

# Technical Note—Sequential Multiproduct Price Competition in Supply Chain Networks

Awi Federgruen

Graduate School of Business, Columbia University, New York, New York 10027, [af7@columbia.edu](mailto:af7@columbia.edu)

Ming Hu

Rotman School of Management, University of Toronto, Toronto, Ontario, Canada M5S 3E6, [ming.hu@rotman.utoronto.ca](mailto:ming.hu@rotman.utoronto.ca)

We analyze a general model in which, at each echelon of the supply process, an arbitrary number of firms compete, offering one or multiple products to some or all of the firms at the next echelon, with firms at the most downstream echelon selling to the end consumer. At each echelon, the offered products are differentiated and the firms belonging to this echelon engage in *price* competition. The model assumes a general set of piecewise linear consumer demand functions for all products (potentially) brought to the consumer market, where each product's demand volume may depend on the retail prices charged for all products; consumers' preferences over the various product/retailer combinations are *general* and *asymmetric*. Similarly, the cost rates incurred by the firms at the most upstream echelon are general as well. We fully characterize the equilibrium behavior under linear price-only contracts, and we show how all equilibrium performance measures can be computed via a simple recursive scheme. Moreover, we establish how changes in the model parameters, in particular, exogenous cost rates or intercept values in the demand functions, impact the system-wide equilibrium. These comparative statics results allow for the quantification of cost pass-through effects and the measurement and characterization of the firms' brand value. Lastly, we illustrate what qualitative impacts various changes in the structure of the supply chain network may bring forth.

**Keywords:** Stackelberg game; price competition; differentiated products; product assortment; sequential oligopoly; multiechelon; supply chain network.

**Subject classifications:** games/group decisions: noncooperative; marketing: pricing; mathematics: matrices.

**Area of review:** Operations and Supply Chains.

**History:** Received January 2015; revisions received June 2015, September 2015; accepted September 2015. Published online in *Articles in Advance* January 8, 2016.

## 1. Introduction

Economists and marketing and operations management researchers have developed many models of competition and coordination within supply chains. Most studies have focused on models where competition arises only at one echelon of the supply process. However, oligopolistic competition prevails at all echelons of the market.

We analyze a general model in which, at each echelon of the supply process, an arbitrary number of firms compete, offering one or multiple products to some or all of the firms at the next or possibly subsequent echelons, with firms in the most downstream echelon selling to the end consumer. At each echelon, the offered products are differentiated and the firms belonging to this echelon engage in *price* competition. Only a few two-echelon sequential oligopoly models have been addressed, with price competition at both echelons, and with an arbitrary number of competing firms and products; see Villas-Boas and Hellerstein (2006), Villas-Boas (2007b) and Bonnet et al. (2013).

However, to our knowledge, this is the first such model in which the existence of a subgame perfect Nash equilibrium is *proven*, and a full characterization of the equilibrium behavior is provided. Moreover, we provide such a characterization for a supply network with an arbitrary number of echelons. Finally, our model is one in which the product assortment

sold in the market is endogenously determined, along with all associated prices and demand volumes.

Our model assumes a general set of consumer demand functions for all  $N$  products (potentially) brought to the consumer market, where each product's demand volume may depend on the retail prices charged for *all* products. More specifically, the system of consumer demand functions, for all products potentially offered on the market, is based on a system of general *affine* functions.

However, the affine structure cannot be assumed to prevail on the complete price space: after all, outside of a special polyhedron  $P$ , the affine demand functions predict *negative* demand volumes. Shubik and Levitan (1980) stipulated that the generalization of the affine demand functions (on the complete price space, i.e., beyond  $P$ ) must satisfy a simple and intuitive regularity condition: under any given price vector, when some product is priced out of the market, i.e., has zero customer demand, any increase of its price has no impact on the demand volumes. Soon et al. (2009) showed that, under minimal conditions, there exists *one*, and *only one*, such a regular extension.

This consumer demand model has many advantages:

(i) The model allows for general combinations of direct and cross-price elasticities and, in particular, *asymmetric* demand functions.

(ii) The model is parsimonious, nevertheless, as it is fully specified by a single  $N \times N$  matrix of price sensitivity coefficients, and a single  $N$ -dimensional intercept vector.

(iii) Along with variants of the multinomial logit (MNL) model, the most frequently used demand model in marketing, operations management, and industrial organization studies, employs *affine* demand functions or extensions thereof; see, e.g., Federgruen and Hu (2015). Online Appendix C (available as supplemental material at <http://dx.doi.org/10.1287/opre.2015.1443>) discusses analogies and differences when fitting the extended affine versus the MNL model.

(iv) Depending on the set of prices selected by the competing firms, a different subset of all potential products is offered on the market. Thus, the model specifies a *product assortment*, along with specific associated demand volumes. This is in sharp contrast to all other commonly used demand models. For example, under the above variants of the MNL model, *all* products attain some market share, irrespective of their absolute and relative price levels.<sup>1</sup>

We assume that prices are selected sequentially, starting with the firms in the most upstream echelon, followed by a simultaneous price selection by all firms in the next, more downstream echelon, for all of their products, et cetera. In the marketing literature, this type of pricing interaction is referred to as “Manufacturer Stackelberg (MS) models.” Other types of interaction are conceivable, for example, vertical Nash (VN) relationships, in which all firms select their prices simultaneously. Empirical support for (MS) interactions was provided by Sudhir (2001), Che et al. (2007) and Villas-Boas and Zhao (2005).<sup>2</sup>

We initially study a two-echelon sequential oligopoly with competing suppliers, each selling *multiple* products through a pool of multiple competing retailers. We characterize the equilibrium behavior under linear price-only contracts. In the second stage, given wholesale prices selected in the first stage, all retailers simultaneously decide on their product assortment and retail prices to maximize their total profits among all products of all suppliers they choose to do business with. In the first stage, the suppliers anticipate the retailers’ responses when selecting their wholesale prices. We show that in this two-stage competition model, a subgame perfect Nash equilibrium always exists. Multiple subgame perfect equilibria may arise but, if so, all equilibria are *equivalent* in the sense of generating *unique* demands and profits for all firms. This characterization is obtained, as follows: Any choice of wholesale prices by the upstream firms induces a unique set of equilibrium retailer demand quantities, giving rise to an induced set of equilibrium demand functions for the first-stage competition model. Moreover, we show that this set of demand functions is structurally analogous to the demand functions faced by the retailers, thus allowing for a similar characterization of the equilibrium behavior among the suppliers. Finally, we derive a simple computational scheme for the (unique) equilibrium sales volumes and profit

levels of all firms as well as the component-wise lowest equilibrium price vectors at both echelons.

We subsequently generalize our results to supply chain models with an arbitrary set of echelons. The solution scheme is to backward inductively show that at every stage of the Stackelberg game, firms face a demand system uniquely specified by the downstream echelon best-response equilibria. This demand system has the same structural properties across all stages.

Our first and foremost contribution is to characterize the equilibrium behavior of a very general sequential oligopoly model with price competition at every echelon, and show how all equilibrium performance measures can be computed via a simple recursive scheme. Moreover, we establish how changes in the structure of the supply chain network, or changes in the model parameters, in particular, exogenous cost rates, or intercept values in the demand functions, impact the system-wide equilibrium. Changes in the network structure include the elimination of intermediate echelons, or changes in the number of firms in each echelon, or the specific supplier-retailer pairs that different final products are associated with. Our comparative statics results allow for the quantification of cost pass-through effects: more specifically, our model can be used to quickly ascertain what impact changes in raw material and component prices have on the equilibrium prices and product assortments of *all* firms at all echelons of the supply chain network. The channel pass-through problem is of central interest in the marketing literature; see, e.g., Moorthy (2005) and Ailawadi et al. (2010, §4) and the references therein, where it was addressed in a single-echelon setting. Similarly, the comparative statics with respect to the intercept vector, enable the measurement and characterization of the *brand value* of different retailers and suppliers, following the methodology of Golfarb et al. (2009).

The following *qualitative* insights arise from our comparative statics results:

(i) An increase of the marginal cost rate of any of the suppliers’ products is “passed on” to the suppliers’ wholesale prices and, subsequently, the retail prices charged by the retailers. Focusing on the componentwise smallest equilibrium price vectors, we prove that the above exogenous cost increase results in *all* products’ equilibrium wholesale and retail prices to go up.

(ii) While all “direct” and all “cross-brand” pass-through rates are nonnegative, these rates *decline* as a function of the suppliers’ marginal cost rates. In other words, when a supplier experiences an increase in the marginal cost rate for one of his products, the percentage pass-through applied to the equilibrium wholesale price of that product and all substitute products in the market is *lower* when the *absolute* level of the marginal cost rate is higher. The same holds for the pass-through rates that are applied to the equilibrium retail prices.

(iii) An increase of a supplier’s marginal cost rate for any of his products maintains or expands the equilibrium

product assortment: an expansion occurs when the cost increase enables other products that failed to be competitive, to capture a market share, after the cost rate increase.

(iv) As may be expected, when a supplier experiences an increase in the marginal cost rate of one of his products, the equilibrium sales volume of that product declines, but that of all substitute products (whether sold by the same supplier or any of the competitors) increases.

(v) For any of the supplier products' marginal cost rates, we show that increases beyond an easily calculable upper bound leave the equilibrium assortment, sales volumes and prices unchanged.

(vi) The *direct* pass-through rates with respect to the equilibrium wholesale prices are bounded from below by 50%, assuming the product is part of the market assortment, i.e., when the equilibrium sales volume is positive. In that case, the pass-through rates with respect to the equilibrium retail prices are bounded from below by 25%. No such *uniform* lower bounds can be obtained for the *cross-brand pass-through rates*, other than that they are always positive; see above. These threshold results are generally, although not uniformly, consistent with empirical findings; see Besanko et al. (2005) and Dubé and Gupta (2008), *inter alia*.

(vii) An increase in the value of any of the demand functions' intercepts elicits an increase in the equilibrium wholesale and retail prices, demand volumes, and the retailers' and suppliers' profit margins for all products. It also increases each firm's profit level and weakly expands the product assortment. This proves that the Goldfarb et al. (2009) methodology to measure brand values, assigns a *positive* value to any of these brand measures.

Section 2 reviews the related literature. Starting with a supply chain network of two echelons, §3 presents the model and preliminary results. In §4, we characterize the equilibrium behavior in the sequential two-stage competition model, and generalize the model and results to general sequential oligopolies involving *any number* of echelons. Section 5 studies the comparative statics and illustrates how the impact of various changes in the network *structure* can be assessed; see above. Section 6 extends, under minor conditions, all of our results in the two-echelon sequential oligopoly model to *asymmetric* price-sensitivity matrices. Section 7 concludes the paper.

## 2. Literature Review

Four papers initiate the study of *sequential* oligopolies, all with two duopoly echelons. McGuire and Staelin (2008) consider the special case of our model where there are two suppliers each selling a single product exclusively to a dedicated retailer; see §3. Choi (1996) generalizes this model to allow each of the suppliers to sell to both retailers. The demand functions are assumed to be affine on the complete price space. The model is used to compare various channel structures that arise when only some of the possible supplier/retailer combinations are able to trade.

Salinger (1988) assumes that at both echelons, two identical firms engage in Cournot competition for a homogenous good. Ordover et al. (1990) model an upstream duopoly of two *identical* firms that produce a *homogenous* good and engage in *price* competition, combined with a downstream duopoly of two firms each selling a differentiated product and engaging in Bertrand competition as well. If all four firms are independent, in equilibrium, the upstream suppliers sell the product at their (common) marginal cost, so that the model reduces to a standard Bertrand duopoly. Hart and Tirole (1990) consider a variant of the Ordover et al. (1990) model in which the two downstream firms sell a *homogenous* good and engage in *Cournot* competition, allowing for the two upstream firms to incur *different* cost rates. See Chen (2001) and Chen and Riordan (2007) for recent variants of the Ordover et al. (1990) model with sequential *price* competition among two duopolies.

Very few sequential oligopoly papers allow for an arbitrary number of firms at some or all of the echelons: Corbett and Karmarkar (2001) consider a market consisting of any number of echelons, however one in which a single homogenous final good is sold to the end consumer. At the most downstream echelon, firms engage in Cournot competition for the single homogenous good, with an affine demand function. At each echelon, all competing firms are assumed to have identical characteristics and to engage in Cournot competition as well. Cho (2014) investigates the impact of horizontal mergers in the Corbett and Karmarkar model. Saggi and Vettas (2002) consider a two-echelon market with two suppliers each selling a single product via its own dedicated network of retailers. Since all retailers within the same supplier's network sell the same undifferentiated product, they all charge the same price for this product. The prices for the two products are affine functions of the aggregate quantities sold in the market. Both the two suppliers, and the retailers engage, sequentially, in quantity competition.

Adida and DeMiguel (2011) analyzed the following generalization of Saggi and Vettas (2002): their model assumes  $M$  suppliers, each selling the same collection of  $P$  products to a set of  $N$  retailers. The consumers perceive each of the  $P$  products to be identical irrespective of which of the suppliers it is procured from. The demand model is specified by a set of affine inverse demand functions for all retailer/product combinations with multiplicative random noise factors. The retailers engage in quantity competition responding to announced wholesale prices, one for each of the  $P$  products, and optimizing a linear combination of the mean and standard deviation of their profits. The suppliers engage in *homogenous* Cournot competition for each of the  $P$  products, separately, based on the equilibrium aggregate function obtained from the retailer competition game. The equilibrium wholesale prices are those where aggregate retailer demand matches aggregate supplier supply. DeMiguel and Xu (2009) analyze a sequential competition model involving two groups of suppliers ultimately delivering



the same homogenous good to the same consumer market, under a stochastic demand function relating the common product price to the aggregate quantity sold.

Villas-Boas and Hellerstein (2006) and Villas-Boas (2007a, b) allow for an *arbitrary* number of suppliers, products, and retailers. In the latter, the demand functions are generated from a mixed MNL model. A system-wide sequential equilibrium is computed, *assuming* that the price competition game, at each stage, has a unique price Nash equilibrium and that this equilibrium is obtained as the unique solution of the system of First Order Conditions. However, even the equilibrium behavior in the retailer game, under exogenously given wholesale prices, is unknown, as of yet. Aksoy-Pierson et al. (2013) recently developed a set of sufficient conditions for the special case where each retailer sells a single product, but the equilibrium behavior in the multiproduct case is still an open question. This applies, a fortiori, to the suppliers' competition game in which the induced demand functions need to be derived from the equilibrium conditions in the retailer game. The approach in Villas-Boas (2007a) was used by Chu and Chintagunta (2009) and Bonnet et al. (2013) to characterize the U.S. server and the German coffee market, respectively. Villas-Boas and Hellerstein (2006) outline the same approach for a general set of differentiable demand functions; they proceed to illustrate the approach for the case of a two-supplier, two-retailer model with three products (discussed in §3, Figure 2), assuming affine demand functions.

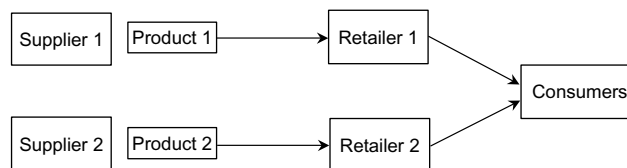
We defer the review of the literature on cost pass-throughs and the measurement of brand values in an equilibrium framework, to §5.

### 3. The Two-Echelon Model

Our base model considers a market where a set  $\mathcal{J} \equiv \{1, 2, \dots, J\}$  of suppliers compete by selling any number of grossly substitutable products, via the same pool  $\mathcal{I} \equiv \{1, 2, \dots, I\}$  of competing retailers. In §4, we generalize this to settings with an arbitrary number of echelons. As a concrete example, consider the market for television sets. Each of the mega brands (Samsung, RCA, Magnavox, Mitsubishi, etc.) sells a line of television types, differentiated by type (LCD or plasma), screen size (19", 27", 32", etc.), and screen resolution (720 versus 1,080 pixels), among other features. Different brands offer different subsets of the collection of all possible combinations; each sells these to some or all of the consumer electronics chains and general department stores.

We denote by  $\mathcal{N}$  the set of all products offered in the market and let  $N \equiv |\mathcal{N}|$ . To differentiate among different products, we employ a triple of indices  $(i, j, k)$ :  $i$  denotes the retailer via which the product is sold,  $j$  the supplier procuring the product. We allow a supplier to sell multiple products through a retailer, and use the index  $k$  to differentiate among the various products sold by supplier  $j$  to retailer  $i$ . We sometimes replace the triple index  $(i, j, k)$  by a single index  $l$ , where  $l$  may range from 1 to  $N$ . Let  $\mathcal{H}(i, j)$  denote the set

**Figure 1.** A two-product channel structure (see McGuire and Staelin 1983, 2008).



Note. We have  $N = 2$  items:  $\mathcal{N} = \{(1, 1, 1), (2, 2, 1)\}$ .

of products offered by supplier  $j$  to retailer  $i$ . A supplier may offer different sets of products to different retailers. For  $i \in \mathcal{I}$ ,  $j \in \mathcal{J}$  and  $k \in \mathcal{H}(i, j)$ , let

$c_{ijk}$  = the constant marginal supply cost of supplier  $j$  for product  $k$  sold at retailer  $i$ ,  
 $p_{ijk}$  = the retail price charged by retailer  $i$  for product  $k$  provided by supplier  $j$ ,  
 $w_{ijk}$  = the wholesale price charged by supplier  $j$  for product  $k$  sold at retailer  $i$ , and  
 $d_{ijk}$  = the consumer demand for product  $k$  provided by supplier  $j$  at retailer  $i$ .

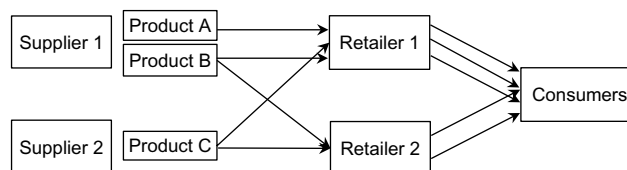
Let  $c$ ,  $p$ ,  $w$ , and  $d$  be the corresponding vectors.

Figure 1 depicts a simple structure with only  $N = 2$  products. This structure was considered by McGuire and Staelin (1983), later reprinted as McGuire and Staelin (2008). The authors compute the sequential equilibrium and compare it with those arising when (i) each supplier merges with his retailer; (ii) only one of the suppliers merges with his retailer. The equilibrium under (i) may be determined by fixing  $w_{111} = c_{111}$  and  $w_{221} = c_{221}$ , with that under (ii) by fixing only  $w_{111} = c_{111}$ .

Figure 2 displays a channel structure in which three products are offered by two suppliers. Since items are differentiated on the basis of the distributing retailer, the channel structure gives rise to  $N = 5$  distinct items. When eliminating product A, one retrieves the channel structure in Choi (1996) and Moorthy (2005), so that  $N = 4$ . (The latter and Villas-Boas and Hellerstein 2006 also consider settings where either product B or C is only offered to one of the retailers, reducing  $N$  to  $N = 3$ .)

Set  $\mathcal{N}(i)$  denotes the set of products offered to any retailer  $i$ , i.e.,  $\mathcal{N}(i) = \{(i, j, k) \mid j \in \mathcal{J}, k \in \mathcal{H}(i, j)\}$ , which is determined by the channel structure. Depending upon the prices selected by the suppliers and retailers, it is possible that only a subset of the products offered to any given

**Figure 2.** Another channel structure.



Notes. We have  $N = 5$  items:  $\mathcal{N} = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (2, 2, 1)\}$ . For instance, the triple  $(2, 1, 1)$  refers to the sole product (product B) that is sold through retailer 2 and provided by supplier 1.

retailer are actually sold there. Indeed, part of the retailers' choices is to determine their product assortment.

Our base model assumes that the suppliers may select arbitrary combinations of wholesale prices. In some settings, these price choices may need to be constrained: for example, in some countries, firms are restricted in terms of their ability to differentiate their prices for an "identical" product sold to different retailers. In Online Appendix E, we discuss the prevalence of such price restrictions, and how the equilibrium analysis and behavior is to be amended when every supplier has to charge an identical price to all retailers, for each of his products.

The demand for each product may depend on the prices of all products offered in the market. As in most supply chain competition models, this dependence is in principle described by general affine functions. In matrix notation, this gives rise to a system of demand equations:

$$q(p) = a - Rp, \tag{1}$$

where  $a \in \mathbb{R}_+^N \equiv \{r \in \mathbb{R} \mid r \geq 0\}$  and  $R \in \mathbb{R}^{N \times N}$ .

The matrix  $R$  is assumed to satisfy two properties: First, we assume that the various products are *substitutes*; this means that any product's demand volume does not decrease when the price of an alternative product is increased: see, however, Federgruen and Hu (2014) for a generalized model, allowing for certain types of complementarities.

ASSUMPTION (Z). *The matrix  $R$  is a Z-matrix, i.e., has nonpositive off-diagonal entries.*

In addition, we assume the following:

ASSUMPTION (P). *The matrix  $R$  is positive definite.*

However, the affine structure (1) can only apply on the polyhedron  $P = \{p \geq 0 \mid q(p) = a - Rp \geq 0\}$ , since, for a price vector  $p \notin P$ , the raw demand functions  $q(\cdot)$  predict negative demand volumes, for some of the products. Shubik and Levitan (1980) suggest that the extension of the demand functions, beyond  $P$ , satisfy the following intuitive regularity condition:

DEFINITION 1 (REGULARITY). A demand function  $D(p): \mathbb{R}_+^N \rightarrow \mathbb{R}_+^N$  is *regular*, if for any price vector  $p$  and product  $l$ , with  $D_l(p) = 0$ , an increase in  $p_l$  does not affect any of the demand volumes.

Soon et al. (2009) showed that there is one, and only one, *regular* extension  $d(p)$  of the affine demand functions  $q(p)$  in (1). Under this extension, the demand volumes generated under an arbitrary price vector  $p$ , are obtained by applying the affine function  $q(\cdot)$  to the *projection*  $\Omega(p)$  of  $p$  onto the polyhedron  $P$ , i.e.,

$$d(p) = q(\Omega(p)), \tag{2}$$

where for any  $p \in \mathbb{R}_+^N$ , the projection  $\Omega(p)$  of  $p$  onto  $P$  is defined as the vector  $p' = p - t$ , with  $t$  the unique solution to the following linear complementarity problem (LCP):

$$d(p) = a - R(p - t) \geq 0, \quad t \geq 0 \quad \text{and} \tag{3}$$

$$t^T [a - R(p - t)] = 0.$$

A unique solution  $t$  exists; see Cottle et al. (1992, Theorem 3.3.7) for the general theory of LCPs.

To simplify the exposition, we initially assume that the matrix  $R$  is symmetric:

ASSUMPTION (S). *The matrix  $R$  is symmetric.*

Empirical studies, e.g., Manchanda et al. (1999), Vilcassim et al. (1999), Dubé and Manchanda (2005), and Li et al. (2015), show that  $R$  is, often, asymmetric. In §6, we extend under weak restrictions all of our results to *asymmetric* price-sensitivity matrices  $R$ .

All vectors in this paper are column vectors and are represented by lowercase symbols. All matrices are denoted by capital letters. The complement set  $\bar{\mathcal{S}} \equiv \mathcal{N} \setminus \mathcal{S}$  for any index set  $\mathcal{S} \subseteq \mathcal{N}$ . For a vector  $a$  and an index set  $\mathcal{S}$ ,  $a_{\mathcal{S}}$  denotes the subvector with entries specified by  $\mathcal{S}$ . Similarly, for a matrix  $M$  and index sets  $\mathcal{S}, \mathcal{T} \in \mathcal{N}$ ,  $M_{\mathcal{S}, \mathcal{T}}$  denotes the submatrix of  $M$  with rows specified by the set  $\mathcal{S}$  and columns by the set  $\mathcal{T}$ . The transpose of a matrix  $M$  (vector  $a$ ) is denoted by  $M^T$  ( $a^T$ ). The symbol  $0$  denotes a scalar, a vector, or a matrix with all entries being zeros, and  $I$  an identity matrix of appropriate dimensions. The matrix inequality  $X = (x_{i,j}) \geq 0$  means that  $x_{i,j} \geq 0$  for all  $i, j$ .

### 3.1. The Retailer Competition Model

To characterize the equilibrium behavior in the sequential oligopoly model, we build on Federgruen and Hu (2015) establishing how the retailers respond to given wholesale prices  $w$ , selected by the suppliers, i.e., the equilibrium behavior in the resulting retailer competition game. To summarize the main results, we define the following vectors and matrices:

$$T(R) = \begin{pmatrix} R_{\mathcal{N}(1), \mathcal{N}(1)}^T & 0 & \cdots & 0 \\ 0 & R_{\mathcal{N}(2), \mathcal{N}(2)}^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{\mathcal{N}(I), \mathcal{N}(I)}^T \end{pmatrix}, \tag{4}$$

$$\Psi(R) = T(R)[R + T(R)]^{-1},$$

$$S \equiv \Psi(R)R = T(R)[R + T(R)]^{-1}R, \quad \text{and}$$

$$b \equiv \Psi(R)a. \tag{5}$$

Federgruen and Hu (2015) show that a pure Nash equilibrium always exists. Often, multiple, sometimes infinitely many, equilibria exist; however, all equilibria are *equivalent* in the sense of generating identical equilibrium sales volumes for all products. The dependence of these equilibrium sales volumes on  $w$  is described by a new set of affine functions:

$$Q(w) \equiv b - Sw, \tag{6}$$

when  $w \in W \equiv \{w \geq 0 \mid Q(w) = b - Sw \geq 0\}$ , or more generally, by the *unique* regular extension  $D(w)$  of this set of affine functions (for arbitrary  $w$ ):

$$D(w) \equiv Q(w') = b - Sw', \tag{7}$$

where  $w' = \Theta(w)$  is the projection of  $w$  onto the *effective wholesale price polyhedron*  $W$ :  $w' \equiv w - t$ , with  $t$  the unique vector such that  $t \geq 0$ ,  $b - S(w - t) \geq 0$  and  $t^T [b - S(w - t)] = 0$ . (When  $R$  is symmetric,  $b \geq 0$  and  $S$  is a  $Z$ -matrix; see Lemma 1; the projection  $\Theta(\cdot)$  is thus well defined with  $\Theta(w) \in W$  for all  $w \in \mathbb{R}^N$ .)

**PROPOSITION 1 (RETAILER COMPETITION MODEL).** Fix  $w \in W$ .

(a) *The retailer competition game has a pure Nash equilibrium.*

(b) *Multiple, pure, Nash equilibria may exist; however, there exists a componentwise smallest equilibrium*

$$p^* = w' + [R + T(R)]^{-1} q(w') \\ = [R + T(R)]^{-1} a + [R + T(R)]^{-1} T(R) w' \in P,$$

where  $w' = \Theta(w)$  is the projection of  $w$  onto  $W$ . (If  $w \in W$ ,  $w' = w$ .)

(c) *All equilibria  $\tilde{p}$  of this game have  $p^*$  as their projection, i.e.,  $\Omega(\tilde{p}) = p^*$ , and share the same sales volumes  $d(\tilde{p}) = q(\Omega(\tilde{p})) = q(p^*) = a - Rp^*$  and the same profit levels for all retailers.*

### 4. The Multistage Competition Model

By Proposition 1, any wholesale price vector  $w$  induces a retailer competition game with an *essentially* unique equilibrium: all equilibria are equivalent in the sense of generating identical sales volumes and profit levels for all retailers. If  $w \in W$ , the equilibrium sales volumes are given by

$$Q(w) = d(p^*(w)) = q(p^*(w)) \\ = a - R w - R [R + T(R)]^{-1} q(w) \\ = \{I - R [R + T(R)]^{-1}\} q(w) \\ = \{[R + T(R)] [R + T(R)]^{-1} - R [R + T(R)]^{-1}\} q(w) \\ = T(R) [R + T(R)]^{-1} q(w) \\ = \Psi(R) q(w) = b - S w, \tag{8}$$

where the second identity follows from  $p^* \in P$  and the third identity from Proposition 1 parts (b) and (c). This confirms (6). Similarly, if  $w \notin W$ , we have by Proposition 1 parts (b) and (c) that (7) is confirmed since

$$D(w) = Q(w') = \Psi(R) q(w') = b - S w'. \tag{9}$$

Thus, the induced demand functions encountered by the suppliers are the (unique) regular extension of the affine functions (6), as long as we can show that the matrix  $S$  is positive definite and has nonpositive off-diagonal elements, i.e., it satisfies properties (P) and (Z), in the same way the original matrix of price sensitivity coefficients  $R$  does. Fortunately, both properties can be shown to apply. This implies

that the supplier competition model is structurally analogous to the retailer competition model. We first characterize the equilibrium behavior in this first-stage competition model: We again define equilibria to be *equivalent* if they result in the same sales volumes for all supplier/product combinations and the same profit values for all suppliers.

**THEOREM 1.** (a)  *$D(\cdot)$  is the unique regular extension of the affine induced demand function; see (6); the matrix  $S = \Psi(R)R$  is a positive definite, symmetric  $Z$ -matrix while  $b \geq 0$ .*

(b) *If a pure equilibrium exists, there exists one and only one equilibrium in  $W$ .*

(c) *Any equilibrium  $w^0 \notin W$ , has  $\Theta(w^0) = w^*$ ; moreover, all equilibria are equivalent.*

Theorem 1 establishes, for the supplier competition model, a major part of the full equilibrium characterization in Proposition 1 (the latter pertaining to the retailer competition model): if a pure Nash equilibrium exists, there exists a componentwise smallest equilibrium  $w^*$ , which belongs to  $W$ ; all other equilibria have  $w^*$  as its projection on  $W$ , and are equivalent to  $w^*$ .

To complete the full equilibrium characterization like Proposition 1, we need to show that a pure Nash equilibrium is guaranteed to exist and to provide an explicit formula for the componentwise smallest equilibrium  $w^*$ . Reorder the products so that their supplier index in the triple of indices  $(i, j, k)$  comes *first*. Let  $\mathcal{M}(1), \dots, \mathcal{M}(J)$  denote the sets of products supplied by supplier 1,  $\dots, J$ , respectively. Define the matrix

$$T(S) \equiv \begin{pmatrix} S_{\mathcal{M}(1), \mathcal{M}(1)}^T & 0 & \dots & 0 \\ 0 & S_{\mathcal{M}(2), \mathcal{M}(2)}^T & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & S_{\mathcal{M}(J), \mathcal{M}(J)}^T \end{pmatrix}$$

and  $\Psi(S) \equiv T(S)[S + T(S)]^{-1}$ . Note that the operator that transforms  $R$  into  $T(R)$  is different from that mapping  $S$  into  $T(S)$ . We nevertheless use the same mapping  $T(\cdot)$  for both operators to simplify the notation. Finally, analogous to the definition of the effective wholesale price polyhedron  $W$ , define the effective polyhedron  $C$  of supplier cost vectors as follows:  $C = \{c \geq 0 \mid \Psi(S)Q(c) \geq 0\}$ . Let  $\Gamma(\cdot)$  denote the projection operator onto  $C$ , which can be defined analogously to the projection operators  $\Omega(\cdot)$  and  $\Theta(\cdot)$ .

**THEOREM 2 (CHARACTERIZATION OF EQUILIBRIA IN THE SUPPLIER COMPETITION GAME).** (a)  *$C \neq \emptyset$ , since  $0 \leq c^0 = S^{-1}b = R^{-1}a \in C$ .*

(b) *If  $c \in C$ , there exists a unique wholesale price equilibrium  $w^*(c)$  in  $W$ . Any equilibrium  $w^0$  outside of  $W$  has  $\Theta(w^0) = w^*(c)$  and is equivalent to  $w^*(c)$ .*

(c) *If  $c \notin C$ , let  $c' = \Gamma(c)$  denote the projection of  $c$  onto  $C$ . Then  $w^*(c')$  is the unique wholesale price equilibrium in  $W$ . Any equilibrium  $w^0 \notin W$  has  $\Theta(w^0) = w^*(c')$  and is equivalent to  $w^*(c')$ .*

Downloaded from informs.org by [128.100.42.73] on 11 February 2016, at 19:38. For personal use only, all rights reserved.

Thus, for any cost rate vector  $c \in \mathbb{R}_+^N$ , there exists a componentwise smallest equilibrium  $w^*(c)$  in the supplier competition game. Moreover, analogous to Proposition 1(b), one can show

$$w^*(c) = c + [S + T(S)]^{-1}Q(c) \\ = [S + T(S)]^{-1}b + [S + T(S)]^{-1}T(S)\Gamma(c). \quad (10)$$

Note that on the polyhedron  $C$ ,  $\Gamma(c) = c$  so that  $w^*(\cdot)$  is an affine function on this polyhedron.

Finally, we can show that the effective price polyhedra  $P$ ,  $W$ , and  $C$  are nested.

PROPOSITION 2.  $P \subseteq W \subseteq C$ .

Online Appendix D illustrates our results with respect to the distribution structure in McGuire and Staelin (2008).

We now discuss the generalization of our two-echelon model to one in which products (potentially) travel through an arbitrary number of distribution/production stages before reaching the end consumer. In the chain of oligopolies, there are  $m$  echelons,  $E_1, \dots, E_m$ , each with an arbitrary number of competing distributors. We number the echelons sequentially, starting with the most downstream echelon of retailers until reaching the most upstream echelon  $m$ . We assume that firms in a given echelon only sell to firms in the next more downstream echelon, i.e., firms in echelon  $e$  only sell to those in echelon  $e - 1$ , and the retailers in echelon 1 sell to the consumer.

Products are partially differentiated by the route  $r$  traveled in the above multipartite network. For any such path  $r \in \mathcal{R}$ , the set of all possible paths, there may be up to  $K$  distinct products. We thus label each distinct product with a pair of indices  $(r, k)$ : product  $(r, k)$  is the  $k$ th product distributed along the route  $r$ ,  $r \in \mathcal{R}$ , and  $k = 1, \dots, K$ .

Our starting point is, again, a set of retailer demand functions  $d^{(1)}(p^{(1)})$ , with  $p^{(1)}$  the vector of retail prices, specified as follows on  $\mathbb{R}_+^N$ :

$$d^{(1)}(p^{(1)}) \\ \equiv \begin{cases} q^{(1)}(p^{(1)}) = a^{(1)} - R^{(1)}p^{(1)}, \\ \quad \text{if } p^{(1)} \in P^{(1)} \equiv \{p^{(1)} \geq 0 \mid q^{(1)}(p^{(1)}) \geq 0\}, \\ q^{(1)}(\Omega^{(1)}(p^{(1)})), \\ \quad \text{if } p^{(1)} \notin P^{(1)}. \end{cases} \quad (11)$$

Here  $a^{(1)}$  and  $R^{(1)}$  are exogenously given, and  $\Omega^{(1)}(p^{(1)})$  is the projection of the vector  $p^{(1)}$  onto  $P^{(1)}$ , as  $\Omega(\cdot)$  defined in §3, with  $a$  and  $R$  replaced by  $a^{(1)}$  and  $R^{(1)}$ . Following the analysis of §3, one verifies that each of the remaining echelons  $e = 2, \dots, m$  experiences equilibrium demand functions of a similar structure. Define, recursively,

$$a^{(e)} \equiv \Psi^{(e)}(R^{(e-1)})a^{(e-1)}, \quad R^{(e)} \equiv \Psi^{(e)}(R^{(e-1)})R^{(e-1)}, \quad (12)$$

where

$$\Psi^{(e)}(R^{(e-1)}) \equiv T^{(e-1)}(R^{(e-1)})[R^{(e-1)} + T^{(e-1)}(R^{(e-1)})]^{-1}, \\ e = 2, \dots, m.$$

The matrix  $T^{(e)}(R^{(e)})$  is obtained from the matrix  $R^{(e)}$  by replacing by zero any entry that corresponds with a pair of products that is distributed via different distributors in echelon  $e$ ; for any pair of products  $(l, l')$  that is distributed via the same distributor in echelon  $e$ ,  $T^{(e)}(R^{(e)})_{l, l'} = R^{(e)}_{l, l'}$ . As shown in §4, after appropriate sequencing of the products, the matrix  $T^{(e)}(R^{(e)})$  is block diagonal, with each block corresponding with a specific distributor in echelon  $e$ . The indirect demand functions for the firms in echelon  $e$  are, again, of the form given by (11):

$$d^{(e)}(p^{(e)}) \equiv \begin{cases} q^{(e)}(p^{(e)}) = a^{(e)} - R^{(e)}p^{(e)}, \\ \quad \text{if } p^{(e)} \in P^{(e)} \equiv \{p^{(e)} \geq 0 \mid q^{(e)}(p^{(e)}) \geq 0\}, \\ q^{(e)}(\Omega^{(e)}(p^{(e)})), \\ \quad \text{if } p^{(e)} \notin P^{(e)}, \end{cases}$$

where  $\Omega^{(e)}(p^{(e)})$  is the projection of the price vector  $p^{(e)}$  onto  $P^{(e)}$ ,  $e = 2, \dots, m$ . As before  $P^{(e)} \neq \emptyset$ , since  $0 \leq R^{(e-1)^{-1}}a^{(e)} = R^{(e-1)^{-1}}a^{(e-1)} = \dots = R^{-1}a \in P^{(e)}$ ,  $e = 2, \dots, m$ .

Applying Theorem 1(a) recursively one verifies that, the matrix  $R^{(e)}$ ,  $e = 2, \dots, m$ , is positive definite. By Proposition 1(a) (or Theorem 1(b)), this, by itself, guarantees the existence of equilibria at any stage of the sequential competition game.

However, to ensure that the indirect equilibrium demand functions for echelon  $e$  are well defined, we, of course, need to establish that a unique sales volume vector arises in equilibrium at any downstream echelon  $l = 1, 2, \dots, e - 1$ , i.e., the equivalency of equilibria in the sense of generating the same sales volumes and profit levels throughout all downstream echelons (if multiple equilibria exist, they all project onto the same price vector in  $P^{(l)}$  for any downstream echelon  $l = 1, 2, \dots, e - 1$ ). By Proposition 1 (or Theorem 2), this is guaranteed as long as each of the matrices  $R^{(1)}, R^{(2)}, \dots, R^{(e)}$  is a Z-matrix, i.e., has nonpositive off-diagonal elements, and  $a^{(1)}, a^{(2)}, \dots, a^{(e)} \geq 0$ . This condition can easily be checked numerically. In addition, by the proof of Lemma 1, the condition can be guaranteed, inductively, when  $R^{(1)} = R$  is a symmetric matrix.

We conclude that as long as we can guarantee that each of the matrices  $R^{(1)}, R^{(2)}, \dots, R^{(m+1)}$  is a Z-matrix and  $a^{(1)}, a^{(2)}, \dots, a^{(m+1)} \geq 0$  (where  $R^{(m+1)}$  and  $a^{(m+1)}$  follow the same type of definition as  $R^{(e)}$  and  $a^{(e)}$ ,  $e = 2, \dots, m$ ), one essentially unique equilibrium exists at each stage of the sequential competition game; the resulting chain-wide equilibrium is a subgame perfect Nash equilibrium. Moreover, for any echelon  $e$ , the values of the unique price equilibrium in  $P^{(e)}$ , and the componentwise smallest price equilibrium among all equilibria, are computed as follows: Let  $c$  denote



the vector of marginal cost rates incurred by the firms in the most upstream echelon:

$$p^{*(m)} = \begin{cases} c + [R^{(m)} + T^{(m)}(R^{(m)})]^{-1} q^{(m)}(c), & \text{if } c \in C \equiv P^{(m+1)}, \\ \Omega^{(m+1)}(c) + [R^{(m)} + T^{(m)}(R^{(m)})]^{-1} q^{(m)}(\Omega^{(m+1)}(c)), & \text{if } c \notin C, \end{cases} \quad (13)$$

where  $P^{(m+1)}(\ni R^{-1}a)$  follows the same type of definition as  $P^{(e)}$ ,  $e = 2, \dots, m$ , and

$$p^{*(e)} = p^{*(e+1)} + [R^{(e)} + T^{(e)}(R^{(e)})]^{-1} q^{(e)}(p^{*(e+1)}), \quad e = m - 1, \dots, 1. \quad (14)$$

To verify (13) and (14), invoke Proposition 1. Moreover, since  $p^{*(m)} \in P^{(m)}$ , it follows from Proposition 1 that  $p^{*(m-1)}$  is given by (14) and  $p^{*(m-1)} \in P^{(m-1)}$ . One thus verifies, by induction, that  $p^{*(e)} \in P^{(e)}$  for all echelons  $e = 1, \dots, m$ , so that (14) applies to all echelons  $e = 1, \dots, m - 1$ .

The computation of all echelons' equilibrium price vector  $\{p^{*(e)} \mid e = 1, \dots, m\}$  is thus confined to the following: first one recursively computes the matrices  $R^{(e)}$  and intercept vectors  $a^{(e)}$  for  $e = 1, \dots, m + 1$ , via (12). Determination of  $p^{*(m)}$  may involve the computation of the unique solution of an LCP—but only if  $c \notin C$ —, which can be achieved by solving a single linear program; see Lemma A.2(c) in the online Appendix A. The remaining computations involve only multiplications and inversions of matrices related to the price sensitivity matrix  $R$ .

Finally, as we move upstream, the sequence of effective price polyhedra expands, converging to a limiting polyhedron.

**PROPOSITION 3.** Assume  $R = R^{(1)}$  is symmetric.

(a) For any  $m \in \mathbb{N}$ ,  $P^{(e)} = \{p \geq 0 \mid a^{(e)} - R^{(e)}p \geq 0\} \subseteq P^{(e+1)}$  for all  $e = 1, 2, \dots, m$ .

(b) For any  $m \in \mathbb{N}$ ,  $P^{(e)} \subseteq H \equiv \{p \mid 0 \leq p \leq R^{-1}a\}$  for all  $e = 1, 2, \dots, m + 1$ .

(c) The sequence  $\{P^{(e)}, e = 1, 2, \dots, m + 1\}$  converges to a limiting polyhedron  $P^*$ .

## 5. Comparative Statics and the Impact of the Network Structure

In this section, we characterize the impact of various model parameters on equilibrium performance measures. We focus, in particular, on the suppliers' cost rate vector  $c$  (§5.1) and the intercept vector  $a$  of the demand functions  $q(\cdot)$  (§5.2), as both relate to important managerial questions. In §5.3 we show how our results can be used to assess the impact of changes in the industry structure.

Much attention has been given to understanding the pass-through rates of exogenous cost changes: when a supplier changes the wholesale price for a given product, how will the retail price of the same product (direct pass-through rate) and

other products (cross pass-through rates) respond? The literature has adopted two approaches: (i) *structural/theoretical models* derive the pass-through rates from a formal market model by characterizing how equilibrium prices depend on exogenous cost rates; and (ii) *reduced form econometric models* stipulate a specific functional relationship between cost rates and equilibrium prices, unsupported by any underlying competition model, and uses empirical data to estimate the parameters in the resulting regression model.

Few papers follow the *first* approach, possibly because of the difficulty to characterize the equilibria in multiproduct multiretailer models. Besanko et al. (2005, §2.2) provided a review of five such papers; all but one assume a retail market with a *single* retailer. Moorthy (2005) addressed the question in a special model with *two* manufacturers and *two* retailers, which arises as a special case of the network structure in Figure 2, without product A. (Moorthy also considers nonlinear demand functions, with several concavity, supermodularity and dominant diagonal properties, to ensure the existence of a unique equilibrium.) Goldberg (1995) characterized the pass-through rates of exogenous wholesale prices in a model with nested logit demand functions.

For the reduced form approach, the seminal paper is Besanko et al. (2005), estimating the pass-through behavior at Dominick's Finer Foods, a U.S. supermarket chain. The study involved 78 products over 11 categories. (Earlier contributions like Chevalier and Curhan 1976 used accounting measures rather than a rigorous econometric study.) Besanko et al. (2005) stipulate either an *affine* dependency of the equilibrium retail prices with respect to wholesale prices or an affine relationship among the *logarithms* of these prices. We will *prove* that the former (affine) structure prevails in our model, but only as long as the wholesale prices are selected within  $W$ . When  $w \notin W$ , the same affine functions need to be applied to  $\Theta(w)$ , its projection onto  $W$ . The statistical validity of the estimation results in Besanko et al. (2005), in particular the significance of cross-brand pass-through rates, was challenged by McAlister (2007). This resulted in a refined study by Dubé and Gupta (2008), confirming that most cross-brand pass-through rates are significant, indeed. To our knowledge, ours represents the first paper in which the impact of exogenous cost changes is characterized in a multiechelon supply network of competing firms, i.e., in a sequential oligopoly.

Goldfarb et al. (2009) have argued that a firm's brand value should be measured in an equilibrium framework. More specifically, consumer demand functions should be modeled as a function of the suppliers' and/or retailers' brands, represented by brand indicator variables.

The brand value of a firm is defined as the *difference* between its profit value when the brand indicator variable equals one (i.e., in the presence of the brand effect), versus a counterfactual equilibrium value, when it is set equal to zero (i.e., in the absence of the brand effect). The authors apply this framework to a sequential two-echelon price competition model, with a single retailer, i.e., with  $I = 1$ ,



but  $J$  and  $K$  arbitrary. (Even so, the authors must assume that the first-stage competition model among the suppliers is well defined and has a unique price Nash equilibrium, arising as the unique solution of the system of First Order Conditions.) The model was then applied to the ready-to-eat breakfast cereal market, with  $J = 5$  national suppliers.

More specifically, Goldfarb et al. (2009) assume that demands for the various products are specified by a mixed MNL model, in which the *intercept* of the utility measure of each product is specified as an affine function of suppliers' brand indicator variables. Following the same approach in our demand model, we specify the intercepts as follows:

$$a_{ijk} = \alpha^T x_{ijk} + \sum_{j'=1}^J \beta_j z_{jj'},$$

all  $i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K,$

where  $z_{jj'} = 1$  if  $j = j'$  and  $z_{jj'} = 0$  if  $j \neq j'$ ; and  $x_{ijk}$  represents a vector of observable attribute values for product  $(i, j, k)$ . The same methodology may be used to measure brand values associated with the different retailers, or with different subbrands, i.e.,  $(j, k)$ -combinations. All of these brand value estimations amount to conducting comparative statics analyses with respect to the intercept vector  $a$ ; this is the subject of §5.2.

### 5.1. Comparative Statics with Respect to the Cost Rates $c$

In this subsection, we characterize the impact of changes in the suppliers' cost rates, with respect to equilibrium prices, sales volumes, and the product assortment. All effects are computable with little effort, requiring at most a few matrix multiplications and inversions and the solution of a single linear program with  $N$  variables and constraints. Moreover, we derive various general first- and second-order monotonicity properties for the relationship between equilibrium retail and wholesale prices, on the one hand, and the cost rates on the other.

**THEOREM 3 (COMPARATIVE STATICS FOR THE COST RATES  $c$ ).** Fix a cost rate vector  $c^0$  and a product  $l = (i, j, k)$ , and consider the impact of an increase of  $c_{ijk}$  from  $c_{ijk}^0$  to  $c_{ijk}^0 + \delta$ .

(a) (Equilibrium demand volumes). There exists a minimal threshold  $\Delta^+ \geq 0$  such that an increase of  $\delta$  beyond  $\Delta^+$  has no impact on any of the equilibrium demand volumes; when  $\delta \leq \Delta^+$ , product  $l$ 's demand volume decreases and the demand volume of all other products increases.

(b) (Equilibrium assortment). An increase of  $\delta$  beyond  $\Delta^+$  has no impact on the equilibrium assortment; when  $\delta \leq \Delta^+$ , the equilibrium assortment remains the same or expands. There exists a second threshold  $\Delta \leq \Delta^+$  such that, for  $\delta \leq \Delta$ , the equilibrium assortment does not change while product  $l$ 's demand volume decreases and that of all other products  $l' \neq l$  increases proportionally with  $\delta$ .

(c) (Equilibrium prices). The componentwise smallest equilibrium retail and wholesale price vectors  $p^*$  and  $w^*$  increase concavely with  $\delta$ .

Beyond these monotonicity properties, our model permits simple expressions of the pass-through rates of cost changes. For any  $\mathcal{A} \subseteq \mathcal{N}$ , let  $R^{\mathcal{A}} \equiv R_{\mathcal{A}, \mathcal{A}} - R_{\mathcal{A}, \bar{\mathcal{A}}} R_{\bar{\mathcal{A}}, \bar{\mathcal{A}}}^{-1} R_{\bar{\mathcal{A}}, \mathcal{A}}$  and  $S^{\mathcal{A}} = \Psi(R^{\mathcal{A}})R^{\mathcal{A}}$ . Let  $(\partial p^*/\partial c)^-$  denote the matrix of left-hand derivatives, which is shown to always exist.

**COROLLARY 1.** (a) In the retailer competition model under a wholesale price vector  $w \in W^\circ$ ,

$$\left(\frac{\partial p^*}{\partial w}\right) = [R + T(R)]^{-1}T(R) \geq 0.$$

(b) Assume  $c \in C^\circ$ .

$$\left(\frac{\partial p^*}{\partial c}\right) = [R + T(R)]^{-1}T(R)[S + T(S)]^{-1}T(S) \geq 0.$$

(c) Fix  $c \in \mathbb{R}_+^N$ . Let  $\mathcal{A}$  denote the unique assortment associated with the price equilibria. Then

$$\left(\frac{\partial p^*}{\partial c}\right)^- = [R^{\mathcal{A}} + T(R^{\mathcal{A}})]^{-1}T(R^{\mathcal{A}})[S^{\mathcal{A}} + T(S^{\mathcal{A}})]^{-1}T(S^{\mathcal{A}}). \tag{15}$$

**REMARK 1.** An expression, similar to (15), provides the matrix of right-hand derivatives  $(\partial p^*/\partial c)^+$ . In fact, as explained, almost everywhere,  $(\partial p^*/\partial c)^+ = (\partial p^*/\partial c)^- = (\partial p^*/\partial c)$ . However, as shown in the proof of Theorem 3(b), it is possible that, for a given product  $l$ , any increase of its cost rate  $c_l$  results in a new product  $l'$ , to be added to the equilibrium assortment  $\mathcal{A}$ , resulting in a new assortment  $\mathcal{A}^+$ . (This corresponds with the case where in Theorem 3(b), the threshold  $\Delta = 0$ .) In that case, the matrix  $(\partial p^*/\partial c)^+$  is given by (15), with  $\mathcal{A}$  replaced by  $\mathcal{A}^+$ .

Although the exact expressions of the cost pass-through rates in (15) are easily computed with a few matrix multiplications and inversions, we derive simpler lower and upper bounds that provide insights into the pass-through rates. For example, the lower bound shows that at least 50% of a reduction in the wholesale price of a product and at least 25% of a reduction in the supply cost rate are passed on to the consumer.

**PROPOSITION 4 (BOUNDS FOR THE COST PASS-THROUGH RATES).** (a) Consider the retailer competition model under a given wholesale price vector  $w$ . Let  $\mathcal{A}$  denote the equilibrium assortment. Then

$$\frac{1}{2} \leq \left(\frac{\partial p_{\mathcal{A}}^*}{\partial w_{\mathcal{A}}}\right)^- = [R^{\mathcal{A}} + T(R^{\mathcal{A}})]^{-1}T(R^{\mathcal{A}}) \leq \frac{(R^{\mathcal{A}})^{-1}T(R^{\mathcal{A}})}{2}. \tag{16}$$

Downloaded from informs.org by [128.100.42.73] on 11 February 2016, at 19:38. For personal use only, all rights reserved.

(b) Fix  $c \in \mathbb{R}_+^N$ . Let  $\mathcal{A}$  denote the equilibrium assortment. Then

$$\frac{1}{4} \leq \left( \frac{\partial p_{\mathcal{A}}^*}{\partial c_{\mathcal{A}}} \right)^{-} = [R^{\mathcal{A}} + T(R^{\mathcal{A}})]^{-1} T(R^{\mathcal{A}}) [S^{\mathcal{A}} + T(S^{\mathcal{A}})]^{-1} T(S^{\mathcal{A}}) \leq \frac{(R^{\mathcal{A}})^{-1} T(R^{\mathcal{A}}) (S^{\mathcal{A}})^{-1} T(S^{\mathcal{A}})}{4}.$$

In Online Appendix D, we apply these results to the McGuire and Staelin (2008) channel structure.

### 5.2. Comparative Statics with Respect to the Intercept Vector $a$

We now derive comparative statics results for the intercept vector  $a$ . We show that all equilibrium retail and wholesale prices increase and that the equilibrium product assortment expands when the intercept vector  $a$  increases. In addition, an increase of one of or more of the intercept values causes all of the suppliers' and retailers' profit margins to grow, as well as their aggregate profit values. One implication is that all brand values, discussed at the beginning of the section, are positive. We write  $p^*$  and  $w^*$ , as well as the projection operator  $\Gamma$ , as  $p^*(w, a)$ ,  $w^*(c, a)$  and  $\Gamma(c, a)$ .

**THEOREM 4 (COMPARATIVE STATICS ON  $a$ ).** Fix  $c \geq 0$  and  $0 \leq a^1 \leq a^2$ . An increase in  $a$  elicits an increase in the equilibrium wholesale and retail prices, demand volumes, and the retailers' and suppliers' profit margins for all products. It also increases each firm's profit level and expands the product assortment. In other words,

- (a) (Wholesale prices).  $w^*(\Gamma(c, a^1), a^1) \leq w^*(\Gamma(c, a^2), a^2)$ .
- (b) (Retail prices).  $p^*(w^*(\Gamma(c, a^1), a^1), a^1) \leq p^*(w^*(\Gamma(c, a^2), a^2), a^2)$ .
- (c) (Demand volumes).  $d(p^*(w^*(\Gamma(c, a^1), a^1), a^1)) \leq d(p^*(w^*(\Gamma(c, a^2), a^2), a^2))$ .
- (d) (Assortment).  $\mathcal{A}(a^1) \subseteq \mathcal{A}(a^2)$ .
- (e) (Retail profit margins).  $p^*(w^*(\Gamma(c, a^1), a^1), a^1) - w^*(\Gamma(c, a^1), a^1) \leq p^*(w^*(\Gamma(c, a^2), a^2), a^2) - w^*(\Gamma(c, a^2), a^2)$ .
- (f) (Wholesale profit margins).  $w^*(\Gamma(c, a^1), a^1) - \Gamma(c, a^1) \leq w^*(\Gamma(c, a^2), a^2) - \Gamma(c, a^2)$ .
- (g) (Profit levels). The profit earned by each firm increases with the intercept vector  $a$ .

Part (g) implies that brand values, as defined in Goldfarb et al. (2009), are always nonnegative.

### 5.3. The Impact of Changes in the Network Structure

In this subsection, we show how our results can be used to evaluate various changes in the network structure. We start with the impact of disintermediation, i.e., when an intermediate echelon of distributors (or "middlemen") is eliminated. We then illustrate how for a given set of final

products, the number of retailers and suppliers as well as the network structure in the industry impact on aggregate profits and the consumer surplus measure. (The network structure specifies which supplier-retailer pair each of the products is associated with.) Under the heading "excessive competition," we exemplify how the simultaneous presence of a very large number of retailers may result in all of them being driven out of the market, in equilibrium, even in the absence of any fixed operating costs. We also illustrate the asymptotic behavior of various cost pass-through rates.

**Disintermediation.** Consider a market with  $J$  manufacturers each selling a group of products to a manufacturer-associated chain of independently owned retailers. Initially, each manufacturer  $j$  sells its products via a dedicated wholesaler at a given price vector  $c^j$ . By Proposition 2,  $P \subseteq W \subseteq C$ . Assume  $c = (c^1, \dots, c^J)^T$  is in the interior of  $C \setminus W$ . What is the impact of disintermediation, i.e., what happens when the retailers can buy the products directly from their manufacturers? On the one hand, it is possible to show that retail prices will come down. More surprisingly, however, product variety *decreases*, all cost efficiencies notwithstanding: in the presence of the intermediary wholesalers, we get  $w^*(c) \in W^\circ$ , hence  $p^*(w^*(c)) \in P^\circ$ , i.e., all products are sold in the market. Without the intermediaries,  $c$  is the vector of "wholesale" prices. Since  $c \notin W$ ,  $p^*(c)$  is on the boundary of  $P$  implying that some products are no longer part of the retailer assortment.

**Network Structure in Two-Echelon Network.** We consider a market with  $N = 120$  substitutable products. Each product  $l = 1, \dots, 120$  has the same intercept value and the same diagonal and off-diagonal elements in the specification of the affine raw demand function  $q_l(p)$ :

$$a_l = \frac{1}{1 + \gamma}, \quad R_{ll} = \frac{1}{1 - \gamma^2}, \quad \text{and} \quad R_{l,l'} = -\frac{\gamma}{(N - 1)(1 - \gamma^2)} \tag{17}$$

for a given parameter  $0 \leq \gamma < 1$ . The parameter  $\gamma$  may be viewed as an index of the degree of homogeneity among the products: when  $\gamma = 0$ , the products are completely heterogeneous with zero cross-price elasticities; as  $\gamma$  increases, cross-price sensitivities and elasticities increase. In the two-product case, this specification was hinted at in Staelin's commentary article accompanying McGuire and Staelin (2008).

A more straightforward specification of the raw demand functions  $q_l(p)$  would be  $q_l(p) = 1 - p_l + \sum_{l' \neq l} (\gamma / (N - 1)) p_{l'}$ . That specification has the following undesirable implications: (i) For any fixed price vector  $p$ , aggregate (raw) demand is  $\sum_l q_l(p) = N - (1 - \gamma)(\sum_l p_l)$ , which *increases* with  $\gamma$ . However, since  $\gamma$  represents the degree of homogeneity among the products, one expects aggregate demand in the market, under a given price vector  $p$ , to *decrease* with  $\gamma$ . (ii) Assuming that each of the products is sold by an independent retailer, it is easily

Downloaded from informs.org by [128.100.42.73] on 11 February 2016, at 19:38. For personal use only, all rights reserved.

verified from (5) in Federgruen and Hu (2015) that under a uniform wholesale price  $w$ , the componentwise smallest price equilibrium  $p^*$  has identical components as well. More specifically,  $(p^*)_i = 1/(2(1 - \gamma)) + w/2$ , as long as  $w \leq (1 - \gamma)^{-1}$ .<sup>3</sup> Here too, we observe that equilibrium prices increase with  $\gamma$ , where a decrease would be expected: as the products become more homogeneous and closer substitutes, competition intensifies and a decrease in the equilibrium prices can be anticipated.

The scaled specification in (17) addresses both problems: (i) under a given price vector  $p$ , aggregate raw demand is given by  $\sum_i q_i(p) = (N - \sum_i p_i)/(1 + \gamma)$ , which decreases in the degree of homogeneity  $\gamma$ ; (ii) similarly, under a uniform wholesale price  $w$  and, again, assuming each product is sold by an independent retailer, the componentwise smallest price equilibrium  $p^*$  has identical components, i.e.,  $(p^*)_i = 1 - (1 - w)/(2 - \gamma)$ , as long as  $w \leq 1$ ;<sup>4</sup> thus, under (17), equilibrium prices decrease with  $\gamma$ , as anticipated.

We vary  $I$ , the number of retailers and  $J$ , the number of suppliers, from 1 to 6. We investigate how aggregate profits in the supply chain, as well as consumer welfare, vary with the number of retailers and suppliers. To assess the latter, we assume, since  $R$  is symmetric, that the extended affine demand functions are derived from a consumer utility maximization problem, where the utility function  $U(d) = (R^{-1}a - p)^T d - \frac{1}{2}d^T R^{-1}d$ ; see Federgruen and Hu (2015, Proposition 1). Consumer welfare is, thus, assessed as the maximal level of the utility function under the prevailing equilibrium price vector  $p$ .

Each of the 120 products is associated with a unique supplier-retailer pair. We assign each product a given supplier-retailer pair as follows. When there are  $I$  retailers and  $J$  suppliers, the first  $N/I$  products are assigned to retailer 1, the next  $N/I$  products to retailer 2, etc. For the supplier assignments, we distinguish between the following two structures: (i) *Fixed network structure*: we assign the first  $N/J$

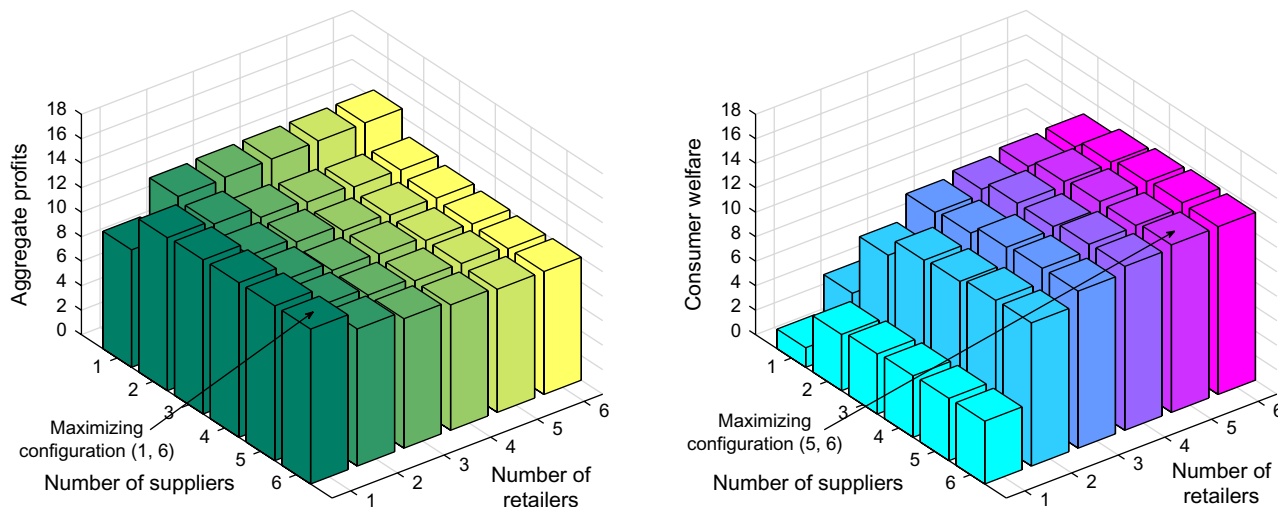
products to supplier 1, the next  $N/J$  products to supplier 2, etc. (ii) *Random network structure*: we randomly select a subset of  $N/J$  products and assign it to supplier 1; we then randomly select a subset of  $N/J$  products from the remaining products and assign it to supplier 2, etc.

Figure 3 displays for all 36 firm combinations  $(I, J)$ , aggregate profits, and consumer welfare when  $\gamma = 0.9$ , i.e., when the products are close substitutes. The aggregate profits are maximized when  $(I, J) = (1, 6)$ , i.e., a single retailer, and a maximum number of suppliers. Figure 4 displays the same measures when  $\gamma = 0.1$ , i.e., when the products are highly heterogeneous. Here, aggregate profits are optimized when  $(I, J) = (5, 6)$ , as opposed to  $(1, 6)$ . In contrast, consumer welfare is maximized under the same configuration  $(I, J) = (5, 6)$  whether  $\gamma = 0.9$  or  $\gamma = 0.1$ .

Under the random network structure and  $\gamma = 0.9$ , the configuration  $(I, J) = (1, 6)$  maximizes expected aggregate profits, whereas expected consumer welfare is maximized when  $(I, J) = (6, 6)$ , i.e., when there is maximum competition at both echelons of the supply chain. When  $\gamma = 0.1$ , the profit maximizing and the consumer welfare maximizing configuration are the same:  $(I, J) = (6, 6)$ .<sup>5</sup>

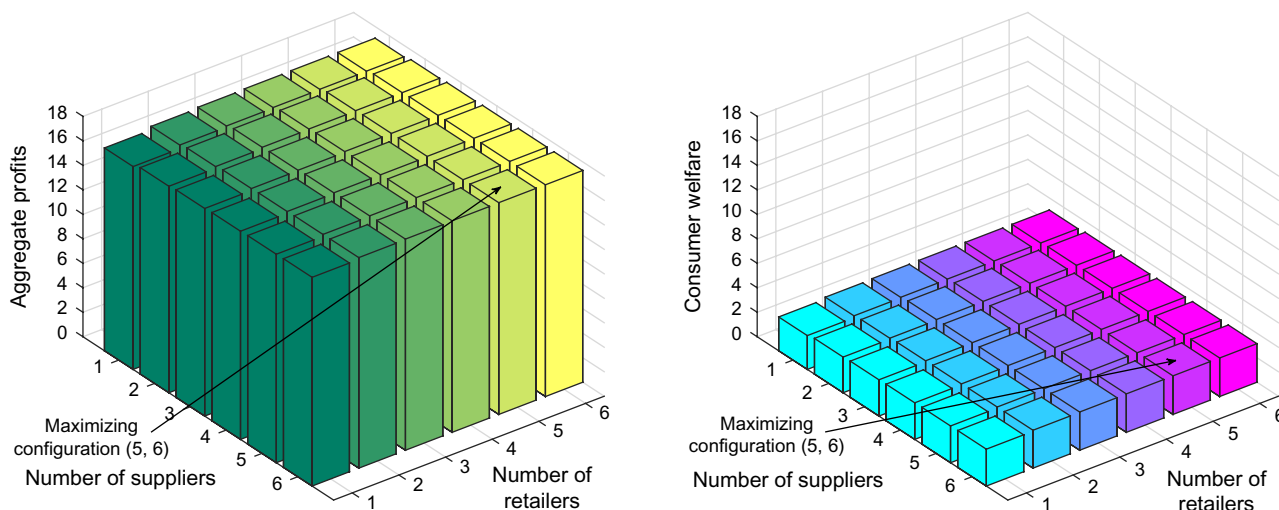
We conclude that aggregate profits in the industry, across both echelons, are maximized when the number of retailers is large, at least when the products are sufficiently heterogeneous (e.g.,  $\gamma = 0.1$ ). This is in sharp contrast to known results in single echelon price competition models. Deneckere and Davidson (1985), later generalized by Federgruen and Pierson (2013), showed that under mild conditions, mergers result in an increase of aggregate equilibrium profits, so that aggregate profits are maximized when the retailer market becomes monopolistic ( $I = 1$ ). The difference in qualitative behavior is based on the following two features: (i) we consider aggregate profits among the suppliers and retailers, as opposed to profits in the retailer echelon, exclusively; (ii) in standard single-echelon competition models, it is assumed that wholesale prices are exogenously given and

**Figure 3.** (Color online) Fixed network structure: Aggregate profits and consumer welfare.



Note.  $\gamma = 0.9$ .



**Figure 4.** (Color online) Fixed network structure: Aggregate profits and consumer welfare.

Note.  $\gamma = 0.1$ .

remain unaltered in response to a structural change in the retailer market, for example, a merger. Indeed, in the context of a single-echelon competition model, no specific adjustment of these wholesale prices can be anticipated. In a sequential oligopoly model, we automatically determine how equilibrium wholesale prices are adjusted in response to such structural changes.

We also note that the optimal industry configurations depend on the degree of heterogeneity among the products ( $\gamma = 0.1$  versus  $\gamma = 0.9$ ), as well as the specific network topology (fixed versus random network structure). This applies both to the aggregate profits and the consumer surplus measure. Finally, the optimizing configurations are quite different from that obtained in Corbett and Karmarkar (2001), dealing with *quantity* competition for a single *completely homogenous* good. In that model, aggregate profits are *always* maximized when  $(I, J) = (2, 3)$  or  $(I, J) = (3, 2)$ , considering all possible  $(I, J)$  pairs in  $\mathbb{Z}^2$ . Note that in Corbett and Karmarkar (2001), both the finished goods and intermediate goods are completely homogeneous and all firms have identical characteristics; the network topology, i.e., which suppliers sell to which retailers, is therefore immaterial.

**Excessive Competition.** In (17), the intercept value  $a_l$  is independent of  $N$ . Alternatively, assume each product's potential market size  $a_l(N)$  decreases, as the variety of differentiated products in the market increases, i.e., as  $N$  increases. Let  $\lim_{N \rightarrow \infty} a_l(N) = a_0 \geq 0$ . Consider the componentwise largest vector  $R^{-1}a$  in the effective price polyhedra (e.g., polyhedron  $C$ ). Because  $\lim_{N \rightarrow \infty} R_{l,l'}(N) = 0$  for all  $l' \neq l$ ,  $\lim_{N \rightarrow \infty} [R(N)]^{-1}a(N) = a_0(1 - \gamma^2) \cdot \mathbf{1}$ . Hence, if the suppliers' cost rates  $c_l > a_0(1 - \gamma^2)$  for all  $l = 1, \dots, N$ , there exists a sufficiently large  $N_0$  such that for  $N > N_0$ ,  $c > [R(N)]^{-1}a(N)$ . Then all products are *driven out of the market* in equilibrium. In other words, even without any fixed

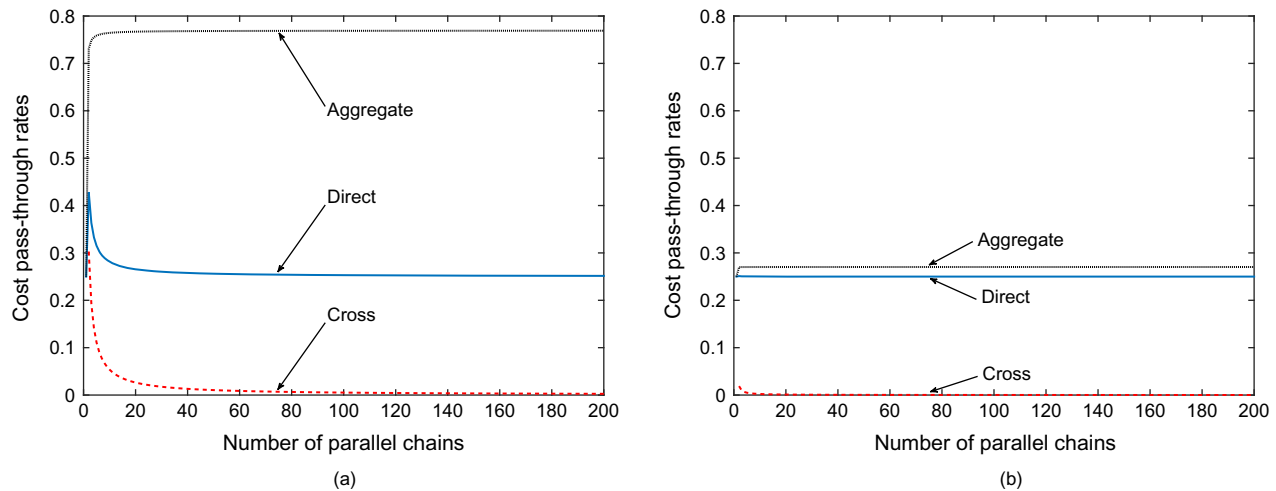
operating costs and when all firms have identical characteristics, competition in the industry may become excessive, resulting in all firms exiting. This is in stark contrast to Corbett and Karmarkar (2001) where all firms *stay in the market*, irrespective of the number of competing firms.

**Asymptotic Cost Pass-Through Rates.** Continuing with the set of raw demand functions in (17), assume there are  $N$  retailers, each selling one of the  $N$  products that is procured from a dedicated supplier; in other words, the industry consists of  $N$  parallel single-product, single supplier-retailer chains, but demand for each of the products depends on *all* prices in the market. In Figure 5, we display *three* cost pass-through rates: (i) the *direct* cost pass-through rate, (ii) the *cross* pass-through rate, and (iii) the *aggregate* pass-through rate, defined as the marginal change in the equilibrium price of a product due to a simultaneous, identical increase of all of the suppliers' cost rates. In view of the symmetry in (17), all three quantities are identical for all products or product pairs. The direct pass-through rate always increases from  $N = 1$  to  $N = 2$ , and decreases thereafter. The cross pass-through rate always decreases as  $N$  increases, starting from  $N = 2$ . The aggregate pass-through rate *increases* as  $N$  increases. Second, for both  $\gamma = 0.1$  and  $\gamma = 0.9$ , the direct pass-through rate converges to 0.25 and the cross pass-through rate to 0, when  $N$  tends to infinity. For  $\gamma = 0.1$ , the aggregate pass-through rate converges to 0.2703 and for  $\gamma = 0.9$ , to 0.7692. Third, cost pass-through rates are larger when  $\gamma = 0.9$  than when  $\gamma = 0.1$ . Intuitively, when products are more substitutable (i.e.,  $\gamma$  is larger), the firms need to pass on more of their savings when experiencing a marginal decrease in the suppliers' cost rates.

## 6. Asymmetric Price-Sensitivity Matrices

Price-sensitivity coefficients often fail to be symmetric. In this section, we show how all of our results can be

**Figure 5.** (Color online) Cost pass-through rates: (a)  $\gamma = 0.9$  (b)  $\gamma = 0.1$ .



extended to *asymmetric*  $R$  matrices, under far less restrictive conditions. Starting with the retailer competition model, all of the characterizations in Proposition 1 continue to apply, under an asymmetric  $R$ -matrix, as long as the following property holds.

**ASSUMPTION (A).**  $b = \Psi(R)a \geq 0$  and  $S$  is a  $Z$ -matrix.

(Indeed, the proof of Proposition 1, in Online Appendix B, is obtained under this Assumption (A), as opposed to the far stronger Assumption (S) of a symmetric  $R$  matrix.)

Assumption (A) is easily verified from the model's primitives (the vector  $a$  and the matrix  $R$ ), with a single matrix inversion and a few matrix multiplications; see (4)–(5). The following lemma provides a strong but broad sufficient condition; see Federgruen and Hu (2015, Proposition 3).

**LEMMA 1.** Assumption (A) applies, if the matrix  $T(R)$  is symmetric.

Symmetry of the matrix  $T(R)$  means that the cross-price sensitivity coefficients are identical for any pair of products sold by the *same* retailer. This symmetry assumption is considerably *weaker* than the global symmetry Assumption (S) for the full matrix  $R$ . (As mentioned, when demand functions  $d(p)$  are derived from a representative consumer maximizing a quadratic utility function, the resulting matrix  $R$  of price-sensitivity coefficients is *always symmetric*, implying that Assumption (A) is automatically satisfied.)

Even, the weak Assumption (A) is only required when  $w \notin W$ . Moreover, even when  $w \notin W$ , many of the results in Proposition 1 can be guaranteed, simply on the basis of properties (P) and (Z) alone; in particular, there exists at most one equilibrium  $p^*$  in  $P$  and if  $\tilde{p} \notin P$  is an equilibrium, then its projection  $\Omega(\tilde{p})$  is an equilibrium as well. Thus, if an equilibrium exists, there is a componentwise smallest equilibrium. Assumption (A) is required to ensure that the projection  $\Theta(\cdot)$  is well defined, in particular that it maps any vector  $w \in \mathbb{R}_+^N$  into a *nonnegative* vector.

**REMARK 2.** Assumption (A) may be replaced by an even weaker condition, referred to as Assumption (NPW) in Federgruen and Hu (2015, see Proposition 3 and Theorem 3).

As far as the two-stage competition model is concerned, the results in Theorems 1 and 2 parts (a) and (b) all continue to apply under Assumption (A). Theorem 2(c), i.e., the characterization of the suppliers' equilibrium choices, when  $c \notin C$ , requires a similar condition to Assumption (A), now to ensure that the projection  $\Gamma(\cdot)$  onto  $C$  is well defined, i.e., any suppliers' cost rate vector  $c \in \mathbb{R}_+^N$  is projected onto a *nonnegative* vector  $\Gamma(c) \geq 0$ :

**ASSUMPTION (A').**  $\Psi(S)b \geq 0$  and  $\Psi(S)S$  is a  $Z$ -matrix.

Condition (A') is, again, easily verified, numerically. As with Assumption (A), it may be replaced by an even weaker although more complex condition; see Remark 2.

Similar to condition (A), a sufficient condition for (A') is that  $T(S)$  be symmetric.

**THEOREM 5.** Assume  $T(R)$  and  $T(S)$  are symmetric. All of the results in Theorems 1 (except for the symmetry of matrix  $S$ ) and 2, and Proposition 2, continue to apply.

Symmetry of  $T(R)$  and  $T(S)$  also suffices to maintain all comparative statics results in §5.

**THEOREM 6.** Assume  $T(R)$  and  $T(S)$  are symmetric. All of the results in Theorems 3 and 4, Corollary 1, and Proposition 4 continue to apply.

## 7. Conclusion

We have analyzed a general sequential oligopoly model, in which, at each echelon of the supply process, an arbitrary number of firms compete by offering a single or multiple products to some or all of the firms in the next echelon. The model assumes sequential noncooperative pricing in the sense that at the first stage of the multistage competition model, the firms of the most upstream echelon select their prices for all products. At the second-stage competition

model, the firms of the next more downstream echelon select their price menu. This process continues until at the last stage, the retailers select all of their retail prices.

We provide a full characterization and simple computational scheme for the equilibria in this model: Consider, for example, a model with two echelons. We show that in this two-stage competition model, a subgame perfect Nash equilibrium always exists. Multiple subgame perfect equilibria may arise but, if so, all equilibria are *equivalent* in the sense of generating *unique* demands and profits for all firms. Indeed, even for a given vector of wholesale prices, the second-stage retailer competition game always has an equilibrium but may possess multiple, possibly infinitely many, equilibria. Nevertheless, these various equilibria are equivalent in the above sense.

We have shown general comparative statics results with respect to the exogenous cost parameters in the model, as well as the intercept vector in the (affine part of) demand functions: these comparative statics results have important implications for the assessment of cost pass-through rates and the measurement of brand values. We have also illustrated what qualitative impacts various changes in the *structure* of the supply chain network bring forth. Additional managerial questions that may be assessed by our model (or extensions thereof), include the following:

- *Entry or exit of firms.* The model provides a convenient tool to assess the market-wide consequences with respect to equilibrium price and assortment choices by all firms, due to the exit or entry of one of them. A firm's *exit* may be modeled by assuming that its products are priced "out of the market," i.e., their price levels are sufficiently high to drive the implied sales volumes down to zero. This modeling paradigm was first employed by Telser (1965) in the context of a single-echelon model. Based on this paradigm, it is possible to derive the set of consumer demand functions resulting from the exit of any given firm and its associated products, or the abandonment of any set of product  $\mathcal{S} \subset \mathcal{N}$ , for that matter: by setting  $q_{\mathcal{S}}(p) = 0$ , it is possible to express the "minimal" exit prices  $\bar{p}_{\mathcal{S}}$  as an affine function of the remaining prices  $p_{\bar{\mathcal{S}}}$ . Substituting  $p_{\mathcal{S}}$  by this affine vector function, we obtain a new price sensitivity matrix on the reduced product space, which is again positive definite and a  $Z$ -matrix, assuming the original matrix  $R$  is. This guarantees that the model resulting from any firm's exit or the elimination of any product set, has the same type of equilibrium behavior as the original model; our efficient procedure to compute the chainwide equilibria, thus allows for a simple comparison of the pre- and post-exit equilibria.

Modeling of the *entry* of a new firm or the adoption of a new set of products is somewhat more involved, since it requires a respecification/reestimation of the expanded intercept vector  $a$  and price sensitivity matrix  $R$ . Thereafter, the model can again be used to compute and compare the pre- and post-entry equilibria.

- *Impact of new direct sales channels.* Upstream suppliers and distributors may initiate direct sales channels, perhaps by

initiating an online sales channel. A firm may choose to offer the same set of products sold via brick-and-mortar retailers via the new direct sales channels, or it may offer distinct and differentiated products. Finally, firms may contemplate shifting complete sets of products from traditional brick-and-mortar channels via independent retailers to direct sales channels. To allow for such channels, we need to extend our results to settings where some of the products skip intermediate echelons; this extension is, indeed, possible. All of the above structural changes may be assessed effectively by comparing the equilibria in two related network structures.

- *Vertical integration.* The economics literature has been interested in identifying the anticompetitive effects resulting from vertical integration, i.e., the merger of an upstream and a downstream firm in a two-echelon model; see, e.g., Riordan (1998, 2008). The existing literature has focused on settings with two firms at both echelons. Our model may be used to test whether the observed effects hold up in more general network structures, with an arbitrary number of firm/product combinations at each echelon. It may also be used to identify new anticompetitive effects.

- *Mergers and spinoffs.* The above observations pertain equally to horizontal mergers or spinoffs of firms belonging to the same echelon.

## Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/opre.2015.1443>.

## Acknowledgments

The authors thank the department editor Dan Adelman, the associate editor, and two anonymous reviewers for their guidance and constructive comments. The authors are also grateful to Guillermo Gallego, Georgia Perakis, Amr Farahat, Victor DeMiguel, Elodie Adida, Omar Besbes, Sridhar Moorthy, and Michael H. Riordan for their helpful comments. The second author is grateful for financial support by the National Natural Science Foundation of China [Grant 71171059] and Natural Sciences and Engineering Research Council of Canada [Grant RGPIN-2015-06757].

## Endnotes

1. The recent survey by Ailawadi et al. (2010, p. 282) opens its section on "product assortment," as follows: "In contrast with the vast amount of research on consumer response to product assortment [ . . . ], there is scant research on how manufacturers and retailers interact to determine the composition of the assortment."
2. Ailawadi et al. (2010, p. 276) describe the MS model as the widely applied "workhorse for modeling manufacturer-retailer interactions."
3. When  $w > (1 - \gamma)^{-1}$ , all products are priced out of the market, in equilibrium.
4. When  $w > 1$ , all products are priced out of the market, in equilibrium.
5. Thus, under a *random* network structure, expected consumer welfare is maximized when the number of firms is maximized. Under a *fixed* network structure,  $(I, J) = (6, 6)$  results in six parallel chains each, exclusively, dealing with 20 products, as opposed to  $(I, J) = (5, 6)$ , where each retailer procures from two suppliers.



## References

- Adida E, DeMiguel V (2011) Supply chain competition with multiple manufacturers and retailers. *Oper. Res.* 59(1):156–172.
- Ailawadi KL, Bradlow ET, Draganska M, Nijs V, Rooderkerk RP, Sudhir K, Wilbur KC, Zhang J (2010) Empirical models of manufacturer-retailer interaction: A review and agenda for future research. *Marketing Lett.* 21(3):273–285.
- Aksoy-Pierson M, Allon G, Federgruen A (2013) Price competition under mixed multinomial logit demand functions. *Management Sci.* 59(8):1817–1835.
- Besanko D, Dubé J-P, Gupta S (2005) Own-brand and cross-brand retail pass-through. *Marketing Sci.* 24(1):123–137.
- Bonnet C, Dubois P, Villas-Boas SB, Klapper D (2013) Empirical evidence on the role of nonlinear wholesale pricing and vertical restraints on cost pass-through. *Rev. Econom. Stat.* 95(2):500–515.
- Che H, Sudhir K, Seetharaman PB (2007) Bounded rationality in pricing under state-dependent demand: Do firms look ahead, and if so, how far? *J. Marketing Res.* 44(3):434–449.
- Chen Y (2001) On vertical mergers and their competitive effects. *RAND J. Econom.* 32(4):667–685.
- Chen Y, Riordan MH (2007) Vertical integration, exclusive dealing, and ex post cartelization. *RAND J. Econom.* 38(1):1–21.
- Chevalier M, Curhan RC (1976) Retail promotions as a function of trade promotions: A descriptive analysis. *Sloan Management Rev.* 18(3):19–32.
- Cho S-H (2014) Horizontal mergers in multitier decentralized supply chains. *Management Sci.* 60(2):356–379.
- Choi SC (1996) Price competition in a duopoly common retailer channel. *J. Retailing* 72(2):117–134.
- Chu J, Chintagunta PK (2009) Quantifying the economic value of warranties in the U.S. server market. *Marketing Sci.* 28(1):99–121.
- Corbett C, Karmarkar U (2001) Competition and structure in serial supply chains with deterministic demand. *Management Sci.* 47(7):966–978.
- Cottle RW, Pang JS, Stone RE (1992) *The Linear Complementarity Problem* (Academic Press, Boston).
- DeMiguel V, Xu H (2009) A stochastic multiple-leader Stackelberg model: Analysis, computation, and application. *Oper. Res.* 57(5):1220–1235.
- Deneckere R, Davidson C (1985) Incentives to form coalitions with Bertrand competition. *RAND J. Econom.* 16(4):473–486.
- Dubé J-P, Gupta S (2008) Cross-brand pass-through in supermarket pricing. *Marketing Sci.* 27(3):324–333.
- Dubé J-P, Manchanda P (2005) Differences in dynamic brand competition across markets: An empirical analysis. *Marketing Sci.* 24(1):81–95.
- Federgruen A, Hu M (2014) Price competition in sequential multi-product oligopolies. Unabridged memo, <http://dx.doi.org/10.2139/ssrn.2437032>.
- Federgruen A, Hu M (2015) Multi-product price and assortment competition. *Oper. Res.* 63(3):572–584.
- Federgruen A, Pierson M (2013) The impact of horizontal mergers and acquisitions in price competition models. Working paper, Columbia University, New York.
- Goldberg PK (1995) Product differentiation and oligopoly in international markets: The case of the U.S. automobile industry. *Econometrica* 63(4):891–951.
- Goldfarb A, Lu Q, Moorthy S (2009) Measuring brand value in an equilibrium framework. *Marketing Sci.* 28(1):69–86.
- Hart O, Tirole J (1990) Vertical integration and market foreclosure. *Brookings Papers on Economic Activity. Microeconomics* (Brookings Institution Press, Washington, DC).
- Li J, Rusmevichientong P, Simester D, Tsitsiklis J, Zoumpoulis S (2015) The practical value of field experiments. *Management Sci.* 61(7):1722–1740.
- Manchanda P, Ansari A, Gupta S (1999) The “shopping basket”: A model for multicategory purchase incidence decisions. *Marketing Sci.* 18(2):95–114.
- McAlister L (2007) Comment: Cross-brand pass-through: Fact or artifact? *Marketing Sci.* 26(6):876–898.
- McGuire TW, Staelin R (1983) An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* 2(2):161–191.
- McGuire TW, Staelin R (2008) An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* 27(1):115–130.
- Moorthy S (2005) A general theory of pass-through in channels with category management and retail competition. *Marketing Sci.* 24(1):110–122.
- Ordover JA, Saloner G, Salop SC (1990) Equilibrium vertical foreclosure. *Amer. Econom. Rev.* 80(1):127–142.
- Riordan MH (1998) Anticompetitive vertical integration by a dominant firm. *Am. Econom. Rev.* 88(5):1232–1248.
- Riordan MH (2008) Competitive effects of vertical integration. Buccirosi P, ed. *Handbook of Antitrust Economics* (MIT Press, Cambridge, MA).
- Saggi K, Vettas N (2002) On intrabrand and interbrand competition: The strategic role of fees and royalties. *Euro. Econom. Rev.* 46(1):189–200.
- Salinger MA (1988) Vertical mergers and market foreclosure. *Quart. J. Econom.* 103(2):345–356.
- Shubik M, Levitan R (1980) *Market Structure and Behavior* (Harvard University Press, Cambridge, MA).
- Soon W, Zhao G, Zhang J (2009) Complementarity demand functions and pricing models for multi-product markets. *Eur. J. Appl. Math.* 20(5):399–430.
- Sudhir K (2001) Structural analysis of manufacturer pricing in the presence of a strategic retailer. *Marketing Sci.* 20(3):244–264.
- Telser LG (1965) A mathematical note on entry, exit, and oligopoly. *Econometrica* 33(2):425–433.
- Vilcassim NJ, Kadiyali V, Chintagunta PK (1999) Investigating dynamic multifirm market interactions in price and advertising. *Management Sci.* 45(4):499–518.
- Villas-Boas JM, Zhao Y (2005) Retailer, manufacturers, and individual consumers: Modeling the supply side in the ketchup marketplace. *J. Marketing Res.* 42(1):83–95.
- Villas-Boas S (2007a) Using retail data for upstream merger analysis. *J. Competition Law Econom.* 3(4):689–715.
- Villas-Boas S (2007b) Vertical relationships between manufacturers and retailers: Inference with limited data. *Rev. Econom. Stud.* 74(2):625–652.
- Villas-Boas S, Hellerstein R (2006) Identification of supply models of retailer and manufacturer oligopoly pricing. *Econom. Lett.* 90(1):132–140.

**Awi Federgruen** is the Charles E. Exley professor of management at the Graduate School of Business at Columbia University.

**Ming Hu** is an associate professor of operations management at Rotman School of Management, University of Toronto. His research interests include revenue management, supply chain management, and service operations management. Currently, he studies operations management in the context of social buying, crowdfunding, crowdsourcing, and two-sided markets.