs because of the difference in incentives between the joint and separate contests–the different amounts of the prizes and the probability of winning that jointly determine the marginal expected gain from a contest. We find that the relative importance of the random factor (i.e., luck) and effort level (i.e., sweat) plays a critical role in the comparison between the two contest mechanisms. Some projects are highly influenced by the random factor (e.g., ideation or art-designing projects), which we call *randomness-based* projects. Some projects rely heavily on the efforts exerted by contestants (e.g., coding competitions), which we call *effort-based* projects. A contestant's incentive to make an effort depends on the relative importance of the effort to the final performance. If a project is randomness based, the contestants tend to make less effort because making much effort will incur a significant cost but gain little. As a result, the difference in incentives will likely produce only a small difference in the effort level between the two types of contests. Hence, the pooling effect is relatively weak and can easily be dominated by the combination effect. As a result, the separate contest tends to be optimal. By contrast, if a project is effort based, contestants tend to make a considerable effort because it is worth doing so; thus, the difference in incentives between the two contests can cause a significant difference in the effort levels. Therefore, the pooling effect may be more likely to dominate the combination effect, and the joint contest tends to be optimal.

We enhance our main insights by further exploring the comparison of the two contest mechanisms in the following directions. First, we investigate the magnitude of the combination effect for different numbers of contestants. We find that a greater number of contestants improves the combination effect but may not affect or may even reduce the pooling effect. Under some conditions, there exists a threshold for the number of contestants above which the combination effect dominates the pooling effect, and the separate contest is optimal, and below which the combination effect is dominated by the pooling effect, and the joint contest is optimal.

Second, we examine the optimal amount of the total prize for the two contest mechanisms. The results depend on the curvature of the cost functions (i.e., the degree of convexity). For each contest mechanism, the optimal total prize is achieved when the marginal benefit of increasing the total prize equals the marginal prize that is one. The marginal benefit for the firm is the marginal effort improvement by contestants with an increase in the prize. Because the cost functions are assumed to be convex, the marginal effort level is decreasing in the total prize. However, the decreasing rate can be different for different cost functions. For exponential cost functions, the degree of convexity is a fixed value, and thus the marginal effort is decreasing at the same rate for the two contest mechanisms. Therefore, both mechanisms have the same optimal total prize. For polynomial cost functions, the degree of convexity is decreasing in the effort level. The marginal effort is decreasing at a slower rate in the joint contest than in the separate contest because contestants make more effort in the joint contest than in the separate one. Therefore, the optimal total prize is higher for the joint contest than for the separate contest.

Finally, we examine contestants who have heterogeneous expertise using a stylized model of two contestants and two (privately known) expertise types for each attribute. Contestants' expertise in attributes can be perfectly positively or negatively correlated. We find that under some conditions, forward-looking contestants behave myopically by truthfully revealing their types in the first subcontest of the separate contest, and for either case of the perfectly positively or negatively correlated expertise, each contestant achieves a higher expected equilibrium effort level in the joint contest than in the separate contest.

2. Literature Review

The modeling of contestants' behavior has been an active research area in economics. It is also gaining traction in operations management as part of managing the crowdsourcing of goods and services. There are many different models in the contest theory, such as the random factor model, the all-pay auction model, the random trials model, and the Tullock contest. Konrad (2007) conducts a comprehensive survey of those models. With the random factor model in which a contestant's performance is made up of his or her effort and a random factor, Lazear and Rosen (1981) show that a contestant's effort depends on the incentive provided by the prize and the cost incurred by exerting effort. Building on the traditional studies, we examine the design of the contest with *multiple attributes*. In our separate contest, each subcontest focuses on one attribute of the project, and each can be viewed as a random factor model. However, in the joint contest, we introduce the multidimensional single-shot contest model in which contestants self-allocate their efforts to multiple dimensions and submit an aggregate solution simultaneously.

One of the main research questions in contest design is how to design the optimal incentive scheme by allocating the total prize to contestants whose performance can be ranked. Rosen (1986) examines the elimination contest and finds that a large enough prize is needed for the best performer. Kalra and Shi (2001) show that the effort made by an individual contestant decreases with the number of contestants or the uncertainty in the contest. If several contestants can be rewarded, the rank-order contest (i.e., a contestant with better performance is awarded a larger prize) dominates the multiple-winner contest (i.e., several top contestants share the total prize equally, even though their performance is different). The authors also consider different risk attitudes of contestants. They show that if contestants are risk neutral, *winner-takes-all* (WTA) (i.e., the best performer is awarded the total prize) becomes optimal. Moldovanu and Sela (2001) use the all-pay auction model to show that if the cost function is concave or not too convex, WTA is optimal. Terwiesch and Xu (2008) show that WTA is always optimal for the random factor model (referred to as a model of an "ideation project" in their paper, p. 1532) but may or may not be optimal for the all-pay auction model (referred to as a model of an "expertise-based project"(p. 1532)). Ales et al. (2017) combine the random trials and random factor models and find that WTA is optimal if and only if the benefit of additional effort for increasing the probability of becoming the winner is higher than that for increasing the probability of attaining other ranks. Furthermore, they show that WTA is optimal if the participation of contestants is guaranteed and the density function of the random factor is log-concave. Stouras et al. (2019) find that whether to use WTA depends on the firm's objective, to motivate participation versus to encourage contestants to make a high effort. Stouras et al. (2015) also study the service contest design in an on-demand service context where agents are ranked based on their service performance, and higher performers receive priority over incoming service requests. The authors show that a coarse partition of priority classes, such as two priority classes, can be optimal. In contrast to one-dimensional contest models, we consider a separate contest as two subcontests, with each focusing on one attribute of the project. We assume that the firm uses the WTA scheme in all subcontests to start with because the WTA scheme has been proved to be optimal in most circumstances, especially under our assumption that the random factor is symmetric log-concave with mean zero.

Another research question is how the number of contestants affects the contestants' behavior. Taylor (1995) examines a contest model in which each contestant conducts random trials to find his or her best shot that can meet a predetermined level. The author shows that an open-entry contest is not optimal because it reduces the effort of contestants in the equilibrium. Fullerton and McAfee (1999) suggest restricting the number of contestants and using the auctioning method to select the two best-qualified contestants to compete. Later, Che and Gale (2003) show that in designing the contest for procuring innovations, it is optimal to let the two most efficient innovators participate and compete and to handicap the more efficient innovator if the contestants are asymmetric. All those studies emphasize the role of random factors in the contest, as we do, and they suggest that the open-entry contest may not be optimal, so the firm needs to restrict the number of contestants. With a different model setup, Terwiesch and Xu (2008) show that more contestants intensify the competition and thus lower the individual effort, but meanwhile, the best performance can be *enhanced* by the diversity of contestants. In keeping with their result, Boudreau et al. (2011) empirically show that there is an effort-reducing effect by adding contestants. Ales et al. (2019) use the random factor model in which the random factor follows a general distribution. They find that the effort-reducing effect may or may not exist depending on the properties of the random factor's distribution. Focusing on multiattribute contests, we identify two opposing forces, one on the effort (the pooling effect) and the other on the random factor (the combination effect), which jointly influence the firm's decision on whether to combine multiple subcontests into one.

Some work examines the contestants' behavior when there is a series of contests. One stream of those studies considers *static* games in which contestants' behavior in different competitions is independent. Rosen (1986) studies a tournament in which the contestants are paired to compete, and the tournament proceeds with $\lceil \log_2(n) \rceil$ stages if there are n contestants. Konrad and Kovenock (2009) examine the equilibrium strategies in a series of competitions in which, in addition to the prize offered for each competition, there is a grand prize for overall performance. DiPalantino and Vojnovic (2009) consider a sequence of crowdsourcing contests in which contestants can choose which contest(s) to enter. Though the latter two papers are similar to ours in that the contest designer splits the contest into several subcontests, their subcontests are designed to evaluate the overall performance over a series of competitions (e.g., English Premier League) or performances for totally different projects (e.g., Yahoo Answers). Moreover, their focus is on characterizing contestants' behavior instead of the firm's decision. The separate contest in our context refers to multiple subcontests dealing with different attributes of a project, and those subcontests can have different cost functions and prizes. In this stream, Körpeoğlu et al. (2018) and Moldovanu and Sela (2006) are the closest to ours. By the all-pay auction model (contestants are heterogeneous and the performance is the effort that is deterministic), Moldovanu and Sela (2006) show that if the firm aims to maximize the expected highest effort, then it is optimal to split the competitors into two divisions and have multiple subcontests. In a different context of multiattribute contests, we find that with the random factor model (contestants are homogeneous and the performance consists of the effort and random factor), whether to raise multiple subcontests depends on the number of contestants and the characteristics of projects. Körpeoğlu et al. (2018) examine the firm's decision on whether to encourage contestants to participate in parallel contests. In our work, from a unique angle, we study the project with various attributes, and the firm can choose to launch multiple subcontests, with each dealing with one attribute, or a joint contest dealing with the whole problem.

Another stream of the literature on multiple competitions focuses on the *dynamic* game in which the contestants' behavior in one competition is related to that in another. Those papers mainly explore the strategic disclosure of information (on the contestants' progress) by the firm or by the contestants themselves in the process of the multistage competition. Some papers study the effects of information disclosure among contestants in the research and development (R&D) competition (e.g., Harris and Vickers 1987, Choi 1991, Malueg and Tsutsui 1997, Yildirim 2005). In this stream, the work most closely related to ours is perhaps on information disclosure by the contest organizer. The contest organizer can set some intermediate prizes as milestones throughout the process so that some contestants' performances are revealed (e.g., Goltsman and Mukherjee 2011, Halac et al. 2017, Bimpikis et al. 2019). Those studies characterize the strategic behavior of contestants in the intermediate stages of the contest and explore the optimal information-disclosure strategies of the firm in anticipation of that strategic behavior. Jiang et al. (2019) conduct counterfactual analysis based on empirical estimations and show that the disclosure of the evaluations of the performance of contestants throughout the contest may not be optimal but that the disclosure of those evaluations at a later time of the horizon may lead to a better overall contest outcome. The subcontests in our separate contest are different from "milestones" in the traditional sense. Each of our subcontests deals with one attribute. Subcontests can be parallel, or later subcontests are built on the best outcome of previous subcontests. Our contest allows contestants to "cooperate" across attributes, whereas the mechanisms in the above-mentioned papers do not. The presence of multiple attributes in the contest is a feature of our model that has not been studied. This unique feature makes it possible to compare two mechanisms: joint versus separate contests. From a different angle, Acemoglu et al. (2014) emphasize that the exact difficulties of innovation tasks may not be known in advance. The authors take a mechanism design approach and show that the solution is a dynamic pricing mechanism that induces workers to self-select into different skill hierarchical layers.

Table 1 summarizes the taxonomy of some relevant literature on multiple contests. Some work compares the joint and separate moves by agents in solving other related management problems. Hausch (1986) considers the situation where the seller can choose between two mechanisms: auctioning two identical objects at once or launching two auctions with each selling one object. The author shows that either mechanism can be optimal depending on the strategic behavior of bidders. In Hausch's auction model, if bidders lose, they will not pay, but in our contest model, no matter whether a contestant wins or loses, the effort has been made, and thus the cost is sunk. A variant of the auction model, the all-pay auction, shares a similarity with our model that contestants (like bidders) incur cost no matter whether they win. However, in a typical all-pay auction model, bidders (analogous to contestants) are assumed to be heterogeneous, but their bid (analogous to the performance of contestants) is a deterministic mapping of their type, whereas in our base model, we examine the homogeneous contestants whose performance consists of the effort and a random factor. In addition, in Section 6, we consider heterogeneous contestants with the sum of their effort and a random factor as their performance. Hu et al. (2013) compare the simultaneous and sequential group-buying mechanisms. They examine a two-period model in which sequential arrivals make sign-up decisions. The firm decides on whether to disclose the number of sign-ups in the first period to the consumers arriving in the second period. They show that the sequential mechanism has a higher chance than the simultaneous mechanism of reaching the predetermined threshold on the number of sign-ups. Clark and Riis (1998) characterize the behavior of

Table 1. Multiple Contests

Static games	Dynamic games
Elimination contest	Information disclosure by players
Rosen (1986)	Choi (1991)
(paired competition with $\lceil \log_2(n) \rceil$ contests for n players)	Harris and Vickers (1987)
Moldovanu and Sela (2006)	Malueg and Tsutsui (1997)
(two divided groups for selecting the best two players)	Yildirim (2005)
Series of independent contests	Information disclosure by the firm
Konrad and Kovenock (2009)	Bimpikis et al. (2019)
(paired competition for all possible combinations)	Goltsman and Mukherjee (2011)
DiPalantino and Vojnovic (2009)	Halac et al. (2017)
(self-selection of participation in different projects)	Jiang et al. (2019) (empirical)
Körpeoğlu et al. (2018)	
(firm's influence on contestants' participation)	
Joint contest with multiattributes and multiple subcontests of separate contest with one on each attribute	Separate contest with multiattributes
This paper	This paper (see Section 6)

heterogeneous contestants and compare the simultaneous and sequential contest mechanisms, similar to our discussion on heterogeneous contestants. There are several differences, though. First, the type of contestants in their study is one dimensional, whereas we consider a multidimensional contest. Second, they assume that the type of contestants is common knowledge—that is, complete information. In Section 6, we consider the learning behavior of contestants during the process of the separate contest—that is, the first subcontest is an incomplete-information game, and the second subcontest is a complete-information game under some conditions. Third, they assume the winner elimination in the sequential contest that could be common for a firm's internal competition. Given our focus on crowdsourcing contests, we assume that contestants can participate in the subsequent subcontests regardless of whether they win in previous subcontests. Chen et al. (2018) employ a similar model to ours to examine whether to raise a joint or separate contest. However, they study which mechanism is optimal if the two dimensions (called "components" in their paper, p. 6) are complements or substitutes. In contrast to their work, our study focuses on how the prize joint with the contest form motivates performance, which depends on the characteristics of the firm's project, such as being randomness based or effort based.

Finally, there are two main criteria for evaluating a contest in the literature: expected *best* performance and expected *average* performance. We focus on the expected best performance criterion. The two opposing effects that we characterize exist when we adopt the expected best performance criterion, whereas only one effect, the pooling effect, exists when we adopt the expected average performance criterion. The criterion of the expected best performance is better suited to innovation contests such as a research or brainstorming contest. In such settings, a single outstanding solution can be more valuable than thousands of mediocre ones.

Our separate contest resembles the *hybrid* structure of idea generation in Girotra et al. (2010), who focus on the best performance criterion. They show with a laboratory experiment that the hybrid structure, in which individuals first work independently and then work together, can generate more and better ideas than the team structure, in which the group works together in time and space. In our separate contest, contestants work independently in each subcontest, and the performances from subcontests can be "assembled" to form a final solution. Other than our unique feature of multiattribute projects, the crowdsourcing contest we study typically faces the general public, who cannot work together, and involves a monetary incentive that is likely absent in brainstorming within an organization.

3. Model Setup

Here we develop a base model to examine contestants' behavior in crowdsourcing contests with multiple attributes. The firm outsources tasks to the public, and contestants make efforts to win rewards. Let A denote the total prize of the project. The number of contestants is $n \geq 2$, which is exogenously given. A contest with a fixed number of contestants is commonly seen in the literature (e.g., Lazear and Rosen 1981; Moldovanu and Sela 2001, 2006). In addition, we derive conditions for the individual participation constraint (nonnegative payoff) to hold for n in Online Appendix C such that our model can be adapted to an endogenized number of contestants under some conditions.

We consider a project with multiple attributes. Without loss of generality, we assume that the project consists of two attributes, indexed by 1 and 2. In each attribute, the performance of a contestant is made up of two additive components. The first component is the *effort level*. The contestant decides his or her effort levels e^1 and e^2, respectively, in those two attributes, depending on the incentives. The second component is the *random factor*. The random factors exist in the projects with unclear standards or projects in which contestants have random performances. For example, a logo-designing project on 99designs may have vague criteria because the judges have undisclosed artistic tastes. Therefore, it is unclear how a submission will be rated. Moreover, the design work could depend highly on a designer's personal experience, random inspirations, or the designing environment; thus, performance itself can be arbitrary.

There are random factors along the two attributes, denoted by ξ^1 and ξ^2, respectively. We assume that the two random factors have the same distribution with the cumulative density function (CDF) $\Psi(\cdot)$ and probability density function (PDF) $\psi(\cdot)$, which is common knowledge. (The qualitative insights would not change for the case in which random factors along the two attributes follow different distributions.) We assume that the random factors along the two attributes are independent and that they are identical and independent across all the contestants. Furthermore, we assume that $\psi(\cdot)$ is symmetric log-concave with mean zero and standard deviation $\sigma > 0$.[5] The symmetric log-concavity holds for commonly used distributions—for example, normal, logistic, and uniform distributions.

The performance in each attribute is the sum of the corresponding effort and random factor. This additive form of individual performance in a contest is commonly seen in the literature; see, e.g., Lazear and Rosen (1981), Kalra and Shi (2001), Terwiesch and Xu (2008), and Ales et al. (2017). We use subscript i to denote a specific contestant and superscript l to denote a specific attribute. If contestant i makes effort $e_i^l \geq 0$, $l = 1, 2$, the performance of contestant i in those two attributes is given by $V_i^l = e_i^l + \xi_i^l$.[6]

The cost of exerting effort can be considered in the form of time/effort consumption or monetary investment. Assume that cost functions along two attributes are, respectively, $C^1(\cdot)$ and $C^2(\cdot)$, which can be different. The cost information is common knowledge. Moreover, we assume that $C^{l\prime}(\cdot) > 0, C^{l\prime\prime}(\cdot) > 0$, $l = 1, 2$, the same as in Stouras et al. (2015). This assumption is also consistent with Ales et al. (2019) and Terwiesch and Xu (2008). They assume a strictly increasing and strictly concave performance function that is equivalent to a strictly increasing and strictly convex cost function.

We assume that all contests adopt the WTA scheme. Previous studies have found that WTA is optimal for a single contest in most circumstances. For example, as mentioned earlier, Moldovanu and Sela (2001) demonstrate that WTA is optimal with the all-pay auction model if the cost function is concave or not convex enough. Terwiesch and Xu (2008) show that WTA is optimal provided that the form of performance is $V = r(e) + \xi$, where $r(e)$ is a concave function of effort, and ξ follows a Gumbel distribution. Based on the same form of performance, Ales et al. (2017) generalize the distribution of the random factor ξ and show that WTA is optimal if the participation of contestants is guaranteed and the PDF of the random factor is log-concave.

Some of our results rely on the specific forms of random factors and cost functions. We present a table of assumptions for our results in Section 7 for easy comparison.

3.1. Separate Contest

In the separate contest, the firm launches two subcontests (indexed by 1 and 2), each of which focuses on one attribute of the project. It allocates the total prize A to two subcontests with an exogenous weight $w \in (0, 1)$; hence, prizes in those two subcontests are $A^1 = wA$ and $A^2 = (1 - w)A$. Recall that the cost functions in the two dimensions can be different. Here we allow the weight w to be arbitrary because the firm may allocate different amounts of the prize to the two subcontests so that contestants can be motivated to make more efforts on one attribute than the other.[7]

Furthermore, contestant i's performances in subcontests 1 and 2 are V_i^1 and V_i^2, respectively. The total performance of contestant i is in the additive form of the performance in each subcontest—that is, $V_i^S = V_i^1 + V_i^2$. The winners in those two subcontests tend to be different. Moreover, the performances along the two attributes may have different levels of importance in the total performance. But, because we do allow different awards and different cost functions in those two subcontests, it is without loss of generality to normalize the relative importance level of performances to one by modifying the allocation of prizes and cost functions. In Online Appendix G, we examine different importance levels for the two attributes and show that our main result still holds. The assumption of the additive form of the aggregate performance requires that the solutions made by different contestants are modular. In practice, the solutions in the two attributes made by the same contestant may have a synergy effect. Intuitively, the synergy effect favors the joint contest. A formal discussion of the synergy effect is relegated to Online Appendix F.

Under the WTA scheme, in subcontest l, $l = 1, 2$, the payoff to contestant i is

$$u_i^l(e_i^l) = \begin{cases} A^l - C^l(e_i^l), & \text{if } i \text{ wins,} \\ -C^l(e_i^l), & \text{if } i \text{ loses.} \end{cases}$$

Note that, in our setting, all the contestants have a priori identical random factors and cost functions. In Online Appendix H, we formally discuss contestants with heterogeneous cost functions.

3.2. Joint Contest

In the joint contest, the firm launches a single contest to collect solutions, so the performance by contestant i, denoted by V_i^J, is the aggregation of performances in two attributes. With a slight abuse of notation, we write $V_i^J = V_i^1 + V_i^2 = (e_i^1 + \xi_i^1) + (e_i^2 + \xi_i^2)$. Unlike the separate contest, contestants make an aggregate submission instead of a solution for each subcontest. Because the contestant with the best performance wins the grand prize A, the payoff to any contestant i is

$$u_i^J(e_i^1, e_i^2) = \begin{cases} A - C^1(e_i^1) - C^2(e_i^2), & \text{if } i \text{ wins,} \\ -C^1(e_i^1) - C^2(e_i^2), & \text{if } i \text{ loses.} \end{cases}$$

We assume that the number of contestants in the two contest mechanisms stays the same. In practice, the contest mechanism may influence the participation of contestants; thus, the number of contestants can be different under the two contest mechanisms (see, e.g., Online Appendix E).

4. Equilibrium

In this section, we derive the contestants' equilibrium behavior in an n-person model. Using the characterized behavior, we then compare the performances of the joint and separate contests. In the literature, the random factor models focus on the symmetric equilibrium; see, e.g., Lazear and Rosen (1981), Kalra and Shi (2001), Terwiesch and Xu (2008), and Ales et al. (2019). Thus, we focus on the symmetric equilibrium as well for the joint and separate contests.

4.1. Subcontests in the Separate Contest

Each subcontest in the separate contest is a one-dimensional contest. The existence of an equilibrium is guaranteed if the expected payoff function is unimodal (i.e., quasi-concave) in effort over the relevant range. However, the expected payoff function may not necessarily be unimodal. In Lazear and Rosen (1981), with the same model setup as ours, a pure equilibrium exists, provided that the standard deviation of the random factor is large enough. Moreover, Dixit (1987) and Terwiesch and Xu (2008) make a stronger assumption—namely that the expected payoff function is concave in effort. In Online Appendix B, we show that a sufficient condition for the existence of symmetric equilibrium is sufficiently high dispersion of the random factor, or a sufficiently high convexity level of the cost functions. The former ensures participation from all (see Online Appendix C).

Denote the equilibrium efforts in subcontests 1 and 2 of the separate contest by $e^{1*}(n)$ and $e^{2*}(n)$, respectively. For subcontest l, $l = 1, 2$, suppose that all the contestants except contestant i make equilibrium effort $e^{l*}(n)$. The winning probability of contestant i, if he or she makes effort e_i^l, is

$$\begin{aligned} &\mathsf{P}(i \text{ wins with effort } e_i^l \mid \text{others make effort } e^{l*}(n)) \\ &\quad = \int_{-\infty}^{+\infty} \Psi(e_i^l - e^{l*}(n) + \xi^l)^{n-1} \psi(\xi^l) d\xi^l. \end{aligned}$$

When $e_i^l = e^{l*}(n)$, one can easily verify that those winning probabilities are equal to $1/n$ because $\int_{-\infty}^{+\infty} \Psi(\xi^l)^{n-1} \psi(\xi^l) d\xi^l = 1/n$. Intuitively, contestants have equal chances of winning because they are ex ante identical. Define $h(\xi^l; n) = \int_{-\infty}^{+\infty} (n-1)\Psi(\xi^l)^{n-2}\psi(\xi^l)^2 d\xi^l$, which measures the marginal change in the winning probability by exerting additional effort beyond the competitor and can be interpreted as the risk taken by the contestant for making an extra effort; see also Ψ_1 in Kalra and Shi (2001) and I_N in Ales et al. (2019). With this function, we have the following characterization.

Lemma 1. *Consider there are n contestants. The equilibrium effort in subcontest l, $l = 1, 2$, of the separate contest is $e^{l*}(n) = C^{l\prime -1}(A^l h(\xi^l; n))$.*[8]

If $n = 2$, Lemma 1 has the same characterization of the contestants' behavior as Lazear and Rosen (1981). It shows that the effort in the equilibrium increases in the prize amount. With a larger prize, contestants have more incentive to exert effort.

4.2. Joint Contest

Contestants in the joint contest make efforts along two dimensions. The contestants' two-dimensional optimization problem can reduce to a single-dimensional optimization problem with the help of the following lemma.

Lemma 2 (Optimal Effort Allocation in Attributes)**.** *The cost function of the aggregate effort level, resulting from the optimal allocation of efforts in two attributes, $C^\circ(e^\circ) = \min_{e^1+e^2=e^\circ}\{C^1(e^1) + C^2(e^2)\}$, is a strictly increasing and strictly convex function. Given the aggregate effort level e°, the optimal effort allocation $(\tilde{e}^1, \tilde{e}^2)$ satisfies $C^{\circ\prime}(e^\circ) = C^{1\prime}(\tilde{e}^1) = C^{2\prime}(\tilde{e}^2)$.*

Lemma 2 characterizes the optimal allocation of efforts made by contestants in the joint contest. It is analogous to the optimal allocation of a fixed budget across products to maximize the total profit in the economics literature. Given a fixed amount of the aggregate effort, contestants optimally allocate the efforts to two dimensions. The marginal costs across the two dimensions are equal in the optimal allocation. Otherwise, say if $C^{1\prime}(e^1) > C^{2\prime}(e^2)$, the contestant can achieve a lower total cost by increasing his or her effort in the second attribute while reducing it in the first attribute. Moreover, we show that the total cost is strictly increasing and strictly convex in the aggregate effort. With Lemma 2, we can transform the two-dimensional contest to a single-dimensional contest so that the derivation of the equilibrium effort level will be the same as in Lemma 1.

Denote the equilibrium effort in the joint contest by $e^{\circ *}(n)$. Suppose that all the contestants except contestant i make equilibrium effort $e^{\circ *}(n)$. Denote $\xi^\circ = \xi^1 + \xi^2$ with PDF $\psi^\circ(\xi^\circ)$ and CDF $\Psi^\circ(\xi^\circ)$. The winning probability of contestant i, if he or she makes an aggregate effort e_i°, is

$$\mathsf{P}\big(i \text{ wins with effort } e_i^\circ \mid \text{others make effort } e^{\circ *}(n)\big) = \int_{-\infty}^{+\infty} \Psi^\circ\big(e_i^\circ - e^{\circ *}(n) + \xi^\circ\big)^{n-1} \psi^\circ(\xi^\circ) d\xi^\circ.$$

As a result, similar to $h(\xi^l; n)$, we can define $h^\circ(\xi^\circ; n) = \int_{-\infty}^{+\infty} (n-1)\Psi^\circ(\xi^\circ)^{n-2}\psi^\circ(\xi^\circ)^2 d\xi^\circ$. By Lemma 2, the total cost $C^\circ(e^\circ)$ is strictly increasing and strictly convex. Then the derivation of the equilibrium effort in the joint contest is analogous to Lemma 1, with the total prize A.

Lemma 3. *Consider there are n contestants. The equilibrium effort for the joint contest is* $e^{\circ *}(n) = C^{\circ\prime -1}(Ah^\circ(\xi^\circ; n))$.

4.3. Comparison

In this section, we compare the *expected best performances* between those two contest mechanisms with n contestants. We denote the highest-order statistic with a sample size n by subscript $_{(n)}$. The expected best performances in the separate and joint contests are denoted by V_n^S and V_n^J, respectively, with $V_n^S = \mathsf{E}((e^{1*}(n) + \xi^1)_{(n)}) + \mathsf{E}((e^{2*}(n) + \xi^2)_{(n)})$ and $V_n^J = \mathsf{E}((e^{\circ *}(n) + \xi^1 + \xi^2)_{(n)})$. Denote the difference by $\Delta_n = V_n^S - V_n^J$. The comparison of the two contests depends on the value of Δ_n. If $\Delta_n \geq 0$, then the separate contest is optimal; otherwise, if $\Delta_n < 0$, the joint contest is optimal. To compare Δ_n with zero, we first perform the following transformation.

Because equilibrium efforts are deterministic (as in the base model, all contestants are homogeneous; we consider heterogeneous contestants later), we have $\mathsf{E}((e^{l*}(n) + \xi^l)_{(n)}) = e^{l*}(n) + \mathsf{E}(\xi^l_{(n)})$, $l = 1, 2$, and $\mathsf{E}((e^{\circ *}(n) + \xi^1 + \xi^2)_{(n)}) = e^{\circ *}(n) + \mathsf{E}((\xi^1 + \xi^2)_{(n)})$. The difference between the expected best performances can then be decomposed into two parts as $\Delta_n = \Delta_n^e + \Delta_n^\xi$, where $\Delta_n^e = e^{1*} + e^{2*} - e^{\circ *}$ and $\Delta_n^\xi = \mathsf{E}(\xi^1_{(n)}) + \mathsf{E}(\xi^2_{(n)}) - \mathsf{E}((\xi^1 + \xi^2)_{(n)})$. The first part, Δ_n^e, is the difference in the equilibrium effort levels. The second part, Δ_n^ξ, is the difference between the expected best random factors.

It is intuitive that $\Delta_n^\xi = \mathsf{E}(\xi^1_{(n)}) + \mathsf{E}(\xi^2_{(n)}) - \mathsf{E}((\xi^1 + \xi^2)_{(n)}) \geq 0$. This can be simply shown as follows. Denote the realizations of ξ^1 and ξ^2 among n contestants by $\{\varepsilon_1^1, \ldots, \varepsilon_n^1\}$ and $\{\varepsilon_1^2, \ldots, \varepsilon_n^2\}$ respectively. Then $\max\{\varepsilon_1^1, \ldots, \varepsilon_n^1\} + \max\{\varepsilon_1^2, \ldots, \varepsilon_n^2\} \geq \max_{i', i'' \in \{1, \ldots, n\}}\{\varepsilon_{i'}^1 + \varepsilon_{i''}^2\}$. Because such inequality holds for any realization, $\mathsf{E}(\xi^1_{(n)}) + \mathsf{E}(\xi^2_{(n)}) \geq \mathsf{E}((\xi^1 + \xi^2)_{(n)})$. To find a clear-cut comparison between the two contests, we need $\Delta_n^\xi > 0$ to hold strictly, which is satisfied by many commonly used distributions. To guarantee $\Delta_n^\xi > 0$, we require that ξ^l, for any $l = 1, 2$, satisfies the regularity condition. This regularity condition has been used by Chakraborty (1999, p. 726) to discuss the multiple objects auctioning strategy.

Assumption 1 (Regularity Condition)**.** *Denote the quantile function of* ξ^l, $l = 1, 2$, *by* $\Psi^{l-1}(u)$ *and the quantile function of* $\xi^\circ = \xi^1 + \xi^2$ *by* $\Psi^{\circ -1}(u)$, $u \in [0, 1]$. *There exists* u_0 *such that* $\Psi^{1-1}(u) + \Psi^{2-1}(u) - \Psi^{\circ -1}(u) < 0$ *if* $u \in (0, u_0)$ *and* $\Psi^{1-1}(u) + \Psi^{2-1}(u) - \Psi^{\circ -1}(u) > 0$ *if* $u \in (u_0, 1)$.

The regularity condition is a condition on the tails of the distributions. It means that a random draw from the convolution of two distributions is more likely to yield a "central" value than random draws from individual distributions. To see this, consider the uniform distribution on $[-1, 1]$. Whereas the individual densities are evenly distributed, the convolution of the two uniform distributions is clustered around the center. Many log-concave distributions satisfy the regularity condition—for example, uniform, normal, logistic, Gumbel, gamma, and so on. A sufficient condition for the regularity condition to hold is available for symmetric distributions: If the hazard function $\psi(\xi)/(1 - \Psi(\xi))$ is nondecreasing, then the regularity condition holds, and $u_0 = 1/2$. Moreover, by corollary 2 of Bagnoli and Bergstrom (2005), if the PDF is log-concave, then its hazard function is increasing over the support. Thus, all the symmetric log-concave distributions satisfy the regularity condition.

Lemma 4. We have $\mathsf{E}(\xi^1_{(n)}) + \mathsf{E}(\xi^2_{(n)}) > \mathsf{E}((\xi^1 + \xi^2)_{(n)})$, $n \geq 2$.

By Lemma 4, the sum of the best realizations of random factors along the two attributes tends to be more extreme than the best realization of the sum—that is, $\Delta_n^\xi > 0$.

Proposition 1 (Expected Best Performance)**.**

a. *Combination effect: The expected best random factor in the separate contest is higher than that in the joint contest—that is,* $\Delta_n^\xi > 0$.

b. *Pooling effect: If* $h^\circ(\xi^\circ; n) > \max\{wh(\xi; n), (1 - w) h(\xi; n)\}$, *the equilibrium effort in the joint contest is higher* *than that in the separate contest—that is,* $\Delta_n^e < 0$. *Moreover, the condition simplifies to* $h^{\circ}(\xi^{\circ};n) > h(\xi;n)/2$ *if one of the following conditions holds:* (i) $w = 1/2$ *or* (ii) $C^1(\cdot) = C^2(\cdot)$, *and their derivatives are weakly convex.*

Proposition 1 characterizes two countervailing forces that determine the superiority between those two contest mechanisms. Part (a) of Proposition 1 is directly implied by Lemma 4. The separate contest has an advantage in selecting the best performances mainly driven by the random factors. Whereas the two subcontests deal with different attributes of the project, each subcontest selects the best performance in each attribute. Given the same effort by all the contestants, the separate contest combines the performances with the best realizations of random factors in those two subcontests. The aggregated best performance in the separate mechanism is made up of the best random realizations along both dimensions and is more likely to have a high value. Moreover, the best solutions in the two subcontests may be provided by different contestants. However, in the joint contest, the contestants' performance depends on the sum of those random factors realized to the same individual; thus, the sum of the random factors across the two attributes is less extreme. Overall, the sum of the most extreme random factors across the two subcontests is stochastically larger than the extreme of the aggregated random factors in the joint contest. As the old saying goes, "Two heads are better than one." Because the separate contest combines the best random realizations from each subcontest, we call this the *combination* effect.

The joint contest encourages a greater effort level in equilibrium under a mild condition in part (b) of Proposition 1—that is, $h^{\circ}(\xi^{\circ};n) > \max\{wh(\xi;n), (1-w)h(\xi;n)\}$. We begin our discussion by considering a special case as in part (b) of Proposition 1, in which the firm evenly allocates prizes into two subcontests—that is, $w = 1/2$. In each subcontest of the separate contest, the return for the winner is one half of the total prize. Because the prize induces contestants to exert an effort, the effort increases with the size of the prize. However, when making efforts, contestants consider the expected gain from each bit of extra effort they make in the process. More specifically, to make it "worthwhile," contestants will choose his or her effort level such that the marginal change in the expected gain is equal to the marginal cost if a marginal amount of extra effort is made. In the joint contest, the term $Ah^{\circ}(\xi^{\circ};n)$ captures such a marginal change in the expected gain in the joint contest if a small amount of additional effort has been exerted in either attribute. And also, the term $Ah(\xi;n)/2$ measures the marginal change in the expected gain in either subcontest of the separate contest.

The overall motivation for contestants to make efforts in an attribute is relatively higher in the joint contest than in the separate contest if $Ah^{\circ}(\xi^{\circ};n) > Ah(\xi;n)/2$. The intuition is that because the joint contest adds together the random factors from two contests, the aggregate variability level is lower than that in each individual contest. Thus, the marginal expected gain from each bit of extra effort on any attribute in the joint contest is more "worthwhile" than that in the corresponding subcontest of the separate contest. As a result, the joint contest pushes contestants to exert more efforts; we call this the *pooling* effect.

For a general prize allocation w, if $w \neq 1/2$, the marginal expected gains in the two subcontests of the separate contest are different because the returns are different in those subcontests—that is, $wAh(\xi;n) \neq (1-w)\, Ah(\xi;n)$. Part (b) of Proposition 1 allows the cost functions in the two dimensions to be different and the prize allocation in the separate contest to be general. For a general prize allocation $w \in (0,1)$, the condition $h^{\circ}(\xi^{\circ};n) > \max\{wh(\xi;n), (1-w)h(\xi;n)\}$ ensures that the induced effort in any attribute of the joint contest is greater than that in the corresponding subcontest of the separate contest.

The equilibrium effort level depends on the marginal cost function, too. Even though the marginal expected gain in the two attributes of the joint contest is the same, the induced effort can be different because of different cost functions in the two attributes. By allowing the cost functions to be identical and their derivative functions to be convex, part (b)(ii) of Proposition 1 shows that a sufficient condition for $\Delta^e < 0$ becomes as simple as $h^{\circ}(\xi^{\circ};n) > h(\xi;n)/2$. The condition that $C^{l\prime}(\cdot)$ is convex holds for many commonly used cost functions, such as the exponential and polynomial functions.

Example 1. To gain intuition, we first examine a two-person example in which those random factors follow a normal distribution. The equilibrium effort levels for the two-person model of the separate and joint contests can be derived from Lemmas 1 and 3. If $\xi^l \sim N(0,\sigma)$, $l = 1,2$, $w = 1/2$, and $n = 2$, then the condition $h^{\circ}(\xi^{\circ};2) > h(\xi;2)/2$ is naturally satisfied because $2\sqrt{2}\sigma < 4\sigma$. The formal derivation can be found in Online Appendix J.

For a normally distributed random factor, if σ is large, the extra effort can enhance the winning probability by only a little, implying that the contestant takes a high risk of failure for making an extra bit of effort. Hence, we can view the value of σ as the amount of risk that contestants bear when making extra efforts. Overall, a contestant's effort in each subcontest depends on the marginal expected gain $A/2 \cdot h(\xi;2) = (A/2)/(2\sqrt{\pi}\sigma)$. The higher the marginal expected gain, the greater incentive the contestant has to make an effort, and in equilibrium, contestants make more efforts.

In the joint contest, the contestant submits an aggregate solution of two attributes in order to win the whole prize. Therefore, if the contestant wins, the return for his or her efforts in each attribute is the whole prize. If random factors are distributed normally with standard deviation σ, the summation of two random factors has the standard deviation $\sqrt{2}\sigma$. Thus, the effort in each attribute relies on the corresponding marginal expected return $A \cdot h^{\circ}(\xi^{\circ};2) = A/(2\sqrt{2\pi}\sigma)$. Because the marginal expected return is higher for each attribute in the joint contest than in the separate contest (i.e., $A/(2\sqrt{2\pi}\sigma) > (A/2)/(2\sqrt{\pi}\sigma)$), the effort for each attribute is higher in the joint contest.

Corollary 1. *If* $\xi \sim N(0,\sigma)$ *and* $w \in (1-\sqrt{2}/2, \sqrt{2}/2)$, $\Delta_n^e < 0$ *for any* n.

Interestingly, if random factors follow a normal distribution (the arguably most commonly used distribution in the natural and social sciences), then $h^{\circ}(\xi^{\circ};n)/h(\xi;n) = 1/\sqrt{2}$ for any number of contestants. Further, if $w \in (1-\sqrt{2}/2, \sqrt{2}/2)$, then $\Delta_n^e < 0$. As a special case, if the firm allocates the prizes equally in the separate contest (i.e., $w = 1/2$), then the joint contest achieves a higher equilibrium effort level. We perform numerical studies on the condition in part (b) of Proposition 1 for normal, logistic, and Gumbel distributions in Online Appendix I. As an extension of Proposition 1(b), Corollary A.1 in Online Appendix A shows that under the same assumption, the joint contest leads to higher total *expected* efforts and performances across two dimensions than the separate contest.

4.4. Luck vs. Sweat

Based on the two effects characterized in the preceding section, we explore how the two forces change depending on the project characteristics. By Proposition 1, we find that the comparison of the two types of contests boils down to a comparison of the combination effect and pooling effect under the assumption that the effort level and random factor have the same weight in the performance. However, the relative importance of the effort level versus the random factor can be different for projects.

We categorize the projects into two types. One type of projects highly relies on the effort exerted by contestants—for example, projects that require experiments or some technical work. Another type of projects highly relies on the inspiration of contestants, and the performance involves high randomness—see, for example, ideation or designing projects.

With a slight modification of the base model, we can characterize the two types of projects by scaling the effort level with a scalar β. Then the cost functions become $C^l(\beta e^l)$, $l = 1,2$, and $C^{\circ}(\beta e^{\circ})$. The parameter $\beta > 0$ measures the *relative importance* of the random factor vs. effort. To see this, by denoting $\beta e = \hat{e}$, we obtain the performance in the form of $V = \hat{e}/\beta + \xi$; therefore, $1/\beta$ is the weight of the effort while the weight of the random factor is normalized to one. If β is large, it is as if the efforts made by contestants contribute little to their performances; thus, contestants tend not to make a great effort because doing so incurs high costs. We call projects with large values of β *randomness-based projects*. By contrast, if β is small, the effort is more important than the random factor in the performance, and contestants can improve their performance significantly with little cost. We call projects with small values of β *effort-based projects*.

We examine the two most commonly used forms of the cost functions. One is the exponential cost function $C^l(\rho^l\beta e^l) = \exp(\rho^l\beta e^l)$, $\rho^l > 0$, $l = 1,2$, and the other is the polynomial cost function $C^l(\beta e^l) = a^l(\beta e^l)^{b^l}$, $a^l > 0$, $b^l \geq 2$.

Proposition 2. *Given that* $h^{\circ}(\xi^{\circ};n) > \max\{wh(\xi;n), (1-w)\, h(\xi;n)\}$, *for the cost function* $C^l(\rho^l\beta e^l) = \exp(\rho^l\beta e^l)$, $\rho^l > 0$, *or* $C^l(\beta e^l) = a^l(\beta e^l)^{b^l}$, $a^l > 0$, $b^l \geq 2$, $l = 1,2$, *there exists a threshold* $\tilde{\beta} > 0$ *such that if* $\beta \geq \tilde{\beta}$, *the separate contest is optimal, and if* $\beta < \tilde{\beta}$, *the joint contest is optimal.*[9]

Proposition 2 shows that the relative importance between the random factor and effort level plays an important role in the comparison between the two contest mechanisms. If the project is randomness based (i.e., β is high), contestants have low effort levels under both contest mechanisms; thus, the difference in their effort levels is small (consider the extreme case that contestants in both contest mechanisms hardly make any effort). As a result, the pooling effect is weak and can be dominated by the combination effect, and the separate contest tends to be optimal. By contrast, if the project is effort based (i.e., β is low), the joint contest tends to be optimal. The reason is that contestants are motivated to make more efforts in both contest mechanisms, and the difference in the expected gains between the two contest mechanisms induces a large difference in efforts. That is, the pooling effect is strong and tends to dominate the combination effect.

4.5. Number of Contestants

Now we compare the two contest mechanisms for different numbers of contestants. The Pentagon's contest brought together specialized military contractors to design a military vehicle. Thus, its number of contestants is expected to be smaller than that for projects on a crowdsourcing platform that do not require sophisticated technique skills, such as InnoCentive and 99designs.

To compare the two contest mechanisms with a different number of contestants, we must know how the combination and pooling effects change with the number of contestants. By Lemma 4, the difference between the expected best random factors Δ_n^{ξ} is positive. We investigate how Δ_n^{ξ} changes with respect to the number of contestants n in Propositions A.1 and A.2 in Online Appendix A. Proposition A.1 shows that if the number of contestants is small, the difference in the expected best random factors Δ_n^{ξ} is increasing in n. And Proposition A.2 shows the asymptotic behavior for a sufficiently large number of contestants. In particular, if the range of random factors is bounded, Δ_n^{ξ} approaches zero, and if those random factors are normally distributed, Δ_n^{ξ} approaches infinity.

Thus, by Proposition A.2, with normally distributed random factors that have unbounded support, the separate contest can benefit more from an increasing number of contestants than the joint contest, even when the pool of contestants is already very large. That is, the combination effect may always be enhanced by more and more contestants.

In general, how the difference between the equilibrium efforts in those contest mechanisms Δ_n^{e} would change with one additional contestant can be ambiguous. However, we can obtain a clear-cut result for normally distributed random factors with the exponential or polynomial cost functions.

Proposition 3 (Expected Best Performance: Number of Contestants). *If $\xi^l \sim N(0, \sigma)$, $l = 1, 2$, and $w \in (1 - \sqrt{2}/2, \sqrt{2}/2)$, for the exponential or polynomial cost functions, there exists a threshold $\tilde{n} \geq 2$ on the number of contestants above which the separate contest is optimal and under which the joint contest is optimal.*

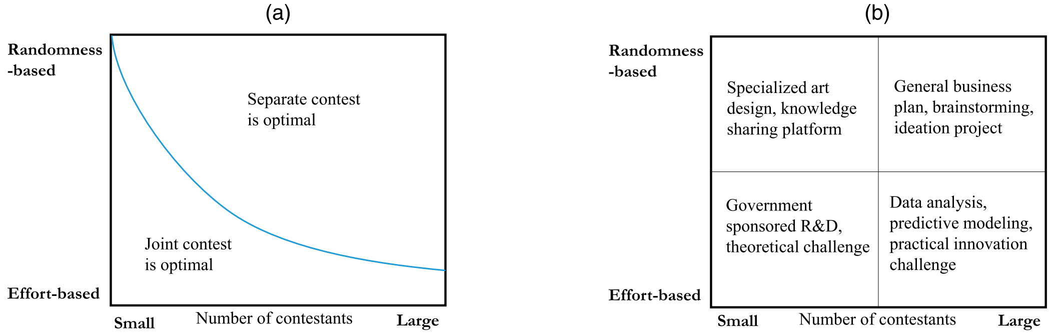
Proposition 3 shows that if random factors follow a normal distribution, the joint contest is optimal when the number of contestants is below a threshold, and the separate contest is optimal when the number of contestants is above that threshold. Ales et al. (2019) study a one-dimensional contest and show that if the random factor follows a symmetric log-concave distribution (a property that the normal distribution satisfies), the effort is decreasing in the number of contestants (see proposition 1 in Ales et al. 2019). This is because a higher number of contestants intensifies the competition and reduces contestants' incentive to expend effort. With this result, the equilibrium effort in both contest mechanisms is decreasing in the number of contestants, but the monotonicity of the difference in the levels of effort may be ambiguous. However, with the normally distributed random factors, if the cost functions are in the exponential form, the difference in the equilibrium effort between the two contest mechanisms is fixed regardless of the number of contestants; and if the cost functions are in the polynomial form, the difference in the equilibrium effort is strictly decreasing in the number of contestants.

Moreover, for these specific cases, we show that the difference in random factors is increasing in the number of contestants, a finding consistent with Propositions A.1 and A.2(ii) for the general case. The combination effect is reinforced by a larger number of contestants, whereas the pooling effect exists but is not influenced or reduced by the number of contestants. Overall, everything else being equal, if the number of contestants is relatively small, the combination effect is weak and dominated by the pooling effect, and thus the joint contest is optimal. Otherwise, if the number of contestants is sufficiently large, the combination effect becomes dominant, and the separate contest is optimal. This may partially explain the Pentagon's switching behavior. Because the number of contestants may be small for a military project and the combination effect is insignificant for a small pool of contestants, the joint contest may perform better than the separate contest. This could be one reason why the Pentagon switched to the joint contest after experimenting with the separate contest. Lastly, when the entry decisions by the contestants are endogenized, the number of entrants would be smaller, and then the joint mechanism may be more likely to be favored.

4.6. Summary of Implications

Figure 2(a) illustrates a comparison between the two mechanisms depending on the randomness of the project (see Proposition 2) and the number of contestants (see Proposition 3) for normally distributed random factors and exponential or polynomial cost functions. Figure 2(b) categorizes the projects into four groups by their level of randomness (high or low) and the number of contestants (large or small). General business plans, brainstorming, and ideation projects are driven by inspiration and have random outcomes with a large pool of participants. For example, yutongo is a platform on which business managers can decompose a project into several subquestions and then combine ideas generated for each subquestion. Government-sponsored R&D and theoretical challenges may only be tackled by a small group of specialists with the outcome highly dependent on the effort level, such as the Pentagon's military project mentioned earlier. Projects that require data analysis, predictive modeling, and practical innovation are often outsourced to the public, sometimes with participating teams in thousands, on platforms such as Kaggle and InnoCentive and are based on the efforts exerted by participants. Projects on specialized art design platforms such as 99designs and knowledge-sharing platforms such as Quora are often outsourced to a relatively small group of specialized agents and may have highly uncertain outcomes. In view of Figure 2(a), we expect that the

Figure 2. (Color online) Comparison Between the Joint and Separate Contests



$\xi^l \sim N(0,\sigma)$, $C^l(e^l) = \exp(\rho^l \beta e^l)$ or $C^l(e^l) = a^l(\beta e^l)b^l$

projects on the diagonal (i.e., in the $(1,1)$ and $(2,2)$ cells) of the 2×2 matrix are more likely to operate under both contest mechanisms than the projects on the off-diagonal (i.e., in the $(1,2)$ and $(2,1)$ cells) of the matrix, for which one of the two contest mechanisms tends to stand out.

5. The Optimal Prize

We have shown that the equilibrium effort levels in the separate and joint contests are highly influenced by the amount of prize. Given a fixed total prize, contestants have a higher effort level in the joint contest than in the separate contest. The difference in equilibrium effort levels between the two contest mechanisms—that is, Δ_n^e—relies on the specific form of the cost functions. In the following analysis, we assume two commonly used forms of the cost functions to compare the two contest mechanisms.[10] Here we focus on the case that the cost functions along the two attributes are identical, and the condition $h^\circ(\xi^\circ; n) > \max\{wh(\xi; n), (1-w)h(\xi; n)\}$ is satisfied. The general case with different cost functions along the two attributes is qualitatively the same because the sufficient condition $h^\circ(\xi^\circ; n) > \max\{wh(\xi; n), (1-w)h(\xi; n)\}$ guarantees that the equilibrium effort level in either attribute is higher in the joint contest than its counterpart in the separate contest, and the difference in the total effort levels can be decomposed to those for each attribute.

Lemma 5. *Consider the difference Δ_n^e of equilibrium effort levels between two mechanisms.*

a. *If $C^l(e^l) = a(e^l)^b$, $l = 1, 2$, $a > 0$, $b \geq 2$, Δ_n^e is strictly increasing in A. The firm has a higher efficiency of inducing effort making in investing in the joint contest than in the separate contest.*

b. *If $C^l(e^l) = \exp(\rho e^l)$, $l = 1, 2$, $\rho > 0$, Δ_n^e is fixed for any A. The firm has the same efficiency of inducing effort making in investing in both contests.*

By Lemma 5, we find that the difference in equilibrium effort levels between the two contest mechanisms has different reactions to a larger prize depending on the cost functions because the degrees of convexity (curvature) of the cost functions are different. Such a degree is measured by the Arrow–Pratt coefficient (see Mas-Colell et al. 1995, p. 190). The degree of the curvature of the cost function has been used to discuss other problems in contest theory. For example, Moldovanu and Sela (2001) show that the Arrow–Pratt coefficient of the cost function determines whether WTA is optimal. The coefficients of the two forms of the cost function are, respectively,

$$\frac{C^{l\prime\prime}(e^l)}{C^{l\prime}(e^l)} = \frac{\rho^2 \exp(\rho e^l)}{\rho \exp(\rho e^l)} = \rho \ \text{(exponential)},$$
$$\frac{C^{l\prime\prime}(e^l)}{C^{l\prime}(e^l)} = \frac{ab(b-1)(e^l)^{b-2}}{ab(e^l)^{b-1}} = \frac{b-1}{e^l} \ \text{(polynomial)},$$
$$l = 1, 2.$$

Because the cost functions along the two attributes are strictly convex, the marginal cost function for each attribute is increasing. The curvature of the cost functions measures the marginal cost increase for additional effort. For example, if the Arrow–Pratt coefficient of a cost function is zero, the cost function is linear, and the marginal cost increase is zero. In equilibrium, the marginal benefit of a higher prize is the inverse of the marginal cost function in effort. Thus, if the marginal cost function is increasing, the marginal effort induced by an additional amount of prize is decreasing in the prize.

For exponential cost functions, at any effort level, the additional effort induced by a marginal increase in the prize is a fixed value. Therefore, the difference Δ_n^e is fixed for any prize A. However, for polynomial cost functions, the decreasing rate of the marginal effort in the prize is decreasing in the effort level. That is, the marginal increase in the prize is more efficient in inducing effort making when the effort level is high. Because the effort level is higher in the joint contest than in the separate contest for any given total prize A, the additional effort induced by the additional prize is higher in the joint contest than in the separate contest.

Thus, the difference Δ_n^e is strictly increasing in A for polynomial cost functions.

Now we characterize the optimal total prize for the joint and separate contests if the prize A is endogenized without any budget constraint. Denote the utility of the firm by U_o^S and U_o^J in the separate and joint contests, respectively. The firm needs to deal with the optimization problems described in Table 2.

The incentive compatibility constraint incorporates the contestants' utility-maximization problem into the firm's problem, and the individual rationality constraint guarantees the participation of contestants. The sufficient condition for the individual rationality constraint to be satisfied is given in Online Appendix C. We have the following results on the optimal total prize.

Proposition 4.

a. *If* $C^l(e^l) = a(e^l)^b$, $l = 1, 2$, $a > 0$, $b \geq 2$, *then, for* $b > 2$, $A^{J*} > A^{S*}$; *for* $b = 2$, *there does not exist a finite optimal total prize for either contest mechanism.*

b. *If* $C^l(e^l) = \exp(\rho e^l)$, $l = 1, 2$, $\rho > 0$, *then* $A^{J*} = A^{S*}$.

Proposition 4 shows a comparison of the optimal prizes between the two contest mechanisms. If the cost functions are polynomial, we find that when $b > 2$, there exists a finite optimal total prize for each contest mechanism. However, the optimal prize in the joint contest is higher than that in the separate contest. For the firm, the marginal benefit of a marginal increase in the total prize is the marginal effort. At the optimal prize, the marginal benefit of the additional total prize must be equal to the marginal increase in the total prize with value equal to one. If the cost functions are exponential or polynomial, the marginal benefit of the additional total prize is weakly decreasing in the total prize. By part (a) of Lemma 5, the joint contest has a higher efficiency in motivating contestants to make efforts than the separate contest; thus, the marginal effort decreases with a larger total prize at a slower rate in the joint contest than in the separate contest. Therefore, the optimal total prize is higher in the joint contest than in the separate contest—that is, $A^{J*} > A^{S*}$. When $b = 2$, the marginal utility of the firm is linear in the total prize for both contest mechanisms; thus, there does not exist a finite optimal total prize. If the cost functions are exponential, by part (b) of Lemma 5, the firm has the same efficiency in investing in the two contest mechanisms. Therefore, the marginal benefit of an additional amount of the total prize decreases at the same rate for the two contest mechanisms. As a result, the optimal total prize for both contest mechanisms is the same—that is, $A^{J*} = A^{S*}$.

In practice, the firm may have a budget constraint that serves as the upper bound on the total award. With Lemma 5 and Proposition 4, in Online Appendix D, we show the optimal choice of the contest mechanism and the amount of prizes for a given budget constraint.

6. Heterogeneous Contestants

The analysis for heterogeneous contestants often resorts to the all-pay auction model, in which contestants' types are their private information and the equilibrium effort level is a function of their types. However, the full analysis for the separate auction with all bidders participating in each, even without random factors, is intractable (see, e.g., Krishna 2009, p. 221). Moreover, Stouras (2018) finds that the closed form for the random factor model with heterogeneous types in general form may be intractable. For tractability, we employ a simple two-person and two-type model. To characterize the forward-looking behavior of contestants in the separate contest, we assume that the expertise "types" that contestants pretend to be in the first subcontest would be fully learned by each other through the disclosed performances in the first subcontest.[11]

6.1. Model Setup

We consider a two-person model (i.e., $n = 2$) with two expertise types (high and low) in each attribute. In contrast to the base model in which all the contestants are assumed to be identical for each attribute, we assume here that contestants are endowed with expertise either x_H or x_L, $x_H \geq x_L > 0$ in each attribute, which is private knowledge to an individual contestant. As common knowledge, the expertise in each attribute follows a two-point distribution. We examine the case where the probability, denoted by η, for one type of contestants to appear is 1/2. The condition $\eta = 1/2$ guarantees the tractability of the model, so that there exists a unique symmetric equilibrium in which contestants have pure strategies in the first subcontest of the separate contest. Otherwise, for the general η, there may only exist a mixed-strategy equilibrium. Even with $\eta = 1/2$, contestants already behave differently from the characterization of the homogeneous contestants.

Table 2. Prize Optimization

Objective and constraints	Separate contest	Joint contest
Objective	$\max_A U_o^S = V_n^S(e^{1*}, e^{2*}) - A$	$\max_A U_o^J = V_n^J(e^{\circ*}) - A$
Incentive compatibility	subject to $e^{l*} = C^{l\prime -1}(A^l h(\xi^l; n))$, $l = 1, 2$,	subject to $e^{\circ*} = C^{\circ\prime -1}(A h^\circ(\xi^\circ; n))$,
Individual rationality	$A^l/n \geq C^l(e^{l*})$, $l = 1, 2$	$A/n \geq C^\circ(e^{\circ*})$

For simplicity, we assume that the two attributes of the project are symmetric such that the two subcontests of the separate contest are symmetric in terms of the random factors, cost functions, and prize allocation.[12] The random factor in each attribute follows the normal distribution $N(0, \sigma)$, and thus the difference in random factors between two contestants i and j is denoted by $\gamma^l = \xi_i^l - \xi_j^l$, $l = 1, 2$, following the normal distribution $N(0, \sqrt{2}\sigma)$ with PDF $g(\cdot)$ and CDF $G(\cdot)$. We assume that the cost functions along the two dimensions are identical in the exponential form $C(\cdot) = C^1(\cdot) = C^2(\cdot) = \exp(\rho e)$. In the first subcontest of the separate contest, every contestant knows only his or her own expertise type and that his or her opponent's expertise is drawn independently from the two-point distribution. For each attribute, if a contestant is type $t = H$, or L, his or her cost function is $C(\cdot)/x_t$. Similar cost functions have been adopted in Fey (2008), Lazear and Rosen (1981), and Moldovanu and Sela (2001) under slightly different model setups.

6.2. Separate Contest

We examine three cases: that the expertise of a contestant is perfectly *positively*, *negatively correlated*, or *independent*. We derive analytical results for the correlated expertise cases, and perform a numerical study for the independent expertise case in Online Appendix H.

Positive Correlation. The contestant who is endowed with high expertise x_H (respectively, low expertise x_L) in the first attribute will have high expertise x_H (respectively, low expertise x_L) in the second attribute.

Negative Correlation. The contestant who is endowed with high expertise x_H (respectively, low expertise x_L) in the first attribute will have low expertise x_L (respectively, high expertise x_H) in the second attribute.

Because there exists a correlation between the expertise of the two attributes, if the firm discloses the performance of contestants after the first subcontest, contestants can infer the type of their opponents and the belief of the prior expertise distribution is updated for the second subcontest. We focus on the case that contestants can learn their opponent's effort level by his or her performance in the first subcontest. This is indeed the case when the randomness comes from the preferences or the private tastes of judges, whereas the quality of a solution depends on the effort level.[13] In the symmetric equilibrium, contestants of different types have different effort levels; therefore, with the disclosed information, contestants can accurately learn their opponent's type.

If contestants are strategic, they may try to hide their type in the first subcontest because their truthful revelation may put them into a disadvantageous position in the second subcontest. The only way for contestants to hide their types is that in the first subcontest both contestants have the same effort level regardless of their true types.

By part (i) of Lemma H.2 in Online Appendix H, we find that if $\eta = 1/2$, contestants have no incentive to hide their types in the first subcontest. Because contestants decide on whether to hide their types at the beginning of the first subcontest, they need to take into account their ex ante utility in the second subcontest. We find that no matter whether contestants hide or do not hide their types in the first subcontest, the ex ante utilities in the second subcontest remain the same if $\eta = 1/2$. As a result, hiding one's type is not beneficial for contestants. Moreover, by Lemma H.1 in Online Appendix H, if $\eta = 1/2$, in the first subcontest, the equilibrium is unique such that contestants with different types make different efforts. Therefore, if contestants hide their types in the first subcontest, they will be worse off because the utilities in the second subcontest are the same no matter whether they hide or not. Thus, contestants' behavior in the first subcontest is characterized by Lemma H.1—that is, they truthfully behave without any strategic behavior. In the second subcontest, contestants know their opponent's type, and the equilibrium efforts are characterized by part (ii) of Lemma H.2.

6.3. Joint Contest

With the model setup of the positively and negatively correlated expertise across the two attributes, we can characterize the contestant behavior in the joint contest. Denote the difference in random factors between contestants i and j along the two dimensions by $\gamma^\circ = \xi_i^1 + \xi_i^2 - \xi_j^1 - \xi_j^2 = (\xi_i^1 - \xi_j^1) + (\xi_i^2 - \xi_j^2) = \gamma^1 + \gamma^2$. By the symmetric property of γ^1 and γ^2, the random variable γ° has a symmetric PDF denoted by $g^\circ(\gamma^\circ)$ and a CDF denoted by $G^\circ(\gamma^\circ)$. Note that $g^\circ(0) > g(0)/2$—equivalently, $h^\circ(\xi^\circ; 2) > h(\xi; 2)/2$—is naturally satisfied by normal distributions. Parts (i) and (ii) of Lemma H.3 in Online Appendix H characterize the equilibrium effort levels in the joint contest with perfectly positively and perfectly negatively correlated expertise, respectively. With Lemmas H.1, H.2, and H.3, we are able to compare the effort levels between two contest mechanisms for each contestant.

Proposition 5. *If contestants' expertise along the two attributes is perfectly positively correlated or perfectly negatively correlated, each contestant has a higher expected effort level in the joint contest than in the separate contest.*

In our model setup of two contestants and two types with an equal probability, forward-looking contestants behave myopically in the first subcontest of the separate contest, and their true types are voluntarily revealed at

Table 3. Summary of Key Assumptions

Key assumptions	Proposition 1	Propositions 2 and 4	Propositions 3 and 5
Cost functions	Strictly increasing and strictly convex	Exponential or polynomial	
Random factors	Symmetric log-concave		Normal

the beginning of the second subcontest. In this heterogeneous-types model, in addition to the pooling and combination effects, there may exist extra forces that push contestants to slack off in the joint contest and in each subcontest of the separate contest.

In either the separate or the joint contest, contestants consider the possibility that their opponent is of a different type, which differs from the homogeneous case. If the two contestants are indeed different (which will be clear in the second subcontest under the separate mechanism), they will slack off because of the heterogeneity. For a low-type contestant, if the contestant knows that his or her opponent is a high-type opponent, it is clear that his or her winning probability is slim; thus, he or she will slack off because making a great effort incurs a high cost but gains little in increasing the chance of winning. For a high-type contestant, if he or she knows that his or her opponent is a low-type opponent who will make little effort, there is no need for him or her to go to great lengths either. Therefore, contestants tend to make less effort in both the joint and separate contests than in the homogeneous-types case. Such slack-off behavior appears in other contest models, such as the all-pay auction model (see Clark and Riis 1998). Moreover, the slack-off behavior is more significant if contestants know their opponent's type in the second subcontest of the separate contest in comparison with the first subcontest and the joint contest, in which contestants only take into account the possibility that their opponent can be of a different type.

For the case of perfectly positive expertise correlation, if the random factors follow the normal distribution, the forces that push contestants to slack off under both mechanisms are dominated by the pooling effect; as a result, each contestant has a higher equilibrium effort level in the joint contest than in the separate contest. Further, for the case of perfectly negative expertise correlation, because contestants who are strong in one attribute must be weak in the other, there exists no slack-off effect in the joint contest given that contestants are ex ante identical in their expertise. However, the slack-off effects resulting from heterogeneity still exist in the separate contest, possibly in each subcontest. Hence, jointly driven by the pooling effect, each contestant always has a higher equilibrium effort level in the joint contest than in the separate contest. The dominance of the joint contest over the separate contest will be reinforced if the contestants' participation is endogenized.

Note that Proposition 5 is on each contestant's effort level. Actually, the contestant with a higher effort level may not be the winner because of random factors. However, if the project is highly effort based and the random factor is negligible, the contestant with a higher effort level will win with a high probability. In the case of a highly effort-based project, for perfectly positively correlated expertise, the expected best effort is higher in the joint contest than in the separate contest because the winner would be the same type in both subcontests—that is, a high-type contestant is strong in both attributes. However, for perfectly negatively correlated expertise, the best effort can be made by different contestants. If the difference between the high and low expertise is small, the expected best effort is higher in the joint contest than in the separate contest because the pooling effect is dominating. If that difference is sufficiently large, the expected best effort is higher in the separate contest than in the joint contest because the separate contest can combine the best effort levels from the two subcontests. A detailed discussion can be found in Section H.3 of Online Appendix H.

7. Conclusion

We compare the joint and separate contest mechanisms of crowdsourcing tournaments for projects with multiple attributes. With the characterization that a contestant's aggregate performance is made up of his or her effort levels and random factors across multiple dimensions, we find that the comparison comes down to comparing two opposing effects, the combination effect and the pooling effect. In addition, we obtain a set of managerial insights. First, the magnitudes of the two effects depend on the relative importance between the effort level and randomness in contestants' performances. With exponential or polynomial cost functions, if the performance has a high level of randomness, the combination effect is likely stronger than the pooling effect, and thus, the separate contest tends to be optimal. If the performance highly relies on the effort, the pooling effect is likely stronger than the combination effect, and the joint contest tends to be optimal. Second, we investigate how the number of contestants affects the comparison. Under some conditions, if the number of contestants is large enough, the separate contest tends to be optimal, and if the number of contestants is small enough, the joint contest tends to be optimal. Third, we compare the optimal prize between the two contest mechanisms. With exponential cost functions, the optimal prizes for the two contest mechanisms are the same. With polynomial cost functions, the optimal prize is higher for the joint contest than for the separate contest. Lastly, as an extension to the base model that assumes that all the contestants are homogeneous, we show that under some conditions, heterogeneous-expertise contestants make more efforts in the joint contest than in the separate contest because the separate contest has an information-disclosure function that may reveal contestants' types and induce slacking off. As, for tractability, we adopt somewhat different assumptions for the results mentioned above, Table 3 summarizes the key assumptions for all the results.

Pooling is a theme widely seen in the operations literature. In the joint contest, pooling of random factors reduces risk in effort making and incentivizes contestants to expend effort. Intuitively, the benefit of pooling, in favor of the joint contest, increases with the number of attributes and the variability in the random factors. However, the combination effect, in favor of the separate contest, is also expected to increase in those two factors. Again, a comparison of the two contest mechanisms comes down to the relative strengths of the combination effect and pooling effect.

There are many limitations to our model. For example, we assume that the random factors along different dimensions follow identical and independent distributions. Moreover, in considering the heterogeneous contestants, we examine a special case in which two contestants are endowed with two expertise types of equal probability. Despite these limitations, our model captures the core trade-off in comparison of joint versus separate contest mechanisms for projects with multiple attributes. It can serve as a base for future studies on designing multiattribute crowdsourcing contests.

Acknowledgments

The second author is the corresponding author of this paper. The authors gratefully acknowledge Gad Allon (department editor), the associate editor, and three anonymous referees for their valuable comments, which helped improve this paper significantly.

Endnotes

[1] Our results are applicable to another type of separate contest, the *progressive contest*, in which subcontests of the separate contest are run sequentially, and the solutions of the subsequent subcontest are built on the best work in progress from previous subcontests. For example, Quirky.com, a platform for crowdsourcing innovations, uses the business model of distributing prizes in subcontests at various stages of turning an idea into a final product. The stages start with idea generation, progress to product design, and may conclude with name and logo designs. A variant of the progressive contest employs elimination: only some contestants with excellent performances in the first subcontest will be selected to participate in the second subcontest. In Online Appendix E, we show that the main trade-off of our results still exists in such a contest form.

[2] The project with deliverables that are ideas, rather than physical products, is more likely to be modular even if created by different contestants because ideas are more fluid than physical products. For example, yutongo builds up a platform for managers to factor a project into different subquestions and collect ideas for each of them. In the idea-collection process, solutions will be rated, and the managers will select the best combination of ideas to build up the final solution. Yutongo's business model is analogous to the crowdsourcing contest for soliciting ideas (see https://www.yutongo.com/how-it-works). However, as the solution, a pure idea can be full of random inspirations, with little effort involved, whereas the solutions we focus on consist of both effort and randomness. When it requires little effort to generate ideas, the separate contest tends to be optimal.

[3] Our results are extendable to joint contests with decomposable solutions. We find that if the firm can decompose the aggregate solution submitted in the joint contest and assemble the best individual parts, the joint contest tends to be optimal. However, this does not imply that any unrelated projects should be run as joint contests just because they are modular and decomposable. Each contest probably requires specific professional skills, and running unrelated contest projects jointly can limit the participation of contestants (because not everyone has all the required skills) and thus lower the professional level of solutions. Such an effect is not captured in our model.

[4] In the absence of random factors, the separate mechanism always weakly dominates the joint mechanism.

[5] Proposition 1, the first and foremost result, does not require the distribution of the random factor to be symmetric, whereas most of our other results depend on the symmetric property. We show that as long as a regularity condition can be satisfied, Proposition 1 holds. All the symmetric log-concave distributions and some commonly used asymmetric log-concave distributions such as the gamma and Gumbel distributions satisfy the regularity condition.

[6] Some studies—for example, Ales et al. (2017), Kalra and Shi (2001), and Terwiesch and Xu (2008)—assume the performance to be in the form of $V = r(e) + \xi$, where $r(\cdot)$ is a concave function. Such a form of performance, together with the linear cost function, guarantees that the first-order condition characterizes the equilibrium strategy. With such a form, these studies assume that the effort e is nonnegative, though $r(e)$ may be negative. In our performance form of $V = e + \xi$, though, e could be negative, which is innocuous in comparing the performances across different schemes and is analogous to $r(e)$, that can be negative in others' performance forms; for consistency, we assume the effort to be nonnegative as well.

[7] If $A^1 > A^2$, A^2 in our context is not the so-called second prize in contest theory. There the second prize refers to a small prize awarded to the contestant whose total outcome ranks in the second place. Those studies are intended to solve the problem of whether WTA or some other reward scheme such as the scheme of multiple winners is optimal for the firm (see, e.g., Kalra and Shi 2001, Moldovanu and Sela 2001, Terwiesch and Xu 2008). In our context, a contestant may win the first subcontest but lose the second one. Whereas the two subcontests focus on different aspects of the project, a contestant who ranks first in the second subcontest wins the prize A^2.

[8] If $C^{l\prime}(\cdot)$, $l = 1, 2$, ranges over a bounded support, the symmetric equilibrium effort may be located at an endpoint of the range—for example, $e^{l*} = [C^{l\prime -1}(A^l g(0))]^+ = 0$, which boils down to a trivial case.

As a result, we restrict our attention to the case in which the equilibrium effort is an interior point.

[9] The result of Proposition 2 relies on a comparison between Δ_n^ξ and Δ_n^e. The difference Δ_n^ξ as a fixed positive value has nothing to do with β, whereas the value of Δ_n^e is strictly increasing in β. The case $\beta \geq \tilde{\beta}$ may not exist. For the exponential cost function (i.e., $C^l(\rho^l \beta e^l) = \exp(\rho^l \beta e^l)$, $l = 1, 2$), the difference in the equilibrium effort levels Δ_n^e has an upper bound because we require $C^{l'^{-1}}(A^l h(\xi; n)/(\rho^l \beta)) = \ln(A^l h(\xi; n)/(\rho^l \beta)) > 0$ and $C^{l'^{-1}}(Ah^\circ(\xi^\circ; n)/(\rho^l \beta)) = \ln(Ah^\circ(\xi^\circ; n)/(\rho^l \beta)) > 0$, $l = 1, 2$. This means that β cannot be arbitrarily large because we have, from above, $A^l h(\xi; n)/(\rho^l \beta) > 1$ and $Ah^\circ(\xi^\circ; n)/(\rho^l \beta) > 1$. Then those values of β such that $\beta \geq \tilde{\beta}$ may not exist because Δ_n^e may have an upper bound such that $\Delta_n = \Delta_n^\xi + \Delta_n^e < 0$ for all the possible values of β, and thus, the joint contest is always optimal. However, for the polynomial cost functions $C^l(\beta e^l) = a^l (\beta e^l)^{b^l}$, the value of Δ_n^e can be arbitrarily large; therefore, either contest mechanism can be optimal depending on the value of β.

[10] For simplicity, we normalize the cost functions such that $\beta = 1$. However, one can always scale the cost functions with a general scalar β.

[11] This assumption holds when the randomness in the performance comes from the preferences or the private tastes of judges, whereas the quality of the solution depends on the effort level.

[12] Our results are not sensitive to this assumption per se.

[13] Because the performance of a contestant in the first subcontest consists of the effort and the random factor, the way contestants infer their opponent's type depends on what kind of signal a contestant can obtain from the first subcontest such as the effort or the performance (i.e., the effort plus the random factor). If contestants can learn the performances in the first subcontest, they may not be able to accurately infer the type of their opponent because of the random factor, which makes the characterization of contestants' behavior extremely complicated.

References

Acemoglu D, Mostagir M, Ozdaglar A (2014) Managing innovation in a crowd. NBER Working Paper No. 19852, National Bureau of Economic Research, Cambridge, MA.

Ales L, Cho SH, Körpeoğlu E (2017) Optimal award scheme in innovation tournaments. *Oper. Res.* 65(3):693–702.

Ales L, Cho SH, Körpeoğlu E (2019) Innovation tournaments with multiple contributors. https://dx.doi.org/10.2139/ssrn.2277215.

Bagnoli M, Bergstrom T (2005) Log-concave probability and its applications. *Econom. Theory* 26(2):445–469.

Bimpikis K, Ehsani S, Mostagir M (2019) Designing dynamic contests. *Oper. Res.* 67(2):339–356.

Boudreau KJ, Lacetera N, Lakhani KR (2011) Incentives and problem uncertainty in innovation contests: An empirical analysis. *Management Sci.* 57(5):843–863.

Chakraborty I (1999) Bundling decisions for selling multiple objects. *Econom. Theory* 13(3):723–733.

Che YK, Gale I (2003) Optimal design of research contests. *Amer. Econom. Rev.* 93(3):646–671.

Chen Z, Mihm J, Schlapp J (2018) Sourcing innovation: Integrated system or individual components? https://dx.doi.org/10.2139/ssrn.3252967.

Choi JP (1991) Dynamic R&D competition under "hazard rate" uncertainty. *RAND J. Econom.* 22(4):596–610.

Clark DJ, Riis C (1998) Competition over more than one prize. *Amer. Econom. Rev.* 88(1):276–289.

DiPalantino D, Vojnovic M (2009) Crowdsourcing and all-pay auctions. *Proc. 10th ACM Conf. Electronic Commerce* (ACM, New York), 119–128.

Dixit A (1987) Strategic behavior in contests. *Amer. Econom. Rev.* 77(5):891–898.

Fey M (2008) Rent-seeking contests with incomplete information. *Public Choice* 135(3–4):225–236.

Fullerton RL, McAfee RP (1999) Auctionin entry into tournaments. *J. Political Econom.* 107(3):573–605.

Girotra K, Terwiesch C, Ulrich KT (2010) Idea generation and the quality of the best idea. *Management Sci.* 56(4):591–605.

Goltsman M, Mukherjee A (2011) Interim performance feedback in multistage tournaments: The optimality of partial disclosure. *J. Labor Econom.* 29(2):229–265.

Halac M, Kartik N, Liu Q (2017) Contests for experimentation. *J. Political Econom.* 125(5):1523–1569.

Harris C, Vickers J (1987) Racing with uncertainty. *Rev. Econom. Stud.* 54(1):1–21.

Hausch DB (1986) Multi-object auctions: Sequential vs. simultaneous sales. *Management Sci.* 32(12):1599–1610.

Hu M, Shi M, Wu J (2013) Simultaneous vs. sequential group-buying mechanisms. *Management Sci.* 59(12):2805–2822.

Jiang Z, Huang Y, Beil DR (2019) The role of feedback in dynamic crowdsourcing contests: A structural empirical analysis. https://dx.doi.org/10.2139/ssrn.2884922.

Kalra A, Shi M (2001) Designing optimal sales contests: A theoretical perspective. *Marketing Sci.* 20(2):170–193.

Konrad KA (2007) Strategy in contests: An introduction. https://dx.doi.org/10.2139/ssrn.960458.

Konrad KA, Kovenock D (2009) Multi-battle contests. *Games Econom. Behav.* 66(1):256–274.

Körpeoğlu E, Körpeoğlu CG, Hafalir IE (2018) Parallel innovation contests. https://dx.doi.org/10.2139/ssrn.2924817.

Krishna V (2009) *Auction Theory* (Academic Press, Burlington, MA).

Lazear EP, Rosen S (1981) Rank-order tournaments as optimum labor contracts. *J. Political Econom.* 89(5):841–864.

Lohr S (2012) Pentagon pushes crowdsourced manufacturing. *New York Times* (April 5), https://bits.blogs.nytimes.com/2012/04/05/pentagon-pushes-crowdsourced-manufacturing/.

Malueg DA, Tsutsui SO (1997) Dynamic R&D competition with learning. *RAND J. Econom.* 28(4):751–772.

Mas-Colell A, Whinston MD, Green JR (1995) *Microeconomic Theory* (Oxford University Press, New York).

Moldovanu B, Sela A (2001) The optimal allocation of prizes in contests. *Amer. Econom. Rev.* 91(3):542–558.

Moldovanu B, Sela A (2006) Contest architecture. *J. Econom. Theory* 126(1):70–96.

Rosen S (1986) Prizes and incentives in elimination tournaments. *Amer. Econom. Rev.* 76(4):701–715.

Stouras KI (2018) Crowdsourcing contests with private types and evaluation uncertainty. https://dx.doi.org/10.2139/ssrn.3076067.

Stouras KI, Girotra K, S Netessine (2015) First ranked first to serve: Strategic agents in a service contest. https://dx.doi.org/10.2139/ssrn.2696868.

Stouras KI, Hutchison-Krupat J, Chao RO (2019) The role of participation in innovation contests. https://dx.doi.org/10.2139/ssrn.2924224.

Taylor CR (1995) Digging for golden carrots: An analysis of research tournaments. *Amer. Econom. Rev.* 85(4):872–890.

Terwiesch C, Xu Y (2008) Innovation contests, open innovation, and multiagent problem solving. *Management Sci.* 54(9):1529–1543.

Yildirim H (2005) Contests with multiple rounds. *Games Econom. Behav.* 51(1):213–227.